# CONDUCTIVITY OF AN ION-ACOUSTICALLY TURBULENT PLASMA

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**Abstract.** Three different mechanisms are suggested for the non-linear saturation of the ion-acoustic instability. These include the indirect wave-particle interaction, the scattering on density fluctuations and the effects of energy renormalization of the particles. A comparison with the earlier theoretical investigations and the recent observations of Skylab has been made.

## 1. Introduction

Although the problem of anomalous conductivity of a plasma in the presence of ion-acoustic turbulence has received the attention of many workers with proposals of varied nature for the saturation of ion-acoustic instability, there is no complete and comprehensive account of various processes contributing to the anomalous resistivity. The different mechanisms that have been suggested, remain as scattered, disconnected pieces of work. In this paper, contribution of three new mechanisms to the anomalous resistivity has been studied. In addition a comparison with the existing theories, from the point of view of the relative effectiveness of the various mechanisms has been attempted. Finally the validity and applicability of these results to explain the release of large amounts of energy in a solar flare has been investigated. Section 2 deals with the mechanism of indirect wave mediated wave-particle interaction, responsible for the saturation of the ion-acoustic instability. This mechanism contains the effect of the various kinds of ion-acoustic waves present in the turbulent plasma state and brings saturation by decreasing the effective scattering of the ion-sound wave by electrons and increasing the effective scattering of the ion-sound wave by ions. Section 3 deals with the damping of ion sound waves caused by their scattering on the static density fluctuations present in the system.

This process has accounted successfully for the anomalous daming of Langmuir waves, Krishan and Ritchie (1970). In Section 4, the modification in the energy of the non resonant electrons and ions, brought about by the ion-acoustically turbulent field, has been determined. These modified particles then interact resonantly with the wave to bring about saturation. Section 5 provides a critical comparison between the results derived here and the earlier ones. Finally in section 6 the value of the anomalous conductivity obtained from section 4 is used to estimate the magnitude of magnetic field gradients present in the region of the magnetically neutral sheet in the solar corona.

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## 2. Wave Mediated Wave-Particle Interaction

Figure 1 shows the direct wave-particle interaction where a particle with momentum $\hbar(\mathbf{k}'+\mathbf{q})$ in a state $|\hbar(\mathbf{k}'+\mathbf{q})\rangle$ is scattered to a state $|\hbar\mathbf{k}'\rangle$ with the emission of a

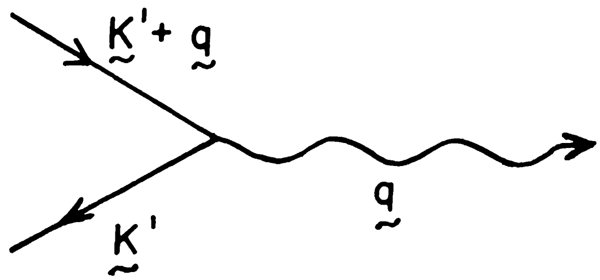


Fig. 1. Direct wave-particle scattering. The solid lines stand for the particles and the wavy lines for the ion-sound waves.

wave with frequency $\omega_{\mathbf{q}}$ and wave vector $\mathbf{q}$. The matrix elements for this process are given by

$$W_{oe} = -\left[\frac{4\pi e^2 \hbar\omega_{\mathbf{q}}}{Vq^2 F_{\mathbf{q}}}\right]^{1/2} \quad \text{for electrons}$$

and

$$W_{oi} = \left[\frac{4\pi e^2 \hbar\omega_{\mathbf{q}}}{Vq^2 F_{\mathbf{q}}}\right]^{1/2} \quad \text{for ions.}$$

Here $V$ is the normalizing volume, $F_{\mathbf{q}} = (\partial/\partial\omega)(\omega\varepsilon)|_{\omega=\omega_{\mathbf{q}}}$ and $\varepsilon$ is the dielectric constant. Figure 2 shows the indirect wave-particle interaction where the particle in the state $|\hbar(\mathbf{k}'+\mathbf{q})\rangle$ gets scattered to an intermediate state $|\hbar(\mathbf{k}'+\mathbf{q}-\mathbf{q}')\rangle$ by the emission of the wave of frequency $\omega_{\mathbf{q}'}$. This particle at $|\hbar(\mathbf{k}'+\mathbf{q}-\mathbf{q}')\rangle$ suffers a second scattering to the state $|\hbar\mathbf{k}'\rangle$ with another wave $\omega_{\mathbf{q}'-\mathbf{q}}$ and these two waves $\omega_{\mathbf{q}'}$

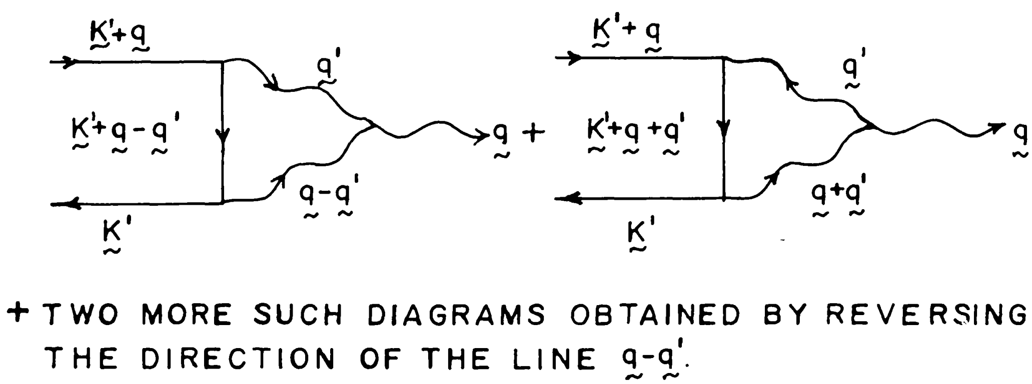


Fig. 2a. Indirect wave-particle scattering. The electron or the ion is scattered with the emission of two waves, which then interact to yield the third wave, which is to be stabilized.

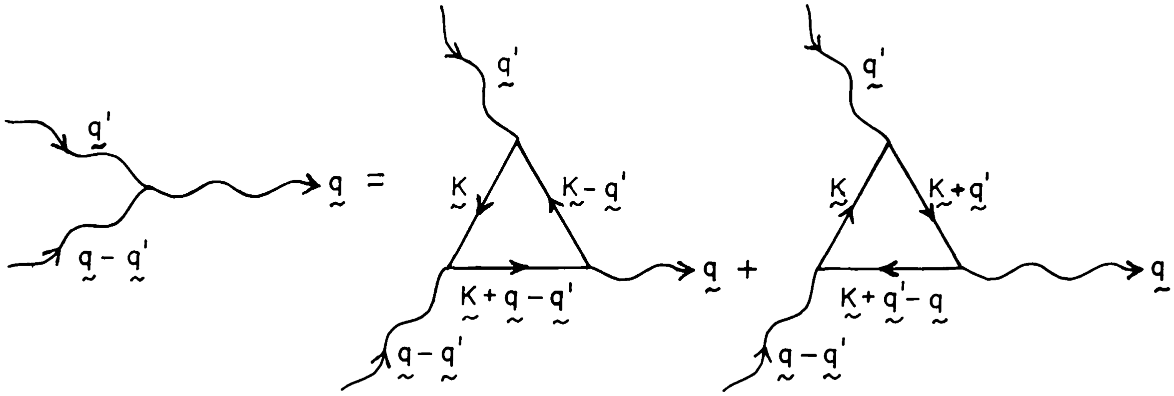


Fig. 2b. The details of the three wave vertex appearing in Figure 2a.

and $\omega_{\mathbf{q}-\mathbf{q}'}$ scatter to the third wave $\omega_{\mathbf{q}}$. So the scattering of the particle $|\hbar(\mathbf{k}'+\mathbf{q})\rangle$ by the wave $\omega_{\mathbf{q}}$ is indirect in the sense that the particle is first scattered by the other waves which then interact with the wave $\omega_{\mathbf{q}}$ and therefore the interaction of particles with the wave is via other waves present due to turbulence. The matrix elements for the indirect wave-particle scattering, though a bit cumbersome, can be written down using the well known techniques given by Dubois (1967), Harris (1969), and Krishan and Selim (1972). Let $W^e$ represent the matrix elements for the indirect electron-ion sound wave interaction, then

$$W^e = \sum_j \sum_{\mathbf{q}\neq\mathbf{q}'} \left[\frac{4\pi e^2 \hbar\omega_{\mathbf{q}}}{Vq^2 F_{\mathbf{q}}}\right]^{1/2} \left[\frac{4\pi e^2 \hbar\omega_{\mathbf{q}'}}{Vq'^2 F_{\mathbf{q}'}}\right]^{1/2} \left[\frac{4\pi e^2 \hbar\omega_{\mathbf{q}-\mathbf{q}'}}{V(\mathbf{q}-\mathbf{q}')^2 F_{\mathbf{q}-\mathbf{q}'}}\right]^{1/2} \times$$
$$\times \frac{N_{\mathbf{q}'}}{\hbar} \frac{1}{m} \int d^3\mathbf{V}_j f_j(\mathbf{V}_j)[W_1^e + W_2^e], \tag{1}$$

where

$$W_1^e = \frac{\omega^2_{\mathbf{q}-\mathbf{q}'}}{m_j^2} (\mathbf{q}'^2 - 2\mathbf{q}\cdot\mathbf{q}')\,[(\omega_{\mathbf{q}} - \omega_{\mathbf{q}'})^2 - \omega^2_{\mathbf{q}-\mathbf{q}'}]^{-1} \times$$
$$\times [\omega_{\mathbf{q}'} - \mathbf{q}'\cdot V_e]^{-2}\,[(\omega_{\mathbf{q}} - \mathbf{q}\cdot\mathbf{V}_j)(\omega_{\mathbf{q}'} - \mathbf{q}'\cdot\mathbf{V}_j)(\omega_{\mathbf{q}} - \omega_{\mathbf{q}'} -$$
$$- (\mathbf{q}-\mathbf{q}')\cdot\mathbf{V}_j)]^{-1} \times$$
$$\times \left[\frac{\mathbf{q}\cdot\mathbf{q}'(\mathbf{q}-\mathbf{q}')^2}{\omega_{\mathbf{q}} - \omega_{\mathbf{q}'} - (\mathbf{q}-\mathbf{q}')\cdot\mathbf{V}_j} + \frac{\mathbf{q}'\cdot(\mathbf{q}-\mathbf{q}')\,q^2}{\omega_{\mathbf{q}} - \mathbf{q}\cdot\mathbf{V}_j} + \frac{\mathbf{q}\cdot(\mathbf{q}-\mathbf{q}')q'^2}{\omega_{\mathbf{q}'} - \mathbf{q}'\cdot\mathbf{V}_j}\right], \tag{2}$$

$$W_2^e = \frac{\omega^2_{\mathbf{q}+\mathbf{q}'}}{m_j^2} (\mathbf{q}'^2 + 2\mathbf{q}\cdot\mathbf{q}')[(\omega_{\mathbf{q}} + \omega_{\mathbf{q}'})^2 - \omega^2_{\mathbf{q}+\mathbf{q}'}]^{-1} \times$$
$$\times [\omega_{\mathbf{q}'} - \mathbf{q}'\cdot\mathbf{V}_e]^{-2}[(\omega_{\mathbf{q}} - \mathbf{q}\cdot\mathbf{V}_j)(\omega_{\mathbf{q}'} - \mathbf{q}'\cdot\mathbf{V}_j)(\omega_{\mathbf{q}} + \omega_{\mathbf{q}'} -$$
$$- (\mathbf{q}+\mathbf{q}')\cdot\mathbf{V}_j)]^{-1} \times$$
$$\times \left[\frac{\mathbf{q}\cdot\mathbf{q}'(\mathbf{q}+\mathbf{q}')^2}{\omega_{\mathbf{q}} + \omega_{\mathbf{q}'} - (\mathbf{q}+\mathbf{q}')\cdot\mathbf{V}_j} + \frac{\mathbf{q}'\cdot(\mathbf{q}+\mathbf{q}')q^2}{\omega_{\mathbf{q}} - \mathbf{q}\cdot\mathbf{V}_j} + \frac{\mathbf{q}\cdot(\mathbf{q}+\mathbf{q}')q'^2}{\omega_{\mathbf{q}'} - \mathbf{q}'\cdot\mathbf{V}_j}\right]; \tag{3}$$

$j$ stands for the species of the particles,

$m_j$ $= m$ for electron mass,

$= M$ for ion mass,

$\omega_{pj}$ $= [4\pi n_0 e^2/m_j]^{1/2}$ is the plasma frequency,

$n_0$ is the electron density,

$f_j(\mathbf{V}_j)$ is the velocity distribution function for $j$ species,

$C_s$ is the ion-sound speed,

$\omega$'s are the frequencies for various wave vectors $q$'s,

$N_{\mathbf{q}'}$ is the number of phonons in the ion sound wave of wave vector $\mathbf{q}'$,

$\mathbf{V}_e$ is the velocity of the electron with momentum $\hbar\mathbf{k}'$.

Here the expression for the matrix elements has been written intentionally in an explicit form to facilitate the recognition of the various factors appearing. As for example, one easily notices the presence of five wave-particle interaction vertices and $W_1^e$ and $W_2^e$, arising from the particle and ion-sound propagators. More precisely $W^e$ can be written as

$$W^e = \sum_j \sum_{\mathbf{q}\neq\mathbf{q}'} |W_{\text{oe}}| A_1 I_{\mathbf{q}'}(t) \frac{1}{mN_0} \int \mathrm{d}^3\mathbf{V}_j f_j(\mathbf{V}_j)[W_1^e + W_2^e], \tag{4}$$

where

$$A_1 = \frac{(\chi T_e)^2}{4N_0},$$

$I_{\mathbf{q}'}(t) = \hbar N_{\mathbf{q}'}\omega_{\mathbf{q}'}$ is the energy in the mode $\mathbf{q}'$, $T_e$ is the electron temperature, and $N_0$ is the number of particles.

Similarly the matrix elements for indirect ion-ion sound wave scattering can be written as

$$W^i = \sum_j \sum_{\mathbf{q}\neq\mathbf{q}'} |W_{\text{oe}}| A_1 I_{\mathbf{q}'}(t) \frac{1}{NN_0} \int \mathrm{d}^3\mathbf{V}_j f_j(\mathbf{V}_j)[W_1^i + W_2^i], \tag{5}$$

where

$$\begin{aligned} W_1^i &= [(\omega_{\mathbf{q}'} - \mathbf{q}'\cdot\mathbf{V}_e)/(\omega_{\mathbf{q}'} - \mathbf{q}'\cdot\mathbf{V}_i)]^2\, W_1^e, \\ W_2^i &= [(\omega_{\mathbf{q}'} - \mathbf{q}'\cdot\mathbf{V}_e)/(\omega_{\mathbf{q}'} - \mathbf{q}'\cdot\mathbf{V}_i)]^2\, W_2^e, \end{aligned} \tag{6}$$

and $\mathbf{V}_i$ is the velocity of ion in whose scattering one is interested. To perform the integration over particle velocity, the following coordinate system has been chosen. Let $Z$ axis be along $\mathbf{q}$ and $(\theta, \psi)$ are the polar and azimuthal angles of $\mathbf{q}'$. Then

$$\mathbf{q}'\cdot\mathbf{V}_j = q' V_{j\perp} \sin\theta \cos(\psi - \eta) + q' V_{j\parallel} \cos\theta,$$

where $V_{j\perp}$ and $V_{j\parallel}$ are the components of velocity $\mathbf{V}_j$ perpendicular and parallel to $\mathbf{q}$ respectively and $\eta$ is the azimuthal angle of $\mathbf{V}_j$. Now one notices that due to the presence of such factors as

$$[(\omega_{\mathbf{q}}+\omega_{\mathbf{q}'})^2-\omega^2_{\mathbf{q}+\mathbf{q}'}]^{-1}=[2qq'c_s^2(1-\cos\theta)]^{-1}$$

in the matrix elements, the dominant contribution comes from $\theta \ll 1$. Since the electrons are the adiabatic species i.e. electron thermal speed is much larger than the ion sound speed, the velocity integral for electrons will be obtained in the asymptotic limit. One needs to satisfy the inequality

$$\omega_{\mathbf{q}'}-\mathbf{q}'V_{\text{oe}}\sin\theta\cos(\psi-\eta)\ll\mathbf{q}'V_{\text{oe}}\cos\theta$$

which is ensured if $\theta$ is chosen such that

$$\sin\theta\simeq\frac{C_s}{V_{\text{oe}}}=\sqrt{\frac{m}{M}}.$$

Here $V_{\text{oe}}=\sqrt{\chi T_e/m}$ is the electron thermal speed. If $\mathbf{u}$ is the drift velocity of the electrons, then the velocity distribution function for the electrons and the ions is given as

$$f_e(\mathbf{V})=N_0\left[\frac{m}{2\pi\chi T_e}\right]^{3/2}\exp\left[-\frac{m}{2\chi T_e}(\mathbf{V}-\mathbf{u})^2\right],$$

$$f_i(\mathbf{V})=N_0\left[\frac{M}{2\pi\chi T_i}\right]^{3/2}\exp\left[-\frac{M}{2\chi T_i}V^2\right].$$

With all these simplifications, the velocity integration can be carried out easily. Summation over $\mathbf{q}'$ has been performed assuming that the energy spectrum $I_{\mathbf{q}'}(t)$ is not very sensitive to its wave vector dependence. After going through the above procedure, one gets the effective matrix elements for the direct and the indirect electron-ion sound scattering and ion-ion sound scattering respectively as

$$W^e_{\text{eff}}=-|\mathbf{W}_{\text{oe}}|\left[1-\frac{2\pi I(t)}{3N_{\text{D}}m}\int\frac{\mathrm{d}\psi}{[(m/2M)C_s-\sqrt{(m/M)}V_{e\perp}\cos(\psi-\varphi_e)]^2}\right], \tag{7}$$

$$W^i_{\text{eff}}=+|W_{\text{oe}}|\left[1+\frac{2\pi I(t)}{3N_{\text{D}}M}\int\frac{\mathrm{d}\psi}{[(m/2M)C_s-\sqrt{(m/M)}V_{i\perp}\cos(\psi-\varphi_i)]^2}\right], \tag{8}$$

where $N_{\text{D}}$ is the number of electrons in the Debye sphere, $V_{e\perp}$, $V_{i\perp}$ are the components of electron and ion velocity perpendicular to $\mathbf{q}$, $\varphi_e$ and $\varphi_i$ are the azimuthal angles of $\mathbf{V}_e$ and $\mathbf{V}_i$ respectively.

Once the effective matrix elements for the wave-particle interaction are known, it is straight forward to write the energy balance equation as

$$\begin{aligned}\left.\frac{\mathrm{d}N_q}{\mathrm{d}t}\right|_j &= \sum_{\mathbf{k}'} \frac{2\pi}{\hbar} |W_{\text{eff}}^j|^2 [(N_{\mathbf{q}}+1) f_j(\mathbf{k}'+\mathbf{q})(1-f_j(\mathbf{k}')) - \\ &\quad - N_{\mathbf{q}} f_j(\mathbf{k}')(1-f_j(\mathbf{k}'+\mathbf{q}))]\delta(\omega_{\mathbf{q}} - \mathbf{q}\cdot\mathbf{V}_j) \\ &= \frac{2\pi V}{\hbar} \int |W_{\text{eff}}^j|^2 \frac{N_q}{m_j} \mathbf{q} \cdot \frac{\partial f_j(\mathbf{V}_j)}{\partial \mathbf{V}_j} \delta(\omega_{\mathbf{q}} - \mathbf{q}\cdot\mathbf{V}_j) \frac{\mathrm{d}^3 V_j}{(2\pi)^3}.\end{aligned} \tag{9}$$

The growth rate of the ion sound wave on electrons is given by

$$\begin{aligned}\gamma_e &= \frac{1}{2N_q} \left.\frac{\mathrm{d}N_q}{\mathrm{d}t}\right|_e \\ &= \sqrt{\frac{\pi}{8}} \frac{C_s}{V_{\text{oe}}} (\mathbf{q}\cdot\mathbf{u} - \omega_{\mathbf{q}}) \exp\left[\frac{-m}{2\chi T_e}(C_s - u)^2\right]\left[1 - \frac{16\pi I(t)\Lambda}{3N_{\mathrm{D}}\chi T_e}\right]\end{aligned}$$

with

$$\Lambda = \int\limits_{1/\sqrt{2}}^{\infty} \frac{e^{-x}}{x} \mathrm{d}x \simeq 0.37\,. \tag{10}$$

The damping rate of the ion sound wave on the ions is given by

$$\gamma_i = \sqrt{\frac{\pi}{8}} \omega_{\mathbf{q}} \left(\frac{T_e}{T_i}\right)^{3/2} \exp\left[-\frac{T_e}{2T_i}\right]\left[1 + \frac{8\pi^2 I(t)}{3N_{\mathrm{D}}\chi T_e \left(\frac{m}{M}\right)^2}\right].$$

The saturation energy of the ion-sound wave is given by the condition

$$\gamma_e + \gamma_i = 0$$

or

$$\frac{4\pi I(t)}{3N_{\mathrm{D}}\chi T_e} = \frac{\frac{\mathbf{q}\cdot\mathbf{u}}{\omega_q} - 1 - \sqrt{\frac{M}{m}}\left(\frac{T_e}{T_i}\right)^{3/2} \exp\left[-T_e/2T_i\right]}{2\pi\left(\frac{M}{m}\right)^{5/2}\left(\frac{T_e}{T_i}\right)^{3/2} \exp\left[\frac{-T_e}{2T_i}\right] + 4\Lambda\left(\frac{\mathbf{q}\cdot\mathbf{u}}{\omega\mathbf{q}} - 1\right)}. \tag{11}$$

Now using the following expression for anomalous resistivity given by Rudakov and Korablev (1966) as

$$\nu_s \simeq \frac{\pi^{3/2}}{\sqrt{2}} \frac{C_s}{V_{\text{oe}}} \int \frac{\mathrm{d}^3\mathbf{q}}{(2\pi)^3} \frac{qI(t)}{mn_0 u}\left(\frac{\mathbf{q}\cdot\mathbf{u}}{\omega_{\mathbf{q}}} - 1\right), \tag{12}$$

and substituting for $I(t)$ from Equation (11), one finds

$$\frac{\nu_s}{\omega_{\rm pe}} = \frac{\pi^{3/2}}{2\sqrt{2}}\frac{u}{C_s}\frac{\left(1-\frac{C_s}{u}\right)^2 - \frac{C_s}{u}\left(1-\frac{C_s}{u}\right)\sqrt{\frac{M}{m}}\left(\frac{T_e}{T_i}\right)^{3/2}\exp\left[-T_e/2T_i\right]}{\left\{\left(\frac{M}{m}\right)^{5/2}\left(\frac{T_e}{T_i}\right)^{3/2}\exp\left[\frac{-T_e}{2T_i}\right]+\frac{2\Lambda}{\pi}\left(\frac{u}{C_s}-1\right)\right\}} \tag{13}$$

and the anomalous conductivity can be calculated from

$$\sigma_s = \omega_{\rm pe}^2/4\pi\nu_s\,.$$

For $T_e/T_i \geqslant 20$, one gets

$$\frac{\sigma_s}{\omega_{\rm pe}} = \frac{\sqrt{2}\Lambda}{\pi^{7/2}}\left(1-\frac{C_s}{u}\right)$$

$$\simeq \frac{1}{82} \quad \text{for} \quad \frac{u}{C_s} = 10\,.$$

## 3. Scattering of the Ion-Sound Waves By Static Density Fluctuations

It has been shown earlier (Krishan and Ritchie, 1970) that electrostatic and electromagnetic waves could get scattered by the static density fluctuations in the plasma. This scattering of the waves in an inhomogeneous plasma system leads to damping of the waves or even conversion from one type of waves to others. The causes of fluctuations in the density could be many and varied, as for example due to the impurities, volume defects or even some periodic variations may be present. This is more so in the regions of solar corona, where apart from the variations in the density from one layer to the other, impurities in the form of ionized and unionized state are quite probable. Figure 3a shows the scattering of ion-sound waves on static density variations. Figure 3b shows the details of the scattering of the ion-acoustic wave on the static density variations produced by the presence of heavy impurities. The damping rate of the ion-sound wave is given as (Krishan and Ritchie, 1970)

$$\gamma = \frac{\omega_{\rm pi}^4}{\omega_{q0}^2}\sum_{\mathbf{q}}(\hat{\mathbf{q}}\cdot\hat{\mathbf{q}}_0)^2\,{\rm Im}\left[-\frac{1}{\varepsilon_{\mathbf{q},\omega}}\right]|f_{\mathbf{q}-\mathbf{q}_0}|^2\,, \tag{14}$$

where $\hat{\mathbf{q}}$ and $\hat{\mathbf{q}}_0$ are the unit vectors, $\varepsilon_{\mathbf{q},\omega}$ is the dielectric function for the ion-sound wave and $f_{\mathbf{q}-\mathbf{q}_0}$ is the fourier transform of $\delta n(r)/n_0$ and the density of particles has been written as

$$n(r) = n_0 + \delta n(r),$$

with

$$\delta n(r)/n_0 \ll 1\,.$$

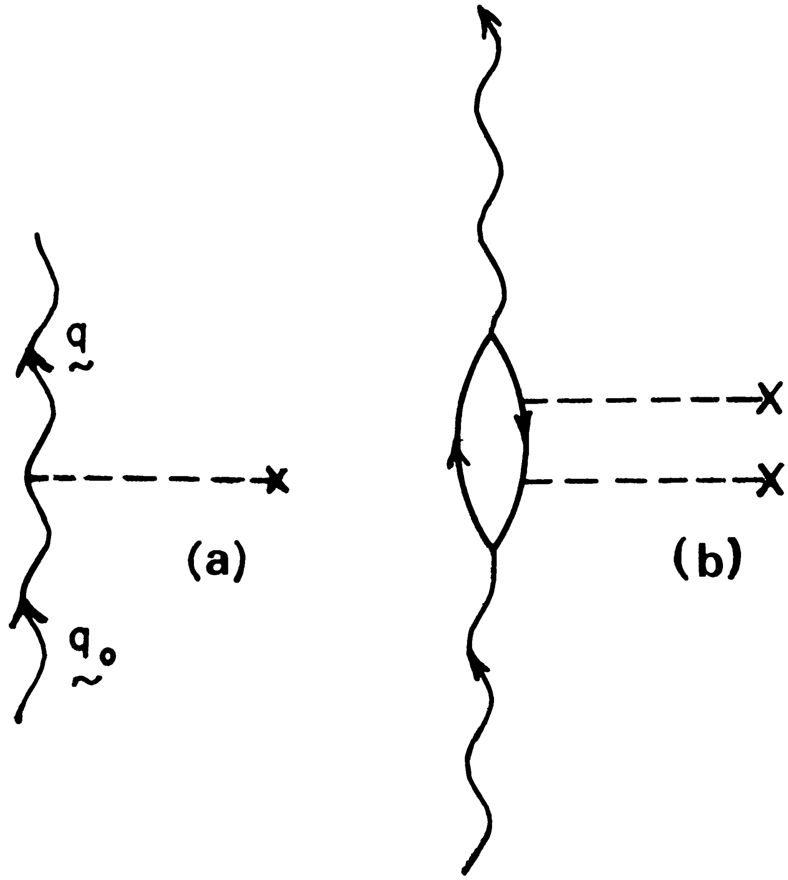


Fig. 3a-b. (a) The elastic scattering of the ion-sound wave on the static density fluctuations. The dashed line represents the static vertex. (b) The cross represents the heavy scattering centre.

Using Poisson equation to relate the fluctuations in the density with the field induced, and

$$\text{Real}\ \varepsilon_{\mathbf{q},\omega_{\mathbf{q}}} = 1 - \frac{\omega_{\text{pi}}^2}{\omega_q^2} + \frac{q_{\text{D}}^2}{q^2}, \tag{15}$$

one finds

$$\gamma = \frac{4\pi^2}{21} \frac{\omega_{\text{pi}}^4}{\omega_{q_0}^3} \left(\frac{q_0}{q_{\text{D}}}\right)^5 \frac{q_0^2 I(t)}{M\omega_{\text{pi}}^2}, \tag{16}$$

where the identity (Harris, 1969))

$$\sum_{\mathbf{q}} \frac{E_q^2}{8\pi} = \sum_{\mathbf{q}} \frac{I_{\mathbf{q}}}{VF_{\mathbf{q}}}$$

has been used and $q_{\text{D}}$ is the Debye wave vector. The saturation energy can be found by equating the damping rate $\gamma$ to the net linear growth rate of the ion-sound wave.

This gives

$$\frac{I(t)}{3N_D\chi T_e}=\frac{42}{32\sqrt{8}}\frac{1}{\pi^{5/2}}\frac{\omega_{q_0}^4}{\omega_{pi}^4}\left(\frac{q_D}{q_0}\right)^7\left\{\sqrt{\frac{M}{m}}\left(\frac{\mathbf{q}\cdot\mathbf{u}}{\omega_\mathbf{q}}-1\right)-\right.$$

$$\left.-\left(\frac{T_e}{T_i}\right)^{3/2}\exp\left[\frac{-T_e}{2T_i}\right]\right\}. \tag{17}$$

Again substituting in Equation (12), the anomalous resistivity is found to be

$$\nu_s=\frac{42\pi}{8}\frac{u}{C_s}\omega_{pe}\sqrt{\frac{m}{M}}\left\{\left(1-\frac{u}{C_s}\right)^2-\right.$$

$$\left.-\sqrt{\frac{M}{m}}\frac{C_s}{u}\left(1-\frac{C_s}{u}\right)\left(\frac{T_e}{T_i}\right)^{3/2}\exp\left[-\frac{T_e}{2T_i}\right]\right\}, \tag{18}$$

which reduces to, for $(T_e/T_i)\gtrsim 20$,

$$\frac{\nu_s}{\omega_{pe}}\approx\frac{42\pi}{8}\frac{u}{C_s}\left[\sqrt{\frac{M}{m}}\right]^{-1}\left(1-\frac{C_s}{u}\right)^2$$

$$\approx 3 \quad \text{for} \quad \frac{u}{C_s}=10.$$

Therefore the contribution of scattering by static density fluctuation is estimated to be

$$\frac{\sigma_s}{\omega_{pe}}\sim\frac{1}{37}. \tag{19}$$

## 4. Resonant Wave – Modified Particle Interaction

Classically, it is known that a particle in the presence of an electrostatic or electromagnetic field does not move as a free particle and the wave field acts on the particles resulting in a change in the trajectory of the particle. Alternatively one can say that the energy of the particle is modified in the presence of wave-field. These self energy corrections can be calculated easily by using many body perturbation techniques which apart from being algebraically simple, also include the long time averages over the particle orbits. So one first finds the field dependent energy of a particle and then lets this modified particle interact resonantly with the wave. For the case of ion-sound wave, one finds that the electrons are retarded by the wave and ions get a positive increment in their energy. As a result more number of ions can interact with the wave and landau damp it and in addition the reduced energy of the electrons helps in inhibiting the linear growth rate of the ion-sound wave. This is in principle the working of this mechanism.

The dispersion relation for an electrostatic wave can be written in its most general form as

$$1-\sum_j \frac{\omega_{pj}^2}{q^2}\int \frac{\mathbf{q}\cdot\dfrac{\partial f_j(\mathbf{V}_j)}{\partial \mathbf{V}_j}\,\mathrm{d}^3\mathbf{V}_j}{(1/\hbar)(E_{\mathbf{k}+\mathbf{q},j}-E_{\mathbf{k},j})-\omega}, \tag{20}$$

where $E_{\mathbf{k},j}=(\hbar^2k^2)/2m_j$ is the bare particle energy, $\mathbf{q}$ and $\omega$ are the wave vector and the frequency of the ion sound wave.

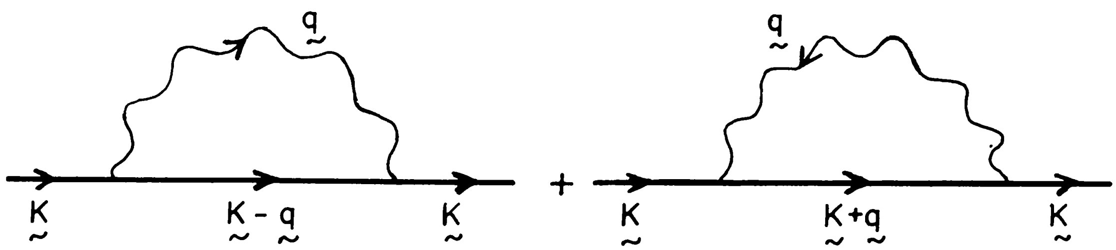


Fig. 4. Non-resonant wave-particle interaction contributing to self-energy corrections.

Now, to the lowest order, the processes contributing to the modification in the energy of the particle are shown in Figure 4. Again using the well known techniques given by Dubois (1967) and Harris (1969), one finds

$$E_{\mathbf{k},e}=\frac{\hbar^2k^2}{2m}-\frac{8\pi^2}{9}\frac{V_{0e}^2}{\left(\dfrac{\hbar k}{m}\right)^2}\frac{I(t)}{N_{\mathrm{D}}} \quad \text{for electrons} \tag{21}$$

and

$$E_{\mathbf{k},i}=\frac{\hbar^2k^2}{2M}+\frac{8\pi^2}{9}\frac{I(t)}{N_{\mathrm{D}}}\left(1+\frac{\hbar^2k^2}{M^2C_s^2}\right) \quad \text{for ions}. \tag{22}$$

Substituting these values of the energies, the modified dispersion relation becomes

$$1-\frac{\omega_{\mathrm{pi}}^2}{\omega^2}(1+A)+\frac{q_{\mathrm{D}}^2}{q^2(1-A)}=0, \tag{23}$$

where

$$A=\frac{16\pi^2 I(t)}{9N_{\mathrm{D}}\chi T_e}.$$

In arriving at Equation (23), the usual approximations like $\mathbf{q}\cdot\mathbf{V}\gg\omega$ for electrons and $\mathbf{q}\cdot\mathbf{V}\ll\omega$ for ions have been used.

To solve Equation (23), let

$$\omega=\omega_0+\omega_1$$

and

$$\omega_0 = \Omega_0 + i\gamma_0 ,$$

$$\omega_1 = \Omega_1 + i\gamma_1 ,$$

where $\omega_0$ is the solution of the unmodified dispersion relation

$$1 - \frac{\omega_{pi}^2}{\omega_0^2} + \frac{q_D^2}{q^2} = 0 .$$

Assuming $\omega_1 \ll \omega_0$, one finds the value of $\gamma_1$ to be

$$\frac{\gamma_1}{\gamma_0} = -\frac{A\gamma_0}{(1+A)\Omega_0\left(\frac{1}{3}+\frac{3\gamma_0^2}{\Omega_0^2}\right)}\left[\frac{3\gamma_0}{2\Omega_0\left(1-\frac{6\gamma_0^2}{\Omega_0^2}\right)} + \frac{A\gamma_0}{2\gamma_0(1-A)} + \frac{\Omega_0}{3\gamma_0}\right]. \tag{24}$$

One observes that $\gamma_1$ is negative, therefore saturation is achieved when $|\gamma_1| = |\gamma_0|$.

Solving for the saturation energy from the above condition, substituting in the Equation (12) for anomalous restivity, one gets

$$\nu_s \simeq 2.77\omega_{pe} \quad \text{for} \quad \frac{u}{C_s} = 10$$

and

$$\frac{\sigma_s}{\omega_{pe}} \sim \frac{1}{33} . \tag{25}$$

## 5. Comparison with the Earlier Results

Surveying the literature, one finds that the ion-nonlinear Landau damping as the saturation mechanism for the ion-acoustic instability suggested by Kadomtsev (1964) cannot overcome the electron-nonlinear Landau growth, as was pointed out by Sloan and Drummond (1970). Kadomtsev and Petviashvili (1963) considered the ion-sound wave scattering by ions, in which the waves propagating in a direction opposite to that of the current, suffer scattering. But such waves have relatively smaller growth rates than those propagating along the direction of the current. Therefore this process is effective only near the threshold of the instability or in other words the quasi-linear relaxation is sufficient to maintain the system near the threshold and from there onwards the ion-ion sound wave scattering takes over. This is all fine, but does not conform to the observed results.

Sagdeev and Galeev (1969) have considered the mode-mode coupling as the major saturation mechanism. Although their result has been in use quite frequently, the restriction of the wave vector dependence of the turbulent field to two lines in the wave vector space does not have a physical basis. The value of anomalous resistivity found by Sagdeev and Galeev (1969) is

$$\frac{\nu_s}{\omega_{\text{pe}}} \sim \sqrt{\frac{m}{M}} \frac{u}{C_s} \frac{T_e}{T_i} \tag{26}$$

which agrees with the results of sections three and four for $T_e/T_i \sim 12$ and with the result of section two for $T_e/T_i \sim 25$.

Tsytovich (1971) has considered the process of one ion sound wave scattering into two ion sound waves. The requirements of conservation of energy in this case presumes the existence of a phenomenon producing the frequency shifts in the ion-sound waves. Thus the applicability of the three wave scattering becomes restrictive in the sense that only those waves can take part in the interaction which have the right amounts of frequency shifts to ensure the energy conservation. Moreover, the resonant three wave interaction is a higher order effect, as can be easily seen by looking at the energy conserving Dirac delta function. Including the effect of resonance broadening, the modified energy conservation law becomes $\delta(\omega_1+\omega_2-\omega_3+\Delta\omega)$ which can be further written as

$$\delta(\omega_1+\omega_2-\omega_3+\Delta\omega) \simeq \delta(\omega_1+\omega_2-\omega_3)+\Delta\omega\delta'(\omega_1+\omega_2-\omega_3)\,.$$

Now the first term does not contribute and in the second term, $\Delta\omega$ is a function of the field of the ion-sound wave. Tsytovish (1971) has retained the first term and neglected the second one, thereby relying heavily on the energy conservation law which is only approximately satisfied, whereas in principle, it is the second term which should be contributing and thus reducing the result by an amount $\Delta\omega/\omega$. The wave mediated indirect wave particle interaction, considered in Section 2, also includes the three wave scattering, but here only its non resonant contribution has been taken into account. the resonant part can be obtained by taking the imaginary part of the contribution of this process, but as mentioned already, it is small. It should be mentioned that the process of indirect wave mediated wave particle interaction does not include the non linear electron and ion Landau damping as such, but these interactions do form a part of the total process which comprises of two wave particle scattering vertices and one three wave interaction vertex. The phenomenon of nonlinear Landau damping by itself has been shown to be ineffective in bringing about the saturation, Sloan and Drummond (1970).

## 6. Application to Solar Flares

The presence of plasma turbulence in the regions close to the magnetically neutral sheet in the solar corona has been established fairly well. The impulsive release of large amounts of energy is associated with the anomalously large reduction in the conductivity of the plasma which in turn is a consequence of the plasma turbulence. Therefore the knowledge of turbulent conductivity is very crucial for the determination of the magnetic field gradients near the magnetically neutral sheet in the solar corona. Values of $\sigma_s$ ranging anywhere from $\omega_{\text{pe}}$ to 100 $\omega_{\text{pe}}$ have been used in this context under the assumptions of large electron-ion temperature ratios (Kuperus, 1976; Spicer, 1977). In the light of the fresh data available from Skylab (Spicer, 1977), a new estimate of the magnetic field gradient can be made using the results of the previous sections. One notices that the rate of release of energy in a solar flare is given by

$$\frac{\partial E}{\partial t}=\frac{C^2}{(4\pi)^2}\frac{B^2}{l^2\sigma_s}\Delta V \simeq 10^{28}\ \text{ergs s}^{-1} \tag{27}$$

for a small flare.

Here $C$ is the speed of light, $B$ is the magnetic field, $l$ is the thickness of magnetically neutral sheet and $\Delta V$ is the volume of energy release.

Using $\sigma_s \sim \omega_{pe}/82$ from Section 2, $\Delta V \sim 2.7\times10^{24}\ \text{cm}^3$ from Skylab, $\omega_{pe} \sim \sim 5.5\times10^{10}\ \text{s}^{-1}$ for $n_0 \sim 10^{12}\ \text{cm}^{-3}$, one finds

$$\frac{B}{l}\sim 6.3\times10^{-4}\ \text{G cm}^{-1} \tag{28}$$

which is in surprisingly good agreement with the observed value of $B/l \sim \sim 6\times10^{-4}\ \text{cm}^{-1}$. If one uses $\sigma_s \sim (\omega_{pe}/33)$, one gets

$$\frac{B}{l}\sim 9.9\times10^{-4}\ \text{G cm}^{-1}\,. \tag{29}$$

## 7. Conclusion

After considering these different mechanisms, which may be responsible for the saturation of the ion-acoustic instability, one could ask, at this point, whether some or all of these varied mechanisms are operative jointly and whether one should add the contribution from every one of them, or they occur in exclusion to one another. Although one could not say with certainty that out of all the nonlinear wave-wave and wave-particle interactions, one or the other is more important, it does seem possible to conclude from the results presented in this paper that the mechanism of wave-mediated indirect wave particle interaction contributes most generously to the determination of the anomalous resistivity. The processes of density fluctuation scattering and the energy renormalization together could compete with the results of Section 2.

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