

## A central gravitational redshift model for the absorption redshift systems in quasi-stellar objects

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**Summary.** Static, spherically symmetric configurations of core and envelope type have been considered in General Relativity as central gravitational redshift models for the quasi-stellar objects (QSOs) and an attempt has been made to explain the observed absorption features in the QSO spectra on the basis of these models. It is argued that the absorption systems occur in thin shells of densities negligible as compared to the typical densities of the model. The effect of radiation pressure due to a small central source of luminosity on such shells has been investigated and it has been shown that stable equilibrium positions are possible due to a balance between the gravitational and radiation forces. For the QSOs in which the absorption redshift exceeds the emission redshift, models with collapsing absorbing shells have been suggested. Finally some illustrative calculations have been done for the QSO PHL 938 and some other QSOs.

### 1 Introduction

Central gravitational redshifts and other physical properties of highly collapsed massive objects in General Relativity have recently been investigated in a series of papers by Das and Narlikar (Das & Narlikar 1975; Das 1975; Narlikar & Das 1976; Das 1976. Hereinafter we shall refer to these as Papers I, II, III and IV respectively). On the basis of these investigations it was shown that it is possible to construct models of static, bound, massive spheres in stable equilibrium giving rise to fairly high values of internal gravitational redshifts without making undue demands on the equations of state.

One of the motives of these investigations was to explore the plausibility of these objects as gravitational redshift models for the quasi-stellar objects (QSOs). It was shown that these objects could serve as models for the QSOs and account for a major part of a QSO redshift (in particular of a high redshift QSO) as well as its other physical properties. Thus, for example, these models could explain the emission-line features in the spectrum of a typical high redshift QSO quite satisfactorily.

But many QSOs (generally all those with high emission redshifts,  $z_{\text{em}} \gtrsim 2.2$ ) show absorption features, with varying degrees of complexity, in addition to the emission lines in their spectra. In the present paper we take up the problem of explaining the absorption

features on the basis of our models. In doing this we shall proceed as follows, we start with a brief sketch of the models developed in the earlier papers. Then we develop the methods to be adopted to explain the absorption systems on the basis of our models. In the next section we give some illustrative calculations. Finally, the main conclusions will be summarized.

## 2 The model

We consider a static, spherically symmetric configuration of mass  $M$  and radius  $R$  (in Schwarzschild coordinates) in hydrostatic equilibrium. We use geometrized units ( $c = 1$ ,  $G = 1$ ) unless otherwise mentioned.

Outside the body ( $r > R$ ) the usual exterior Schwarzschild solution holds. Inside the body ( $r \leq R$ ) the line element is

$$ds^2 = \exp(\nu) dt^2 - \exp(\lambda) dr^2 - r^2 (d\theta^2 + \sin^2\theta d\phi^2), \quad (1)$$

and the energy momentum tensor  $T^i_k$  is

$$T^i_k = \text{diag}(-p, -p, -p, \rho), \quad (2)$$

where  $p$ ,  $\rho$ ,  $\nu$  and  $\lambda$  are functions of  $r$  only.

Following Bondi (1964), *cf.* Paper I for greater details, we consider a class of core-envelope-type configurations with the following equations of state:

core: isothermal  $p = k\rho$  ( $k < 1$ ),

envelope adiabatically stable  $dp/d\rho = 1/n$ , ( $1 \leq n < k^{-1}$ ). (3)

We denote the values of various physical quantities at the surface, core-envelope boundary and the centre by the subscripts  $s$ ,  $b$  and  $c$  respectively.

For a specified equation of state, that is for a given pair of  $n$  and  $k$ , the march of various dimensionless physical variables such as  $\nu$ ,  $u = m(r)/r = [1 - \exp(-\lambda)]/2$ ,  $v = 4\pi r^2 p(r)$ ,  $p/p_c$ ,  $\rho/\rho_c$ ,  $a = \sqrt{(4\pi/3)}\rho_c r$  and also the gravitational redshifts  $z_c$ ,  $z_b$  and  $z_s$ , of light leaving respectively from the centre, the core-envelope boundary and the surface and received by a distant observer can be numerically determined. Thus to quote some typical redshift values, for  $k^{-1} = 3$  and  $n = 1$ ;  $z_c = 2.85$ ,  $z_b = 1.29$  and  $z_s = 0.62$  whereas for  $k^{-1} = 10$ ,  $n = 9.9$ ;  $z_c = 1.19$ ,  $z_b = 0.77$  and  $z_s = 0.15$ .

## 3 Analysis of the absorption lines

### 3.1 QUALITATIVE PICTURE

The model proposed in Papers I, II and IV is the so-called Hoyle-Fowler cluster model (Hoyle & Fowler 1967). It may be described briefly as follows.

We assume the object to consist of a small, spherical, optically thin emitting region at the centre, surrounded by a spherically symmetric distribution of a large number of comparatively compact, highly collapsed subunits with dimensions much smaller than the overall dimensions of the object. The central region produces the continuum and the emission lines which all get redshifted by approximately the same amount  $z_c (= z_{em})$  due to the gravitational field produced by the subunits. As the subunits are quite small compared to the overall size of the object they allow most of the centrally emitted light to escape to the surface.

To incorporate the absorption-line systems in this scenario we assume them to be intrinsic to the object and as arising due to the absorption taking place in ion or atom shells. The densities of these shells should be negligible as compared to the typical densities of the object so as not to alter the overall structure of the object significantly. The forces acting on these shells are gravity and the radiation pressure due to the central source of luminosity. As we shall show later, stable equilibrium positions are possible, in principle, for the shells due to a balance between these two forces. The absorption lines produced in such static shells would have redshifts,  $z_{\text{abs}}$ , less than the emission redshift,  $z_{\text{em}} (= z_c)$ , as is usually observed. Also, as different ion shells may be situated at different radial distances,  $z_{\text{abs}}$  may be different for different ions corresponding to a single  $z_{\text{em}}$ , thus accounting for the often observed multiple absorption redshift systems. For the few QSOs which have well-established  $z_{\text{abs}} > z_{\text{em}}$  systems, we can argue in terms of shells for which gravity dominates over the radiation pressure resulting in an inward fall and a Doppler contribution to the gravitational redshift.

### 3.2 $z_{\text{abs}} < z_{\text{em}}$

Let us consider a QSO with emission redshift  $z_{\text{em}}$  and one absorption redshift  $z_{\text{abs}}$  with  $z_{\text{abs}} < z_{\text{em}}$ . We choose our model  $(n, k)$  – that is core  $p = k\rho$  and envelope  $dp/d\rho = 1/n$  – such that  $z_c = z_{\text{em}}$  and  $z_s < z_{\text{abs}}$ . We assume that the absorption line is produced by a static shell of radius  $r$  and thickness  $\Delta r$  interior to the object.

Let  $\lambda$  = actual wavelength of the absorption line,  $\lambda_0$  = observed wavelength of the absorption line =  $\lambda(1 + z_{\text{abs}})$ ,  $W$  = observed equivalent width of the line,  $\omega$  = observed line width,  $\tau_\lambda$  = optical depth at  $\lambda$ .

Then from the usual definition we get

$$W = \int_{\omega} [1 - \exp(-\tau_\lambda)] d\lambda_0. \quad (4)$$

We assume that the entire absorption occurs at  $\lambda$  and that the broadening is entirely due to the potential gradient across the thickness  $\Delta r$  of the shell. In this case  $\tau_\lambda$  is a constant given by

$$\tau_\lambda = NA_\lambda. \quad (5)$$

where  $N(\text{cm}^{-2})$  = column density of the absorbing ions.

$$A_\lambda(\text{cm}^2) = \frac{4\pi e^2}{m_e c} \frac{f_{\text{abs}}}{\gamma}. \quad (6)$$

$e$  = electronic charge in esu,  $m_e$  = electron mass in gm,  $\gamma$  = damping constant in  $\text{s}^{-1}$ ,  $f_{\text{abs}}$  = absorption oscillator strength. If  $NA_\lambda \gg 1$ , that is the line is sufficiently dark or optically thick, we get

$$W = \omega. \quad (7)$$

If the absorbing shell is confined between the spheres of radii  $r$  and  $r + \Delta r$  we have, in an obvious notation

$$(\lambda + \Delta\lambda + \omega)/\lambda = \exp[-\nu(r)/2],$$

and

$$(\lambda + \Delta\lambda)/\lambda = \exp[-[\nu(r) + \Delta\nu(r)]/2],$$

or

$$\frac{\omega}{\lambda + \Delta\lambda} \approx \frac{\Delta\nu}{2}.$$

Using relevant equations of structure to eliminate  $\Delta\nu$  – *cf.* Paper I for details – we obtain

$$\Delta r = f_1(z_{\text{abs}}) \sqrt{\frac{3}{4\pi\rho_c}} q, \quad (8)$$

where

$$q = \frac{\omega}{\lambda + \Delta\lambda} = \frac{W}{\lambda + \Delta\lambda},$$

and  $f_1(z_{\text{abs}})$  is a known dimensionless function of  $z_{\text{abs}}$ .

We must ensure that the imposition of an absorbing shell on the model does not alter the equations of structure of the model significantly. For this the density of the shell must be negligible as compared to the density of the model around  $r$ . Hence we must satisfy

$$\frac{Nm_0}{\Delta r \exp(\lambda/2)} \ll \rho_0(r), \quad (9)$$

where  $m_0$  = proper mass of the absorbing ion,  $\rho_0(r)$  = proper density of the model at  $r = \rho_c f_2(z_{\text{abs}})$  where  $f_2(z_{\text{abs}})$  is a known dimensionless function of  $z_{\text{abs}}$  (*cf.* Paper IV for details).

Combining equations (8) and (9) we obtain

$$N \ll \frac{q}{m_0} \sqrt{\frac{3\rho_c}{4\pi}} F_1(z_{\text{abs}}), \quad (10)$$

where again  $F_1(z_{\text{abs}})$  is a known dimensionless function of  $z_{\text{abs}}$ .

In the right-hand side of inequality (10)  $q$  is known from observation,  $m_0$  is the proper mass of the absorbing ions,  $F_1(z_{\text{abs}})$  is a known function of  $z_{\text{abs}}$  and the central density  $\rho_c$  is determined once the total mass  $M$  or the radius  $R$  of the object is specified (*cf.* Paper I for details). Thus the right-hand side of inequality (10) can be taken as a known quantity which puts an upper limit on  $N$  whereas the assumption of an optically thick line puts a lower limit on  $N$ , namely  $N \gg A_\lambda^{-1}$ . Hence we are provided with a check on the consistency of the model. It remains to apply this method to any particular QSO. We postpone it until the next section.

### 3.3 EFFECT OF RADIATION PRESSURE

In this subsection we obtain the equation of motion of a particle under the combined influence of radiation pressure and gravity in order to show that, in principle, positions of stable equilibrium are possible.

Consider a point source of luminosity  $L$  at the centre and a particle at a radial distance  $r$  moving with a coordinate velocity  $V = dr/dt$ . We first obtain an expression for the effective redshift,  $z_{\text{eff}}$ , of a photon emitted at the centre and observed in the frame of the particle.

Consider a photon of coordinate period  $\Delta t_c$  emitted radially at the centre. Its coordinates measured at two successive points of equal phase will be  $(t_c, 0, \pi/2, 0)$  and  $(t_c + \Delta t_c, 0, \pi/2, 0)$ . If the observer in the frame of the particle measures the coordinate period to be  $\Delta t$  the corresponding coordinates will be  $(t, r, \pi/2, 0)$  and  $(t + \Delta t, r + \Delta r, \pi/2, 0)$ .

The equation of motion of a photon travelling radially outwards in the space-time described by equation (1) is

$$\frac{dr}{dt} = \exp [(\nu - \lambda)/2]$$

which yields

$$\Delta t_c = \Delta t \left[ 1 - \frac{V}{\exp [(\nu - \lambda)/2]} \right]$$

for

$$\Delta r = V \Delta t.$$

The proper time intervals corresponding to the coordinate time intervals are given by

$$\Delta \tau = \Delta t \sqrt{\exp(\nu) - \exp(\lambda) V^2} = \Delta t_c \frac{\sqrt{\exp(\nu) - \exp(\lambda) V^2}}{\|1 - V/\exp [(\nu - \lambda)/2]\|}, \quad (11)$$

and

$$\Delta \tau_c = \exp(\nu_c/2) \Delta t_c.$$

By definition  $z_{\text{eff}}$  is given by

$$(1 + z_{\text{eff}}) = \frac{\Delta \tau}{\Delta \tau_c}.$$

Using equation (11) and putting  $\exp(-\nu_c/2) = (1 + z_c)$  and  $\exp(-\nu/2) = (1 + z)$  where  $z$  and  $z_c$  are the gravitational redshifts at  $r$  and at the centre we obtain

$$(1 + z_{\text{eff}}) = \frac{(1 + z_c)}{(1 + z)} \sqrt{\frac{\exp [(\nu - \lambda)/2] + V}{\exp [(\nu - \lambda)/2] - V}}. \quad (12)$$

The equation of motion of the particle in the Schwarzschild coordinates frame is given by

$$\frac{d^2 x^i}{ds^2} + T_{jk}^i \frac{dx^j}{ds} \frac{dx^k}{ds} = a^i, \quad (13)$$

or

$$Q^i = a^i(\text{say})$$

where  $T_{jk}^i$ s are the Christoffel symbols and  $a^i$  is the four acceleration due to radiation.

Let primed quantities denote the local Lorentz frame of the particle. In this frame only  $a^{1'}$  is non-zero where

$$a^{1'} = \frac{\epsilon L r_0^2}{4r^2 (1 + z_{\text{eff}})^2 m_0}, \quad (14)$$

where  $r_0$  = proper radius of the particle,  $m_0$  = proper mass of the particle,  $\epsilon$  = efficiency factor for radiation pressure.

From the transformation laws between the two frames we get

$$Q^{1'} = \frac{\exp [(\lambda + \nu)/2]}{\sqrt{\exp(\nu) - \exp(\lambda) V^2}} [Q^1 - V Q^4] = a^{1'}.$$

Substitution of the expressions of  $T_s$  and those of  $a^{1'}$  and  $z_{\text{eff}}$  and a straightforward calculation finally yields

$$\begin{aligned} \exp [(3\nu - 2\lambda)/2] r^2 \frac{[d/dr(V^2/2) + (V^2/2)(d\lambda/dr) - V^2(d\nu/dr) + \exp[(\nu - \lambda)]/2(d\nu/dr)]}{[\exp[(\nu - \lambda)/2] + V]^{1/2} [\exp[(\nu - \lambda)/2] - V]^{5/2}} \\ = \frac{\epsilon L r_0^2}{4m_0(1 + z_c)^2}. \end{aligned} \quad (15)$$

Equation (15) giving  $V$  as a function of  $r$  is the required equation of motion of the particle.

The condition for a static solution is given by

$$r^2 \exp [(2\nu - \lambda)/2] \frac{d\nu}{dr} = \frac{\epsilon L r_0^2}{2m_0(1 + z_c)^2}, \quad (16)$$

or

$$F_2(z) = \frac{\epsilon L r_0^2}{4m_0} \sqrt{\frac{4\pi}{3}} \rho_c \text{ (say)} \quad (17)$$

where  $F_2(z)$  is a known dimensionless function of  $z$ .

For a given situation (specified  $\epsilon$ ,  $r_0$ ,  $m_0$ ,  $z_c$ ,  $\rho_c$  and  $L$ ) the right-hand sides of equations (15), (16) and (17) are constants. We can thus study the motion of the particle by integration of equation (15) or check whether a static solution exists at a point  $r$  by an explicit evaluation of the left-hand side in equations (16) or (17). But even without solving equations (16) or (17) it may be seen that the situation here is different from what one obtains in the case of normal stellar atmospheres. In the latter case both radiation pressure and gravity have the same ( $1/r^2$ ) dependence on the radial distance  $r$  and hence cannot be balanced by a change of  $r$ . In equations (16) or (17) the right-hand side is a constant whereas the left-hand side starts increasing from zero at  $r = 0$ . Hence positions of stable equilibrium are possible in principle. Of course a definite statement in a specified case can be made only after explicit computation. We shall illustrate this in the next section.

### 3.4 $z_{\text{abs}} > z_{\text{em}}$

We now consider the case in which the absorption redshift exceeds the emission redshift. At present there are several QSOs in which this has been confirmed (Strittmatter & Williams 1976).

In order to explain this case on the basis of our model we assume that the radiation pressure force is insufficient to balance the gravitational force on the absorbing shell and hence it is falling inwards that is, receding from the observer. For an infalling shell the column density  $N$  increases as  $1/r^2$  and, hence, even if  $N$  was initially too small to produce an observable line it may attain a sufficiently high value at a later stage. Also the total redshift as observed by a distant observer will have a Doppler contribution in addition to the gravitational, and may exceed the emission redshift. Rigorously speaking we have to obtain the total redshift of a photon emitted from a particle which is falling under the combined influence of gravity and radiation pressure due to the central source of luminosity and this would essentially involve a numerical integration of the equation of motion given by equation (15). But here we present a simplified calculation by ignoring radiation pressure and assuming that the particle is in a free fall. The results obtained will hence be only approximate but will suffice to give a qualitative idea.



The radial geodesic motions of a photon and a material particle in the space-time described in equation (1) are respectively given by

$$\frac{dr}{dt} = \exp [(\nu - \lambda)/2], \quad (18)$$

for a photon travelling radially outwards and

$$\frac{dr}{dt} = - \exp [(\nu - \lambda)/2] \llbracket 1 - \exp [(\nu - \nu_1)] \rrbracket^{1/2} \quad (19)$$

for a material particle falling radially inwards starting from rest at  $r = r_1$ .

We denote the coordinates of the emitter and those of the observer by the subscripts  $e$  and  $o$  respectively and calculate the total redshift  $z_t$  of a photon emitted by the material particle at  $r_e$  as seen by a remote stationary observer at  $r_o$ .

We use equations (18) and (19) to calculate the coordinate and proper time intervals at the emitter and the observer as was done in Section 3.3. A straightforward calculation yields

$$(1 + z_t) = (1 + z_e) \left[ \frac{(1 + z_e)}{(1 + z_1)} + \sqrt{\frac{(1 + z_e)^2}{(1 + z_1)^2} - 1} \right] \quad (20)$$

where  $z_t$  is the total redshift and  $z_e, z_1$  are the gravitational redshifts at the points  $r_e$  and  $r_1$  respectively.

We may solve (20) for either  $z_e$  or  $z_1$ ;

$$(1 + z_e) = \frac{(1 + z_t)}{2(1 + z_t)/(1 + z_1) - 1}, \quad (21)$$

and

$$(1 + z_1) = \frac{2(1 + z_t)}{(1 + z_t)^2/(1 + z_e)^2 + 1}. \quad (22)$$

The locally measured velocity of the infalling particle at  $r_e$  is given by

$$V_e = \left| \frac{\exp(\lambda/2) dr}{\exp(\nu/2) dt} \right|_{r=r_e} = \frac{z_t - z_1}{(1 + z_t)}. \quad (23)$$

For a given  $z_t$ ,  $z_e$  is an increasing function and  $V_e$  is a decreasing function of  $z_1$ . Hence

$$(V_e)_{\max} = \frac{z_t}{(1 + z_t)},$$

and

$$(1 + z_e)_{\min} = \frac{(1 + z_t)}{\sqrt{2z_t + 1}}, \quad \text{for } z_1 = 0. \quad (24)$$

In order to apply the above method to our models we adopt the following procedure.

For a given QSO with  $z_{\text{abs}} > z_{\text{em}}$  we choose a model  $(n, k)$  which has  $z_c = z_{\text{em}}$  and put  $z_{\text{abs}} = z_t$ . For the model to work we must satisfy the condition

$$(z_e)_{\min} < z_c.$$

That is the value  $z_{\text{abs}}$  should be reached before the particle reaches the centre. Hence

$$\frac{(1 + z_{\text{abs}})}{\sqrt{2z_{\text{abs}} + 1}} < (1 + z_{\text{em}}). \quad (25)$$

If (25) is satisfied we calculate  $z_e$  and  $V_e$  using (21) and (23) for a suitable starting point  $z_1$ .

#### 4 Illustrative calculations

In this section we shall perform some illustrative calculations based on the techniques developed in the previous section.

##### 4.1 PHL 938

We illustrate our analysis for the  $z_{\text{abs}} < z_{\text{em}}$  case with the example of the QSO PHL 938.

##### 4.1.1 Relevant observational data

$z_{\text{em}} = 1.9552$  (mainly  $L\alpha$ , C IV etc.). The flux observed in  $L\alpha$  emission,  $s = 3.105 \times 10^{-13}$  erg cm<sup>-2</sup> s<sup>-1</sup>.

The width of  $L\alpha$  emission  $\omega_E = 428$  Å.

Total observed flux  $s_{\text{tot}} \approx 10^{-12}$  erg cm<sup>-2</sup> s<sup>-1</sup>  
(obtained from Oke, Neugebauer & Becklin 1970)

$z_{\text{abs}} = 1.9064$  (mainly  $L\alpha$ ),  
= 0.6128 (Mg II, Fe II).

As the equivalent widths for only the  $z_{\text{abs}} = 0.6128$  system are available (Tung Chan & Burbidge 1971) we shall consider only that system. Of course, similar calculations will also hold for the  $z_{\text{abs}} = 1.9064$  system.

##### 4.1.2 The model

We put  $z_{\text{em}} = z_c$  to determine the model. We choose  $k^{-1} = 5$ ,  $n = 4.4$ . This has

$$z_c = 1.9306, z_b = 1.0477, z_s = 0.3368, a_s = 2.3420, u_s = 0.2202. \quad (26)$$

From the  $L\alpha$  emission-line data we determine the mass of the object for an assumed distance  $d$  by the following formula (*cf.* Paper I for details).

$$M = u_s a_s \sqrt{\frac{3k+1}{2q_E} \left\{ \frac{3s(1+z_c)^2}{\beta \alpha N_e^2} \right\}^{1/3}} d^{2/3} \quad (27)$$

where  $q_E$  = observed width of the emission line/observed wavelength,  $s$  = flux observed in the line (erg cm<sup>-2</sup> s<sup>-1</sup>),  $N_e$  = average ion or electron density on the emitting region,  $\alpha N_e^2$  = volume emissivity of the emitting region (erg cm<sup>-3</sup> s<sup>-1</sup>),  $\beta$  = fraction of the total emission which escapes. Putting  $\alpha \approx 10^{-22}$ ,  $N_e \approx 10^8$  and  $\beta \approx 0.5$  we obtain

$$\frac{M}{M_\odot} \approx 1.02 \times 10^{11} \left( \frac{d_{\text{Mpc}}}{100} \right)^{2/3} \quad (28)$$



where  $d_{\text{Mpc}}$  is the distance of the object in megaparsecs. Once the values of  $M$  is determined, the values of other physical parameters, e.g. radius  $R$ , central density  $\rho_c$  etc., can be scaled.

#### 4.1.3 Analysis of the absorption lines

We assume that the Fe II, Mg II absorption lines are produced due to absorption in static shells situated at radial distances given by their redshift. We get

$$a_{(\text{Fe II})_{\text{abs}}} = a_{(\text{Mg II})_{\text{abs}}} = 1.496. \quad (29)$$

We give further calculations only for the Fe II lines, for similar considerations will hold for the Mg II lines.

We first check whether the Fe II shell can remain static. Putting  $z = z_{\text{abs}} = 0.6128$ , we obtain in equation (17)  $F_2(0.6128) = 1.705$ . In the right-hand side the values of  $r_0$ ,  $m_0$ , the radius and mass of the Fe<sup>56</sup> atom, respectively, are known but the values of  $L$  and  $\rho_c$  are fixed only when the distance  $d$  is specified. Noting that  $L\sqrt{\rho_c} \propto d^{4/3}$  we can choose a value of  $d$  so as to match the two sides. It is seen that  $d = 3.51$  Mpc gives the desired result (assuming  $\epsilon \approx 1$ ). For this value of  $d$  we obtain

$$\begin{aligned} M &\approx 1.09 \times 10^{10} M_{\odot}, \\ R &\approx 7.35 \times 10^{15} \text{ cm}, \\ L &\approx 2.39 \times 10^{40} \text{ erg/s}, \\ \rho_c &\approx 3.28 \times 10^{-4} \text{ gm cm}^{-3}. \end{aligned} \quad (30)$$

If the efficiency factor for radiation pressure is small ( $\epsilon \ll 1$ ), we have to take a suitably larger value of  $d$  which will yield higher values for  $M$  and  $L$  and a smaller value for  $\rho_c$ . We may also note that the distance  $d$  is large enough for tidal perturbations of our galaxy due to the QSO to be negligible.

We now have to check the consistency of the model through the inequality (10). For the Fe II lines  $A_{\lambda}s$  can be calculated from equation (6) by using the tabulated values of  $\gamma$  and  $f_{\text{abs}}$  (Morton & Hayden Smith 1973). It is seen that  $10^{-12} \lesssim A_{\lambda} (\text{cm}^2) \lesssim 10^{-10}$ . Hence the assumption of large optical depth will be valid if  $N(\text{cm}^{-2}) \gtrsim 10^{13}$ .

Substitution of values of  $\rho_c$ ,  $m_0$ ,  $q \approx 10^{-3}$  and  $F_1(0.6128) = 0.1446$  in the inequality (10) yields

$$N(\text{cm}^{-2}) \ll 1.6 \times 10^{30}.$$

which is easily satisfied for  $N \gtrsim 10^{13} \text{ cm}^{-2}$ . Thus the condition for consistency is satisfied.

For  $N \approx 10^{13} \text{ cm}^{-2}$  we obtain thickness of the shell  $\Delta r \approx 1.07 \times 10^{13} \text{ cm}$  and the density of absorbing material  $\approx 6.59 \times 10^{-23} \text{ gm cm}^{-3}$ . These values may be compared with the values of  $R_1\rho_c$  in equation (30) and  $\rho_0 \approx 1.05 \times 10^{-5} \text{ gm cm}^{-3}$ , the proper density of the model around the shell.

Thus we see that the Fe II absorption system in PHL 938 can be adequately explained by assuming that the absorption takes place in a shell which is held in equilibrium due to a balance between gravity and radiation, and whose thickness and density are negligible compared to the typical dimensions and densities of the object.

#### 4.2 $z_{\text{abs}} > z_{\text{em}}$

In this section we apply the analysis of Section 3.4 to the QSOs with  $z_{\text{abs}} > z_{\text{em}}$  (Burbidge, Crowne & Smith 1977).

Table 1. For explanation of the various symbols see Sections 3.4 and 4.2 in the text.

| QSO                   | $z_{em}$ | $z_{abs}$        | $k^{-1}, n$ | $z_s$  | $z_b$  | $z_c$  | $z_e$<br>( $z_1 = z_s$ ) | $V_e$<br>( $z_1 = z_s$ ) | $z_e$<br>( $z_1 = z_b$ ) | $V_e$<br>( $z_1 = z_b$ ) |
|-----------------------|----------|------------------|-------------|--------|--------|--------|--------------------------|--------------------------|--------------------------|--------------------------|
| 0835 + 580 (3CR 205)  | 1.534    | 1.5380           | 6, 5.2      | 0.3022 | 0.8476 | 1.5261 | 0.4909                   | 0.4869                   | 0.9200                   | 0.2720                   |
| 0736-063              | 1.905    | 1.9123<br>1.9299 | 5, 4.3      | 0.3455 | 1.0181 | 1.8883 | 0.5962<br>0.5996         | 0.5380<br>0.5408         | 1.1205<br>1.1235         | 0.3070<br>0.3112         |
| 0151 + 048 (PHL 1222) | 1.910    | 1.9340           | 5, 4.3      | 0.3455 | 1.0181 | 1.8803 | 0.6003                   | 0.5414                   | 1.1243                   | 0.3122                   |
| 0199-046 (4C 04.04)   | 1.955    | 1.9650           | 5, 4.4      | 0.3368 | 1.0476 | 1.9306 | 0.5996                   | 0.5491                   | 1.1533                   | 0.3094                   |
| 2251 + 244 (4C 24.61) | 2.328    | 2.3630           | 4, 3.1      | 0.4335 | 1.1780 | 2.3159 | 0.7502                   | 0.5737                   | 1.3273                   | 0.3524                   |
| 0123 + 257 (4C 25.05) | 2.358    | 2.3682           | 4, 3.2      | 0.4250 | 1.2045 | 2.3562 | 0.7446                   | 0.5769                   | 1.3492                   | 0.3455                   |

As explained in Section 3.4 for a given QSO, we choose a model  $(n, k)$  which has  $z_c = z_{em}$ , then check the applicability of the model using equation (25). If equation (25) is satisfied we put  $z_{abs} = z_t$  and calculate  $z_e$  and  $V_e$  by using equations (21) and (23) for a different starting point  $z_1$ . Two convenient starting points in our model are naturally the surface ( $z_s$ ) and the core—envelope boundary ( $z_b$ ). The results of this procedure are given in Table 1. It is seen that:

- (1) For all the QSOs listed in Burbidge *et al.* (1977), which have  $z_{abs} > z_{em}$ , our models can be applied.
- (2) For  $z_1 = z_s$  (the absorbing shell starts falling from the surface) the absorption occurs in the envelope ( $z_e < z_b$ ) and the velocity of infall  $V_e$  is  $\sim 0.5$  to  $0.6 c$ .
- (3) For  $z_1 = z_b$  the range of  $V_e$  is  $\sim 0.25$  to  $0.35 c$ .

Once  $z_e$  is thus determined we may carry out the consistency analysis similar to that given in Section 3.2, provided relevant observational data are available. But it should be borne in mind that the above treatment is not rigorous as we have neglected the effect of radiation pressure. Hence the results quoted in Table 1 should be taken only as approximate.

## 5 Conclusions

In the present paper we have attempted to provide a satisfactory explanation to the observed absorption redshift systems of the QSOs on the basis of core—envelope-type Hoyle—Fowler models. We have shown that the absorption systems can occur within the object in shells of negligible thicknesses and densities and that such shells can attain positions of stable equilibrium due to a balance between radiation pressure and gravity. In the case of the QSOs, in which the absorption redshift exceeds the emission redshift, an explanation on the basis of collapsing shells, falling towards the centre with speeds within a fraction of speed of light, has been given.

To summarize, in the earlier papers (Papers I, II and IV) we had shown that the emission features and various other physical properties of a high redshift QSO can be satisfactorily explained on the basis of the central gravitational redshift models proposed by us. The present work shows that the observed absorption features can also be adequately explained by these models.

The Redshift Controversy – whether redshifts are cosmological or not – is far from settled to date and continues to be a major problem in astronomy. In this context we feel that the possibility that a major part of the redshift of a high redshift QSO may be gravitational merits serious consideration.

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