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Optical Colours and Polarization of a Model Reflection Nebula. II. Silicate and Graphite Grains in a Nebula with the Star in the Rear

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The colour difference between the star and the attendant reflection nebula and polarization both caused by silicate and graphite grains have been given. The properly normalized size distribution function for each type of grains has been considered within homogeneous plane parallel slab-model of the reflection nebula with the star in the rear. Contrary to some earlier results, it has been possible to show that silicate grains can certainly play a role in the phenomena of reflection nebulae.

Die durch Silikat- und Graphitteilchen verursachte Farbdifferenz zwischen einem Stern und dem zugehörigen Reflexionsnebel sowie die Polarisation werden angegeben. Die geeignet normalisierte Verteilungsfunktion der Teilchengrößen für jeden Partikeltyp wird in einem homogenen planparallelen Schichtmodell eines Reflexionsnebels mit dem leuchtanregenden Stern im Hintergrund diskutiert. Im Gegensatz zu einigen früheren Ergebnissen war es möglich nachzuweisen, daß Silikatteilchen durchaus eine Rolle in den Reflexionsnebeln spielen können.

1. Introduction

In this communication some interesting results on colours and polarization of light from a reflection nebula with the star in the rear have been considered. The submicron sized scattering grains responsible for reflection nebula phenomena are taken to be composed of silicate and graphite materials which are often mentioned in connection with interstellar extinction and polarization and infrared emission and absorption features from certain cosmic objects (see, for example, DORSCHNER 1976, ZELLNER 1973 and SHAH 1976). Although the geometry of the model resembles that of GREENBERG and ROARK (1967), we have treated the analytical and numerical parts somewhat independently as shown earlier by SHAH (1974, hereafter referred to as Paper I). For example, GREENBERG and ROARK (1967) and GREENBERG and HANNER (1970) do not take into account the finite aperture of the telescope. The new expression for the exact nebular scattering volume (Paper I) has been used here; this volume element has some application in the study of X-ray halo around the Crab pulsar, Sco-X1 and other celestial sources. An approximate volume element is untenable in such study. The size distribution function has also been properly normalized so as to render it useful even for mixtures of grains and polydispersions of grains.

Regarding the silicate grains in reflection nebula, HANNER (1971) has concluded that "no match to the observed $(U-B)$ and $(B-V)$ colours in the Merope Nebula could be found by using the MIE scattering functions for silicates and a simple homogeneous geometric model". However, in what follows, it has been shown that it is possible to match the observed $(B-V)$ and $(U-B)$ colours to models with size parameter $a_0 \cong 0.3$ μm and polarization with $a_0 \cong 0.2$ μm both in the case of silicate grains within the reflection nebula with the star in the rear. The difference in values of a_0 for final fits to observations of colours and polarization has been accepted by GREENBERG and HANNER (1970). However, they have considered only pure ice grains with fixed index of refraction. Note that we normally refer to the observations of colours and polarization for the Merope reflection nebula (see, for example, ELVIUS and HALL 1966, 1967).

2. Theoretical consideration

The model of the reflection nebula consists of a plane-parallel homogeneous slab with the illuminating star behind the nebula. The nebula is divided into a number of elementary parallel slabs. The new expression (Paper I) for exact nebular volume element intercepted by the telescope and the elementary slab has been used. The contributions of scattered light towards the observer from all the elementary scattering volumes along the line of sight (or rather a cone of sight) at a given offset angle have been integrated. This involves double integration because of the size distribution function for radii of the grains. The emergent intensity is then integrated further over the range of wavelength corresponding to each filter bandpass. All integrations have been performed using SIMPSON's rule. The intensities in various colour bands have been calculated separately for two orthogonal states of polarization in order to derive the colour excess differences between the star and the nebula as well as the percentage of polarization as function of offset angle. The extinction within the nebula has been considered on the basis of the observations (BOGGESS and BORGMAN 1964); the empirically determined values of extinction, chosen as $A_U = 1.86$, $A_B = 1.66$ and $A_V = 1.2$ mag/pc for U, B and V filter bands, respectively, have been used to evaluate the optical depth for the light trajectory within the nebula. The models thus computed agree well with the corresponding ones using the wavelength dependent extinction curve.

The notation and values of the geometrical parameters are as follows:

T = thickness of the nebula = 1.0 pc,
D = distance of the illuminating star from the observer = 160 pcs,
H = perpendicular distance from the star to the front surface of the nebula = 1.25 pc,
Φ = offset angle,
β = semi-vertex angle of the telescope = 0.0015 radian,
λ = wavelength of light,
a_0 = parameter in the size distribution function,
N_0 = number of grains per cm^3.

The properly normalized size distribution function has the form

$$\mathrm{d}n(a) = \frac{5^{1/3}}{\Gamma(\frac{4}{3})} \frac{N_0}{a_0} \exp\,[-5(a/a_0)^3]\,\mathrm{d}a\,, \tag{1}$$

which gives the relative number of grains per cm^3 in the size range a and $a + \mathrm{d}a$.

The detailed description of definitions and notation is given by GREENBERG and ROARK (1967) and SHAH (1974) Here we recall the $(B-V)$ and $(U-B)$ colour differences between the star and the nebula as defined in terms of magnitudes as follows:

$$(B-V)_{*-N} = (B-V)_* - (B-V)_N\,,$$
$$(U-B)_{*-N} = (U-B)_* - (U-B)_N\,.$$

The exact MIE theory of scattering (MIE 1908, VAN DE HULST 1957, DORSCHNER 1970) has been used throughout for calculating the scattering phase functions. Single scattering has been assumed to hold. The multiple scattering is assumed to play no significant role, although it may be important in case of some optically thick reflection nebulae as, for instance, NGC 7023 (VANÝSEK and SŎLC 1973). A useful numerical technique for MIE theory of scattering by smooth spheres has been summarized in the appendix. It has been useful for the calculations of scattering parameters like extinction, scattering and absorption efficiencies, amplitude scattering functions, radiation pressures etc. for arbitrary size of the sphere and wavelengths of interest in the infrared, optical as well as X-ray astronomies. The corresponding Fortran computer program has been found to be valuable in practice especially for very odd values of complex refractive indices.

3. Calculated colours and polarization for graphite and silicate grains

The observations pertaining to colour difference and polarization of the Merope Nebula have been taken from ELVIUS and HALL (1967) and ROARK (1966). The dashed curves, representing smooth runs through appropriate observations, are drawn in some figures for direct comparison with the models. In what follows all the results are for silicate and graphite grains within the reflection nebula with the star in the rear. The abscissae in all figures represent offset (viewing) angle Φ in minutes of arc measured from the direction to the star. So far no observations of U-band polarization as function of Φ seem to have been reported. However, the calculated results are included for future comparison. The complex index of refraction is denoted by $m = m' - im''$. The representative values of m for silicate are based on work by DORSCHNER (1968, 1971) and HUFFMAN and STAPP (1971). For m of graphite we refer to TAFT and PHILLIP (1965) and FRIEDEMANN and SCHMIDT (1966).

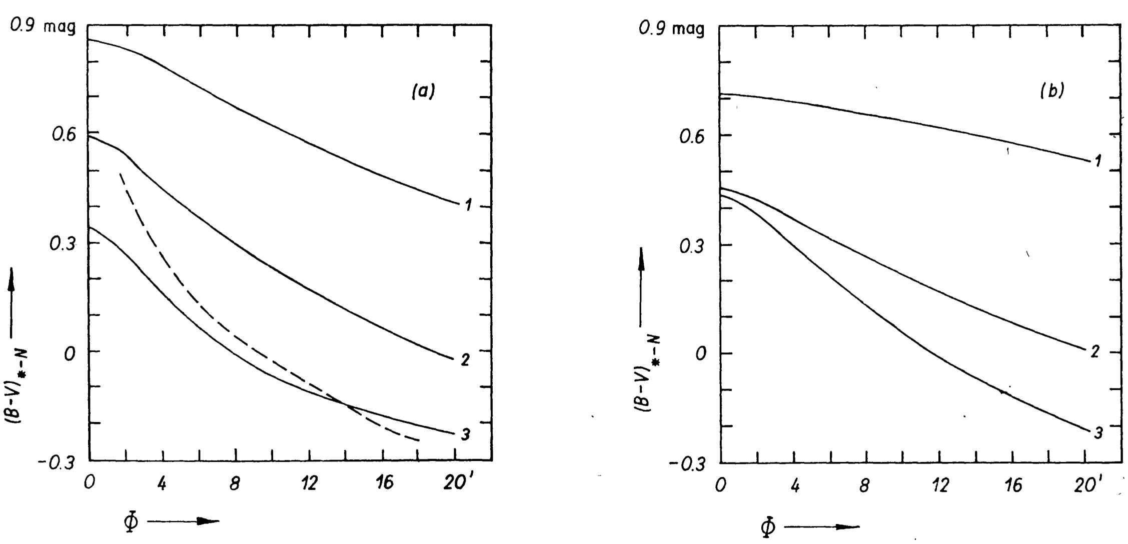

Fig. 1a. Colour difference in $(B-V)$ between star and nebula for dirty silicate grains with refraction index $m = 1.66 - 0.05i$, size parameter $a_0 = 0.2\,\mu$m, 0.3 μm, and 0.4 μm are shown in the curves 1, 2 and 3, respect vely.

Fig. 1b. Same as in Fig. 1a for graphite with $m = 2.46 - 1.45\,i$. The size parameter $a_0 = 0.1\,\mu$m, 0.2 μm and 0.3 μm corresponds to the curves 1, 2 and 3, respectively.

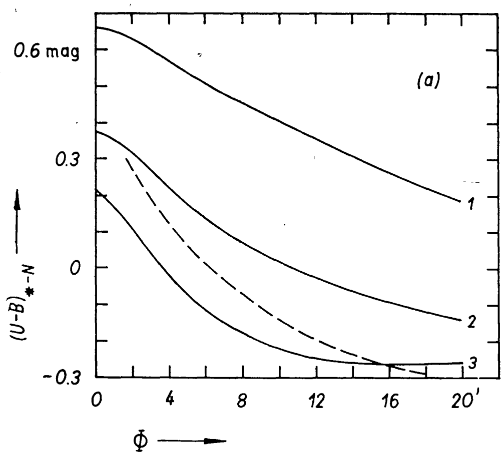

Fig. 2a. Same as in Fig. 1a, except that the ordinate represents the colour difference in $(U-B)$ between star and nebula.

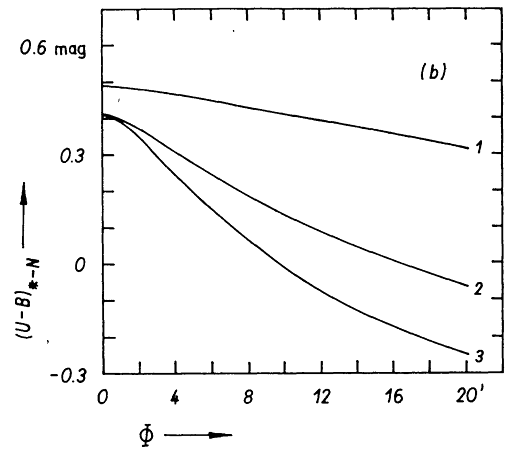

Fig. 2b. Same as in Fig. 1b, except that the ordinate represents the colour difference in $(U-B)$ between star and nebula.

The results on colour differences between the star and the nebula, viz. $(B-V)_{*-N}$ and $(U-B)_{*-N}$ expressed in magnitudes, are shown in Figures 1 and 2, respectively. The figures designated "a" are for silicate ($m = 1.66-0.05i$) and those designated "b" are for graphite ($m = 2.46-1.45i$). In the case of silicate in Figures 1a and 2a, the likely size parameter giving colours nearest to the observations turns out to be $a_0 = 0.4$ μm. Therefore, the conclusion of HANNER (1971) mentioned earlier in the introduction is unwarranted. For graphite also, the $(B-V)$ and $(U-B)$ colour differences shown in Figures 1b and 2b favour a value of a_0 slightly larger than 0.3 μm. The crossover between observational and theoretical curves is though disconcerting. In paper I, a model calculation with $m = 1.3-0.1i$ has been shown to fit observations of colours without such crossover. However, one cannot exclude the silicate and graphite grains at least as a small admixture because some other unforeseen considerations may apply for certain reflection nebulae. For instance, looking at the curves 3 for silicate and graphite, one can say that the observations show steeper slope for small Φ compared to the theoretical models. This holds for dielectric grains as well. It may be due to the presence of a predominantly small sized particle distribution in the vicinity of the star and a larger sized particle distribution in regions away from the star. Therefore, the inner regions where one may find very effective scatterers of blue light, will have positive colours increasing faster as one approaches the star. However, in the outer regions, one will have not only less efficient colour selective scatterers but comparatively larger grains also. The latter

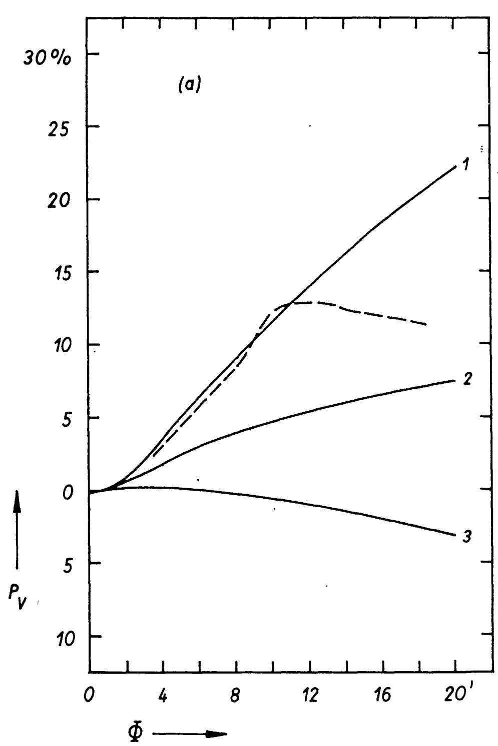

Fig. 3a. Same as in Fig. 1a, but the ordinate represents the polarization P_V.

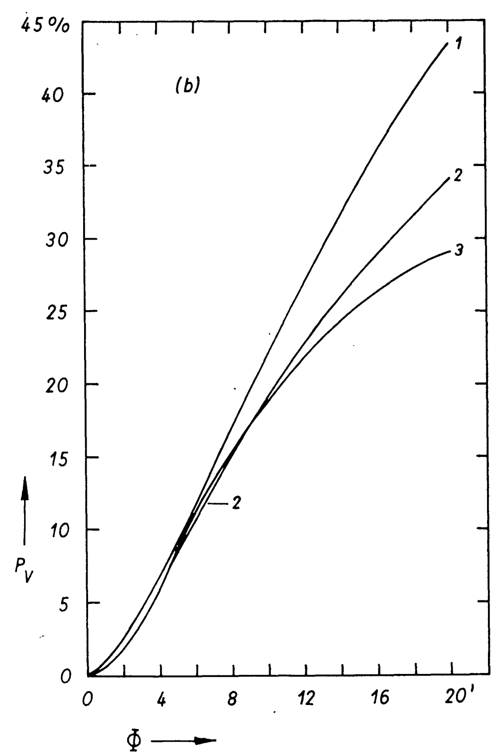

Fig. 3b. Same as in Fig. 1b, but the ordinate represents the polarization P_V.

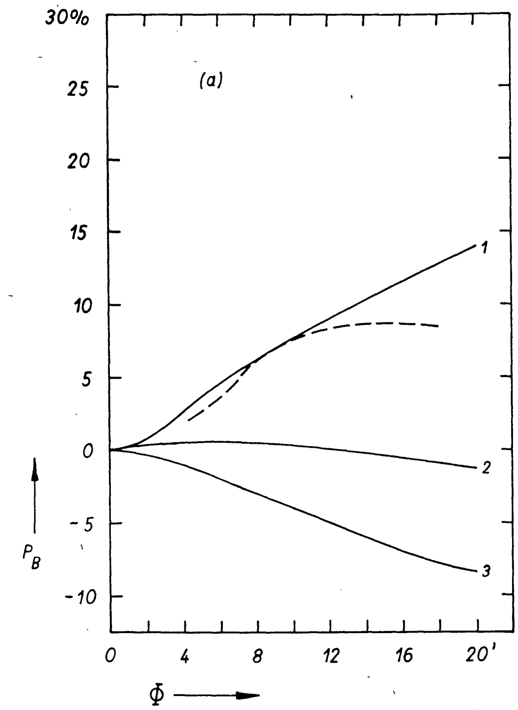

Fig. 4a. Same as in Fig. 1a, but the ordinate represents the polarization P_B.

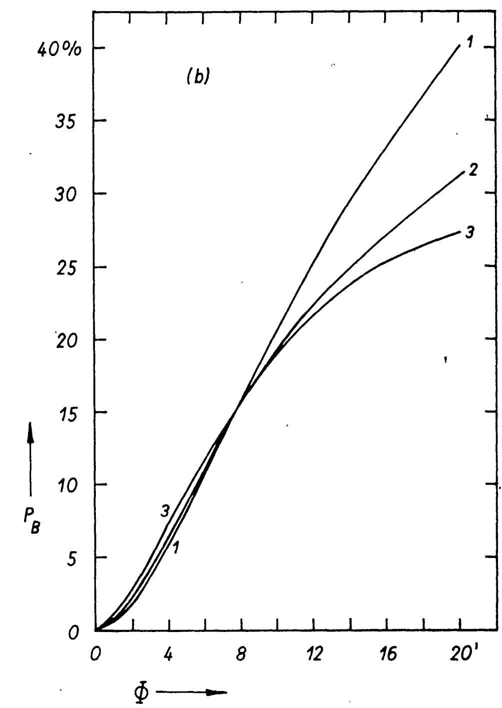

Fig. 4b. Same as in Fig. 1b, but the ordinate represents the polarization P_B.

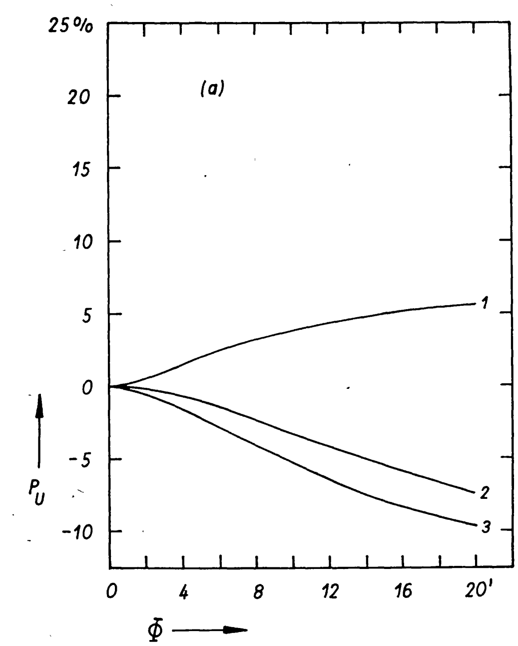

Fig. 5a. Same as in Fig. 1a, but the ordinate represents the polarization P_U.

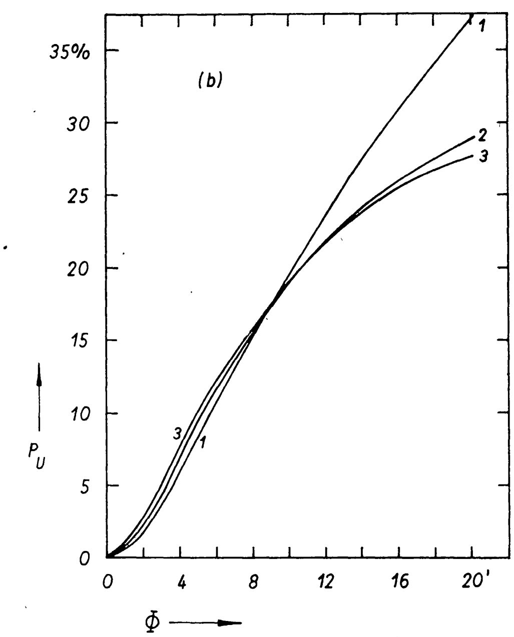

Fig. 5b. Same as in Fig. 1b, but the ordinate represents the polarization P_U.

would act as neutral scatterers. They may also provide thermal emission of radiation mainly in the infrared and far infrared wavelength range. This will cause the slope to slow down and thus produce slowly varying negative colours. This implies that the nebula will appear redder compared to the star as the offset angle becomes sufficiently large assuming, of course, that the grains are present there. The essential point mentioned here is that the particle size distribution may have radial dependence. As one moves away from the neigh-bourhood of the star, the effective size of the grains may attain a certain maximum value in the intermediate region and farther out one may again expect slowly decreasing effective sizes. The latter is important in view of the observational fact that the polarization vectors in the Merope nebula tend to be parallel to the direction of the filaments. It may be noted that the small sized particles can be aligned with ease. These considerations need to be scrutinized in future calculations including nonspherical particles with possible alignment.

The results on polarization by silicate and graphite grains are shown in Figures 3a and b, respectively, for the visual band, V. Similarly, Figures 4a, b and 5a, b are for B and U colour bands, respectively. GREENBERG and HAN-

NER (1970) state that "MIE theory predicts large negative polarization for submicron silicate spheres". Their conclusion is not necessarily true in view of the results set out in Figures 3a, 4a and 5a in the case of silicate grains. In particular, for $a_0 \cong 0.2$ µm, polarization is positive for all offset angles. In the case of silicate grains with $a_0 = 0.2$ µm, the polarization very nearly match with the observations (dashed curves) up to $\Phi = 10$ minutes of arc. But as mentioned above the fit to colours is favoured by the size parameter $a_0 = 0.4$ µm. The graphite grains give far too high a polarization. However, one may wonder if the same grains which produce the colour difference between the star and associated reflection nebula are responsible for polarization as well. The question is: can one accept different values of a_0 for fitting the models to the observations of colours and polarization? The answer lies partly in understanding the role of possible orientation mechanism in the nebula. Another consideration should be the contribution of nonselective scatterers of large sizes which will effectively dilute the degree of polarization produced by a hitherto unknown mixture of interstellar grains in the reflection nebula.

4. Conclusion

We propose to consider other geometric configurations with the star within the nebula and the star in the front. These studies are necessary for drawing reliable conclusions regarding the nature of grains and the geometry involved in various reflection nebulae. Here we summarize in Table 1 the general trends of the variation of colour difference between the star and the nebula as well as polarization as function of the real and imaginary parts of the index of refraction and the size distribution parameter a_0. This table is compiled on the basis of model calculations on dielectric (Paper I), silicate and graphite grains.

Table 1. General trends of nebular colours and polarization

Calculated quantity	Effect of increase in m'	Effect of increase in m''	Effect of increase in a_0
$(B-V)_{*-N}$	↓	↓	↓
$(U-B)_{*-N}$	↓	↓	↓
P_V	↓	↑	↓ **
P_B	↓	↑	↓ **
P_U	↓	↑	↓ **

** At cross-over point this trend is reversed.
↑ = Increase, ↓ = Decrease

Appendix. Numerical technique for MIE theory of scattering of electromagnetic waves by a sphere

The usual expression for the MIE coefficients (see, for example, MIE 1908, DEBYE 1909, VAN DE HULST 1957, DORSCHNER 1970) can be given the following representation:

$$a_n = \frac{A_n(z) - mB_n(x)}{A_n(z)\,G_n(x) - mH_n(x)}, \tag{1}$$

$$b_n = \frac{mA_n(z) - B_n(x)}{mA_n(z)G_n(x) - H_n(x)}, \tag{2}$$

where

$$A_n(z) = \frac{S_{n-1}(z)}{S_n(z)} - \frac{n}{z}, \tag{3}$$

$$B_n(x) = \frac{S_{n-1}(x)}{S_n(x)} - \frac{n}{x}, \tag{4}$$

$$G_n(x) = 1 + i\,\frac{C_n(x)}{S_n(x)}, \tag{5}$$

$$H_n(x) = B_n(x) + i\,\frac{C'_n(x)}{S_n(x)}. \tag{6}$$

The following notation has been used:
size-to-wavelength parameter,

$$x = \frac{2\pi a}{\lambda}, \tag{7}$$

index of refraction,

$$m = m' - im'' \tag{8}$$

$$z = mx, \tag{9}$$

radius of the sphere a,
wavelength of incident and scattered wave λ.

The prime denotes the derivative with respect to the argument concerned. But primes on m, viz. m' and m'', represent real and imaginary parts of refractive index.

$S_n(x)$ and $C_n(x)$ are the RICCATI-BESSEL functions of first kind and second kind, respectively. Their definitions and recursion relations may be found in standard text books (see, for example, WATSON 1964).

The procedure of evaluating higher integral orders of BESSEL functions of the first kind and real argument is discussed by BROUWER and CLEMENCE (1961). It is essentially based on continued fraction applied to downward recursion. It was extended by SHAH (1967) to derive the higher integral order BESSEL functions with complex argument in connection with the scattering by infinite cylinder at oblique incidence. We present here a procedure for simultaneous numerical evaluation of the RICCATI-BESSEL function of the first kind and its logarithmic derivative in a single downward recursion.

It may be mentioned that $A_n(z) = S_n'(z)/S_n(z)$ is a logarithmic derivative first introduced by INFELD (1947). ADEN (1951) used this definition and showed a method of computing it by upward recursion which is not stable for $n > |z|$. KATTAWAR and PLASS (1967) have employed almost the same numerical scheme of ADEN with the difference that they apply downward recursion on one log derivative. However, this does not permit one to evaluate the necessary RICCATI-BESSEL function $S_n(z)$ simultaneously. Another downward recursion would consume extra computer time. Also, they have to perform an additional upward recursion on second log derivative [$G_n(x)$ in their notation] which is a composite function of $S_n(x)$ and $C_n(x)$ both. However, it has been known that for reliable and accurate evaluation of the latter functions, one should perform downward recursion on $S_n(x)$ and an upward one on $C_n(x)$. Therefore, the computation of the composite function of KATTAWAR and PLASS by upward recursion may run into error propagation trouble as some stage of computation especially for large n and x in the geometrical optics region. In the present procedure as shown below, the recursions on such composite functions are not required at all.

It is convenient to define the coefficients P_n and Q_n such that

$$\frac{S_n(z)}{S_{n-1}(z)} = P_n + iQ_n. \tag{10}$$

From standard recursion relation one can obtain

$$\frac{S_n(z)}{S_{n-1}(z)} = \frac{2n-1}{z} - \frac{S_{n-2}(z)}{S_{n-1}(z)}. \tag{11}$$

Using equation (10), this reduced to

$$[P_{n-1} + iQ_{n-1}]^{-1} = [(2n-1)/z] - [P_n + iQ_n] \tag{12}$$

Let

$$y_1 = xm', \quad y_2 = -xm'', \quad y = y_1^2 + y_2^2, \quad z = y_1 + iy_2.$$

Define:

$$\alpha_n = [y_1(2n-1)/y] - P_n, \tag{13}$$

$$\beta_n = [y_2(2n-1)/y] + Q_n. \tag{14}$$

Then one can derive the following recursion relations for the complex argument:

$$P_{n-1} = \alpha_n/(\alpha_n^2 + \beta_n^2), \tag{15}$$

$$Q_{n-1} = \beta_n/(\alpha_n^2 + \beta_n^2). \tag{16}$$

Similarly if

$$R_n = S_n(x)/S_{n-1}(x) \tag{17}$$

the recursion relation for real argument turns out to be

$$R_{n-1} = x/[(2n-1) - xR_n]. \tag{18}$$

Initially n is chosen sufficiently large, say N, so that P_N, Q_N and R_N can be set equal to zero in the first instance. The equations (13), (14), (15), (16) and (18) are used to carry the downward recursions for the complex argument z and the real argument x, respectively. This supplies (P_N, Q_N) and R_N for all orders $n = N-1, N-2, \ldots, 3, 2, 1$ by successive application of the procedure. The initial guess of N, however, may be varied and experimented upon to derive related BESSEL functions, the absolute value of the argument being maintained at the largest value anticipated in practice. N is raised successively until the desired accuracy is achieved. This happens when N higher than a certain value does not affect the accuracy anymore. As a by-product of the coefficients P_n, Q_n and R_n and initial knowledge of $S_0(z)$ and $S_0(x)$, one can evaluate all higher orders of $S_n(z)$ and $S_n(x)$ by using the equations (10) and (17), respectively. Furthermore, the logarithmic derivatives $A_n(z)$ and $B_n(x)$ in equations (3) and (4), respectively, can be expressed directly in terms of the coefficients P_n, Q_n and R_n as follows:

$$A_n(z) = [P_n + iQ_n]^{-1} - (n/z), \tag{19}$$

$$B_n(x) = (1/R_n) - (n/x). \tag{20}$$

Next it remains to compute $C_n(x)$ and $C_n'(x)$ in order to obtain the quantities $G_n(x)$ and $H_n(x)$ in equations (5) and (6), respectively. The usual recourse to upward recursion as in the case of NEUMANN functions has been found appropriate. However, both of these functions can be calculated simultaneously in one upward recursion. The necessary recursion relations, after division by C_{n-1} are

$$C_n/C_{n-1} = [(2n-1)/x] - C_{n-2}/C_{n-1}\,, \tag{21}$$

$$C'_n/C_{n-1} = 1 - (n/x)\,(C_n/C_{n-1})\,. \tag{22}$$

Now by defining

$$\zeta_n = C_{n-1}/C_n\,, \tag{23}$$

equations (21) and (22) reduce to

$$\zeta_n^{-1} = [(2n-1)/x] - \zeta_{n-1}\,, \tag{24}$$

$$C'_n/C_n = 1 - (n/x)\,(\zeta_n^{-1})\,. \tag{25}$$

Initially the functions C_0 and C_1 are calculated from their definitions, viz.

$$C_0(x) = \cos x\,,$$

$$C_1(x) = \frac{\cos x}{x} + \sin x\,,$$

to obtain ζ_1. Now $\zeta_2, \zeta_3 \ldots \zeta_N$ can be calculated successively by equation (24). $C_2, C_3 \ldots C_N$ can be obtained from equation (23). Then $C'_1, C'_2, \ldots, C'_N$ are evaluated from equation (25). Having done this and recalling that $S_n(x)$ was obtained as by-product earlier from equation (17), one can easily compute $G_n(x)$ and $H_n(x)$. This completes the method of computing the MIE coefficients a_n and b_n essential for the calculations of all the scattering parameters connected with the MIE theory.

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