EFFECT OF DYNAMICAL FRICTION ON THE ESCAPE OF A SUPERMASSIVE BLACK HOLE FROM A GALAXY

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Abstract. We have used the impulsive approximation technique to numerically estimate the effect of dynamical friction on the motion of a supermassive black hole (mass $\simeq 10^9 M_\odot$) through a galaxy (mass = $10^{11} M_\odot$) which has recoiled from the center of the latter as a result of anisotropic emission of gravitational radiation or asymmetric plasma emission. We find the effect to be minimal for recoil taking place at a velocity larger than that of escape at the center of the galaxy. There is a certain critical velocity of ejection (slightly larger than the central escape velocity) at which the black hole must be ejected for the recoil to be successful. Otherwise, dynamical friction becomes relatively pronounced and damped oscillatory motion of the black hole in the potential well of the galaxy ensues. The phenomenon of high-velocity recoil although rare, can be astrophysically spectacular in view of the fact that the black hole would carry a substantial amount of gaseous material as well as a very large number of galactic stars. Some recent observations are cited where the recoil phenomenon might be applicable.

1. Introduction

It has been suggested by several authors that ejection of matter from galactic nuclei in the form of compact supermassive bodies: namely, magnetoid, spinar, black hole, and white hole can explain certain high-energy phenomena such as relativistic and non-relativistic jets, radio lobes, and quasar-galaxy associations, etc. (Shklovsky, 1972, 1982; Rees and Saslaw, 1975; Kapoor, 1976; Harrison, 1977; Narlikar, 1984, and references quoted therein). The phenomenon of high-velocity recoil is of astrophysical and stellar dynamical interest in many ways. In this paper, we deal with one aspect of the problem, viz., the ejection of a supermassive black hole (mass $\approx 10^9 M_\odot$) at velocities comparable to that of escape at the center of the ejecting galaxy (mass $\simeq 10^{11} M_\odot$) and its tidal interaction with the latter using the impulsive approximation technique as developed by Alladin (1965) for the study of inter-penetrating collisions of galaxies.

The ejection of the supermassive object is achieved by requiring conservation of linear momentum in most of the processes discussed by the authors cited above. For example, the gravitational sling-shot mechanism is fairly efficient in ejecting supermassive black holes and spinars in singles and in pairs (Valtonen, 1977, 1979). Following Bekenstein (1973), Kapoor (1976) has proposed recoil of a supermassive black hole formed in the nonspherical collapse of a supermassive body in a galactic nucleus as a consequence of anisotropic emission of gravitational radiation. Another scenario for ejection in a similar way invokes supermassive black hole binaries in orbit round each other in some active galactic nuclei proposed to explain the observed bending and apparent precession of radio jets emanating from these objects (Begelman *et al.*, 1980). The binary may form from fragmentation in the collapsing mass caused by rotation or from galactic mergers

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which eventually evolves to the stage when the black holes coalesce into a single black hole. The latter would recoil due to anisotropically emitted gravitational wave burst (Rees referred to in Blandford, 1979; see also Cooperstock, 1977). Ejection may also result from asymmetric accretion onto a supermassive black hole, or asymmetric radiation from a spinar or magnetoid in a galactic nucleus (Shklovsky, 1972; Harrison, 1977). Rees (1982) has mentioned that if one-sidedness of jets of compact radio sources is intrinsic, a flip-flop mechanism is needed to give rise to symmetric double radio-lobes. This may, for instance, be achieved if the central engine has been displaced from the center of the galaxy and oscillates in its potential well. Shklovsky (1982) regards one-sidedness of jets of active galactic nuclei in most cases intrinsic, and interprets these as a relativistic ejection of massive plasma clouds (the plasmoids) from thick accretion disks around supermassive black holes in an asymmetric manner. The black hole recoils in the process. Successive plasmoid ejections build up the recoil to the extent that the accreting black hole can escape from the nucleus.

It is relevant to cite here some observations to which the ejection scenario can be applicable. The deep CCD images of the field around the quasar 3C273 obtained by Tyson *et al.* (1982) reveal an elliptical nebulosity which appears to have an offset location (equivalent to $\simeq 4$ kiloparsecs) from the quasar position. It is less likely to have been caused by the superposition of a galaxy in front, or, by stochastic perturbations of galactic stars in the nucleus of galaxy hosting the quasar (Guzadyan, 1982). Arp *et al.* (1975) have presented evidence for disturbances in the inner isophotes of some galaxies pointing in the direction of quasars they are seen near, which, in the case of Mkn 205, extend down to the nucleus of NGC 4319 (Sulentic, 1983). The disturbances are indicative of gravitational influence of a supermassive body that presumably has been ejected from the center of the galaxy in question. Fricke *et al.* (1983) have recently detected a jet-like feature in the outer regions of Seyfert 1 galaxy Mkn 335, which resembles a weak Seyfert nucleus and could be a consequence of ejection. Wilson *et al.* (1983) have reported the kinematic center of the rotation curve of the Seyfert and X-ray galaxy NGC 2110 displaced by $\simeq 230$ pc from the active nucleus which in their opinion could be orbiting about or oscillating through the kinematic center of the galaxy.

2. The Ejection and Visibility of the Black Hole

Before we present our calculations regarding the tidal interaction between the black hole and the galaxy, we briefly state the visibility of the recoil phenomenon, assuming that a $V\,\mathrm{d}M/\mathrm{d}t$ term does not contribute to the equations of motion. It is, therefore, appropriate to first outline the simple gravitational interaction between the black hole and the galaxy. We regard the latter as a Plummer sphere, a model that has been frequently used to represent density distribution in globular clusters, galaxies, and clusters of galaxies (see, e.g., Toomre, 1977). Except for a high-density cusp in the innermost regions, a general mass and density distribution according to this model is of the form

$$M_1(r) = M_1\left(\frac{r}{\alpha}\right)^3\left[1+\frac{r^2}{\alpha^2}\right]^{-3/2}, \qquad n(r) = n_0\left[1+\frac{r^2}{\alpha^2}\right]^{-5/2}, \tag{1}$$

where $\alpha = (3M_1/4\pi m n_0)^{1/3}$ is the scalelength of the system; m, the mass of a star; and n_0, the central star density. The system, represented by Equation (1), extends to infinity: $M_1(\infty) = M_1$; 99% of its mass lies within 12.96α and 99.63% within 20α. The potential function for the galaxy is

$$\Phi_1 = -\frac{GM_1}{\alpha\left(1 + \frac{r^2}{\alpha^2}\right)^{1/2}}\,, \tag{2}$$

while the internal energy of a Plummer sphere follows from the virial theorem

$$U_1 = \frac{3\pi}{32}\frac{GM_1^2}{\alpha}\,. \tag{3}$$

The supermassive black hole ejected from the center of the galaxy is slowed down by the attraction of a galactic mass $M_1(r)$ according to $\mathrm{d}V'/\mathrm{d}t = -GM(r)/r^2$ so that

$$V'(r) = V_0\left[1 - \frac{V_{\mathrm{esc}}^2(0)}{V_0^2}\left\{\frac{(1 + r^2/\alpha^2)^{1/2} - 1}{(1 + r^2/\alpha^2)^{1/2}}\right\}\right]^{1/2}, \tag{4}$$

where $V_{\mathrm{esc}}(0)$ is the escape velocity at the center of the galaxy (vide Equation (24))

$$V_{\mathrm{esc}}(0) = \left[\frac{2GM_1}{\alpha}\left(1 + \frac{M_2}{M_1}\right)\right]^{1/2}, \tag{5}$$

and M_2 is the mass of the black hole.

Based on some recent numerical estimates, Smarr *et al.* (1983) quote the range of the efficiency of gravitational wave emission in gravitational collapse and black hole merger to be $\varepsilon = \Delta E_{GW}/Mc^2 \lesssim 0.01$–$0.2$. Therefore, in principle, the recoil velocity $V_0 = \varepsilon c \gtrsim 10^3$ km s^{-1}. However, quasi-Newtonian estimates indicate the degree of anisotropy of emission of gravitational radiation to be too small (Bekenstein, 1973; Cooperstock, 1977; Fitchett, 1982) to cause recoils in excess of $\sim 10^2$ km s^{-1}. Perturbation calculations by Oohara and Nakamura (1983) suggest the possibility of somewhat higher velocities. It is thought that exact numerical simulations could enhance these from $\sim 10^2$ km s^{-1} to even $\sim 10^3$ km s^{-1}. Other processes such as those detailed in Harrison (1977), Valtonen (1977), and Shklovsky (1982) can impart recoil velocities of this order and more. In none of these processes would the galaxy recoil in the opposite direction.

The ejection phenomenon can be spectacular if one considers the ejected black hole to be supermassive since it would then be able to carry as well as capture a large number of galactic stars, provided a density cusp in the galactic nucleus has developed prior to the ejection. In such a case, material around the black hole which can be in the form of an accretion disk (or torus), gas clouds and a dense cluster of stars is bound to it within a volume of dimension $\sim 2GM_2/\langle V^2\rangle$, $\langle V^2\rangle$ being the mean square velocity of stars. When the recoil takes place, material within volume of dimension

$r_c \sim 2GM_2/(V_0^2 + \langle V^2 \rangle)$ is carried along. If most of this mass is in stars, their number is roughly

$$N_1 \sim \frac{4\pi}{3} r_c^3 n_0 , \tag{6}$$

which is $\sim 10^8$ if $n_0 \sim 10^5$ pc^{-3}. Mass of the accretion disk (M_g) itself could be of similar order. However, these quantities are model dependent and here we shall assume that $(mN_1 + M_g) < M_2$. In general, the black hole captures galactic stars while on its way out at a rate (Kapoor, 1976)

$$\frac{\mathrm{d}N_2}{\mathrm{d}t} = n(r)V'(r)\sigma_{\mathrm{cap}}(r) , \tag{7}$$

where σ_{cap} refers to the capture cross-section for non-relativistic particles ($m \ll M_2$) falling into the hole from infinity in the hole's frame (Shapiro and Teukolsky, 1983)

$$\sigma_{\mathrm{cap}} = \pi l^2 ; \qquad l = \frac{4GM_2}{V'(r)} , \tag{8}$$

l being the maximum impact parameter for such capture. Integrated, Equation (7) gives

$$N_2 \cong \frac{16\pi G^2 M_2^2 n_0 \alpha}{V_0^2 c^2} I . \tag{9}$$

Most of the captures would be from the nucleus itself. Hence, in the limit $r \ll \alpha$, $I \simeq r_0/\alpha$. If we choose a large central density, $n_0 \sim 10^5$ pc^{-3}, $r_0 \sim r_c \sim 10$ pc, $N_2 \simeq 10^{4-5}$ which is rather small. It may be argued that the black hole should catapult galactic stars, those passing within a distance $\sim r_c$ (but $> l$) of its trajectory, in the direction of its motion in the reference frame of the galaxy. However, it is possible that a star may be captured from within $\sim r_c$ by interacting with collective modes of the cluster around the black hole, rather than requiring just a three-body interaction. Many-body modes of capture may be significant at low velocity – a process intermediate between three-body capture and dynamical friction (W. C. Saslaw, private communication). This could happen particularly when the black hole oscillates through or occupies an orbit in the dense regions of the galaxy. In the present context, however, when the black hole moves beyond the distance $\sim r_0$ itself from the center of the galaxy, the stellar density becomes too low for such captures or accelerations to cause any substantial loss of linear momentum of the hole or lead to any observable effects except than possibly showing up with an anomalous velocity dispersion along the trajectory. Moreover, our calculation suggests the amount of interstellar gas accreted along the trajectory to be $\ll M_2$.

Confined to a region of dimension $\lesssim r_c$ around the black hole, the system is highly compact and includes the gaseous debris left over in the process of collapse and stellar disruptions caused by the strong gravitational wave burst (Lipunov and Sazin, 1982) or, the debris from stellar collisions and coalescences and tidal interactions between stars and the black hole. The gas in its fall onto the black hole would render it luminous. The luminosity could be as low as $\sim 10^{41}$ erg s^{-1} if it is just a bare black hole which accretes gas from interstellar medium while moving through (Shapiro and Teukolsky, 1983) and as high as $\sim 10^{44}$ erg s^{-1} if a star cluster with $N_1 \lesssim 10^8$ accompanies it. Much of the gaseous remnants of the various processes mentioned above would remain bound to the black hole all through since there is virtually no rampressure stripping of the gas accreting onto the black hole during its passage through the interstellar medium of density ρ_I. The momentum transferred per unit time to the gaseous contents of mass M_g of the system around the black hole is $\rho_I V_0^2(\pi r_c^2)$. Stripping takes place when this quantity exceeds the gravitational attraction of the black hole keeping the gaseous contents bound to it:

$$\rho_I V_0^2(\pi r_c^2) > \frac{GM_2}{r_c^2} M_g , \tag{10}$$

where we have neglected the attraction due to the stars bound to the black hole. Writing $M_g = fM_\odot$ ($f > 1$), we have $\rho_I > fM_\odot r_c^{-3} \simeq 10^{-25} f$ g cm^{-3} if $V_0 \sim 10^3$ km s^{-1} and $\simeq 10^{-19} f$ g cm^{-3} if $V_0 \sim 10^4$ km s^{-1} (these values refer to the periphery of the system and if r_c is replaced by l, we get still stringent limits). This is something contrary to the general expectation where ram-pressure stripping becomes more effective with increasing velocity.

3. Energy Transfer from the Object to the Galaxy

In this section we set up the theoretical framework to estimate the transfer of energy from the black hole to the motions of stars in the galaxy. For our purpose, we use the technique employed for a study of the interpenetrating collisions of galaxies in an impulsive approximation (Alladin, 1965). The usefulness of the impulsive approximation is discussed in detail by Ahmed and Alladin (1981) which as a first approximation can give reasonable results even for low velocities (Sastry and Alladin, 1970; Tremaine, 1981).

Treating one of the systems as point mass simplifies this calculation. In what follows, rotation of the galaxy is neglected and motion of the black hole, called the object, along only the z-axis is assumed. When the object is instantaneously at a distance z from the center of the galaxy, their mutual interaction energy is given by

$$W(z) = -\frac{GM_1M_2}{\alpha(1 + r^2/\alpha^2)^{1/2}} = -GM_1M_2\chi(z) . \tag{11}$$

Refer to Figure 1, where we have

$$\mathbf{r}'' = \mathbf{r}' - \mathbf{r} ; \qquad r = (x^2 + y^2 + z^2)^{1/2}$$

and

$$r'' = [(x' - x)^2 + (y' - y)^2 + (z' - z)^2]^{1/2} .$$

The force per unit mass on a representative star P in the galaxy due to the attraction of the object can be written as

$$\mathbf{f}_S = \nabla'' \phi_2(r''), \tag{12}$$

so that $f_{S,\,x-x'} = -GM_2(x - x')/r''^3$, etc. The force per unit mass $\mathbf{f}_G$ on the galaxy due to the attraction of the object is

$$\mathbf{f}_G = -GM_2 \nabla\chi(z). \tag{13}$$

The tidal force per unit mass of the star is written as

$$\mathbf{f}_T = \mathbf{f}_S - \mathbf{f}_G. \tag{14}$$

In the impulsive approximation, the velocity increment induced in the motion of P then amounts to

$$\Delta\mathbf{V}(t) = \int \mathbf{f}_T \,\mathrm{d}t. \tag{15}$$

Since the object moves in the z-direction, $x, y = 0$, and $f_{G,\,x} = f_{G,\,y} = 0$. Hence, the various components of the last equation assume the forms

$$\Delta V_{x'} = GM_2 \int \frac{x'}{r''^3}\,\mathrm{d}t, \tag{16a}$$

$$\Delta V_{y'} = GM_2 \int \frac{y'}{r''^3}\,\mathrm{d}t \tag{16b}$$

and

$$\Delta V_{z'} = \int \left[\frac{GM_2(z' - z)}{r''^3} - f_{G,\,z}\right]\mathrm{d}t. \tag{16c}$$

If V_i and V_f are the initial and final velocities of p (Figure (1)), the change in its kinetic energy per unit mass in the course of the encounter with the object is

$$\tfrac{1}{2}(V_f^2 - V_i^2) = \mathbf{V}_i \cdot \Delta\mathbf{V} + \tfrac{1}{2}(\Delta\mathbf{V})^2. \tag{17}$$

It is reasonable to assume that the stars have random motions in the galaxy and if they do not escape, the mean of the fluctuating term in Equation (17) can be taken to be zero (Ahmed and Alladin, 1981). If we divide the galaxy into a number of concentric shells of stars, each containing X sample stars, then for a shell located at r' the average change in the kinetic energy will be

$$\langle \Delta T_1(r') \rangle = \tfrac{1}{2}\langle (\Delta V(r'))^2 \rangle = \frac{1}{2X} \sum_i^x [(\Delta V_{x'})^2 + (\Delta V_{y'})_i^2 + (\Delta V_{z'})_i^2]. \tag{18}$$

In the impulsive approximation, the change in the kinetic energy equals that in the system's binding energy U; i.e., $\Delta T = \Delta U$. Therefore, a summation over the mass of the

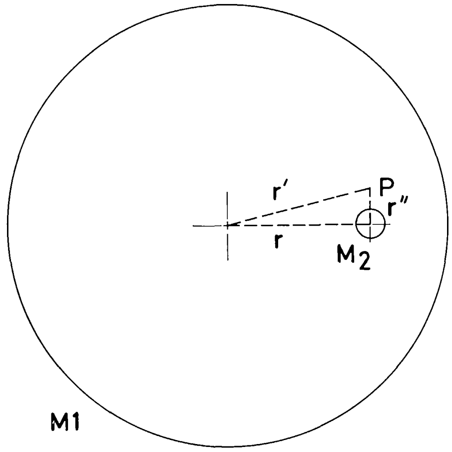

Fig. 1. This shows the instantaneous position $r = z$ of the object with respect to the center of the galaxy, and that of the star P.

galaxy yields

$$\Delta U_1(z) = \int_0^{R_g} \langle \Delta T_1(r') \rangle \frac{\mathrm{d}M_1}{\mathrm{d}r} \, \mathrm{d}r \; ; \tag{19}$$

R_g being the dimension of the galaxy. We write the velocity of the object as

$$V(r) = V(z) = \left\{ \frac{2}{\mu} [E(z) - W(z)] \right\}^{1/2} ; \qquad \mu = \frac{M_1 M_2}{M_1 + M_2} , \tag{20}$$

where $E(z)$ is called the translational energy, given as

$$E(z) = E_i - \Delta U_1(z) \; ; \qquad E_i = \tfrac{1}{2}\mu V_0^2 + W(0) \, . \tag{21}$$

In the impulsive approximation the space derivative of the translational energy is related to deceleration due to the dynamical friction $f_D(z)$ by

$$-f_D(z) = \mu^{-1} \frac{\mathrm{d}E}{\mathrm{d}z} \, . \tag{22}$$

It is convenient to define a velocity function

$$F(z) = \frac{V_{\mathrm{esc}}(z)}{V(z)} = \left(1 - \frac{E(z)}{W(z)} \right)^{-1/2} , \tag{23a}$$

where the escape velocity $V_{\mathrm{esc}}(z)$, is given by

$$\tfrac{1}{2}\mu V_{\mathrm{esc}}^2(z) + W(z) = 0 \, . \tag{24}$$

If $F(z) < 1$ for $0 < z \leq R_g$, the object escapes (successful recoil); otherwise it falls back, retracing its path when $F(z) \to \infty$ (failed recoil). The impulsive approximation can be considered applicable as long as $F(z)$ remains $\lesssim 1$. For comparison, we use Equation (4) to define the velocity function with dynamical friction neglected:

$$F'(z) = \frac{V_{\rm esc}}{V'(z)} . \tag{23b}$$

4. Results and Discussion

We have carried out computations to estimate the change in the internal energy of the galaxy ΔU_1 as a function of the separation between the galaxy and the object, for various velocities of ejection. The main aim is to see the effect of the force of dynamical friction on the motion of the object ejected at a velocity close to that of escape at the center of the galaxy, $V_{\rm esc}(0)$.

The galaxy is divided into twenty shells characterized by radii a such that $\alpha \le a \le 20\alpha = R_g$. Each shell is defined by $X = 14$ stars which are located at various points on it in the following manner: the stars numbered 1 to 6 are placed on the axes of the Cartesian coordinates. Those numbered 7 to 14 are placed on the centers of octans of the sphere of radius a. These positions are given in Table I. Thus taking 280 stars we believe that the galaxy is fairly well approximated. If we express the velocity in units of 1000 km s^{-1}, the calculations for $V(z)$, $\Delta U_1(z)$ and $f_D(z)$ have been performed for $M_1 = 10^{11}\,M_\odot$, $M_2 = 1.1 \times 10^9\,M_\odot$, $\alpha = 2.5$ kpc, and for $V_0 = 0.60$, 0.62, 0.65, 0.70, and 0.80. Note that $V_{\rm esc}(0) = 0.59$.

Starting with $z = 0.05\alpha$, we set $\Delta U_1 = 0$ and calculate velocity increments $\Delta V_{x'}$, $\Delta V_{y'}$, and $\Delta V_{z'}$ for all the stars on a shell for a given value of V_0, which gives $\langle \Delta T(a) \rangle$.

TABLE I

Positions of stars on a shell of radius a

Star no.	x'	y'	z'
1	0	0	$+a$
2	0	0	$-a$
3	0	$+a$	0
4	0	$-a$	0
5	$+a$	0	0
6	$-a$	0	0
7	$+a/\sqrt{3}$	$+a/\sqrt{3}$	$+a/\sqrt{3}$
8	$+a/\sqrt{3}$	$-a/\sqrt{3}$	$+a/\sqrt{3}$
9	$-a/\sqrt{3}$	$+a/\sqrt{3}$	$+a/\sqrt{3}$
10	$-a/\sqrt{3}$	$-a/\sqrt{3}$	$+a/\sqrt{3}$
11	$-a/\sqrt{3}$	$-a/\sqrt{3}$	$-a/\sqrt{3}$
12	$+a/\sqrt{3}$	$-a/\sqrt{3}$	$-a/\sqrt{3}$
13	$-a/\sqrt{3}$	$+a/\sqrt{3}$	$-a/\sqrt{3}$
14	$+a/\sqrt{3}$	$+a/\sqrt{3}$	$-a/\sqrt{3}$

Integrating over the mass of the galaxy, Equation (19) then evaluates $\Delta U_1(z)$. With these values we obtain the velocity $V(z)$ of the object using Equation (20). An iteration is performed till a converged value of $\Delta U_1(z)$ is obtained. The process is repeated for several values of z to trace out the trajectory of the object through the galaxy for various velocities of ejection.

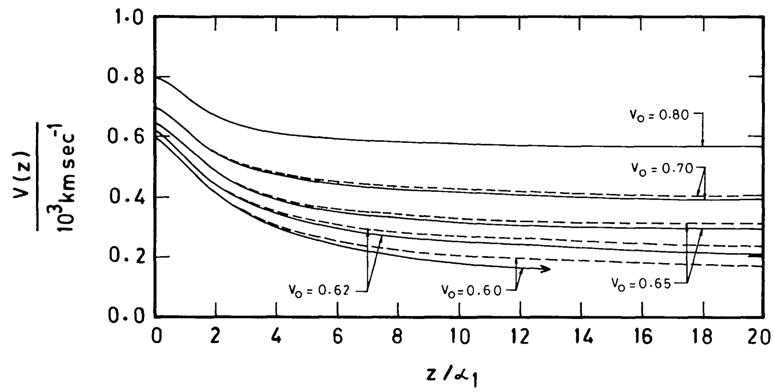

Fig. 2. This shows the decrease in velocity of the object with distance for various velocities of ejection. Solid lines take into account the dynamical friction while dotted lines do not. The velocities are in units of 1000 km s^{-1}.

In Figure 2, velocity of the object, corrected for $\Delta U_1(z)$, is plotted against z. For the sake of comparison we have plotted $V'(z)$ also (dotted lines). The effect of dynamical friction can be seen to be very small, and dwindling, as the ejection velocity is increased, to the extent that for $V_0 = 0.80$, the dotted and the solid lines overlap. The arrow in the $V_0 = 0.60$ curve implies that the iterations become inaccurate beyond this point. The

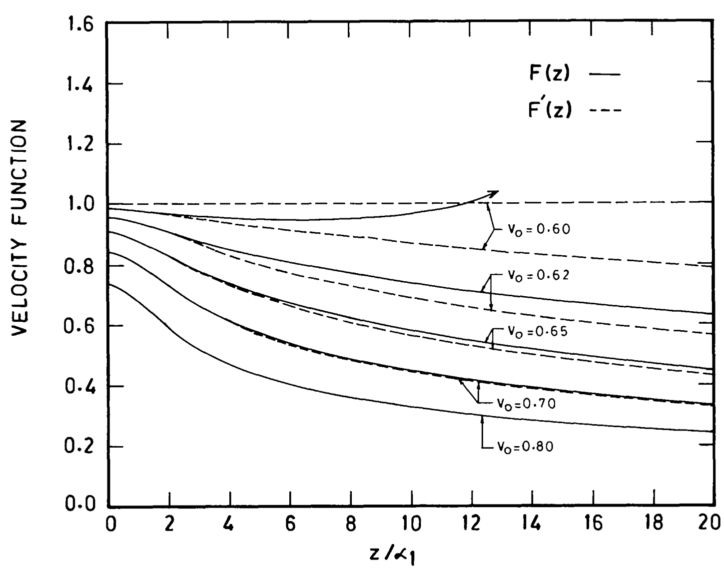

Fig. 3. This shows the behaviour of the velocity function as a function of separation z for various velocities of ejection. Upward bend in the $V_0 = 0.60$ curve implies a failed recoil. Solid lines take into account the dynamical friction but dotted lines do not.

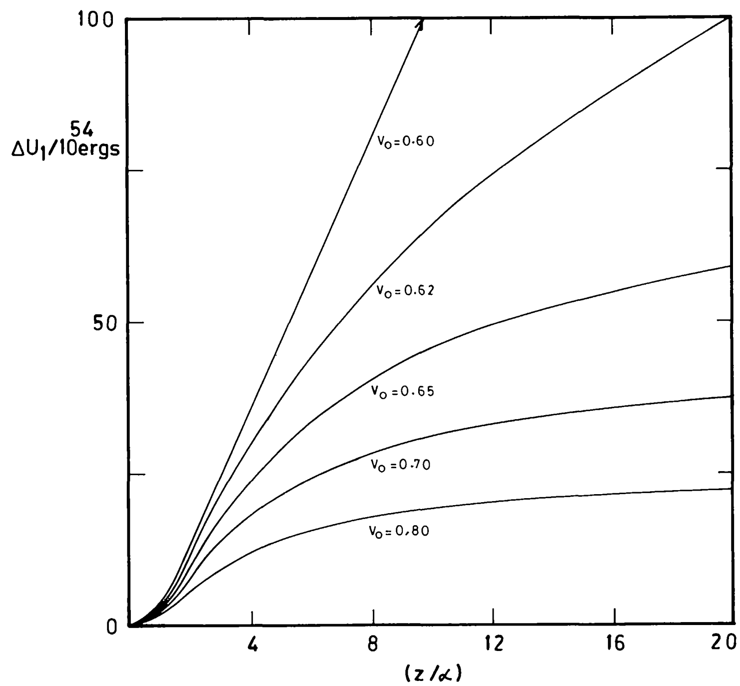

Fig. 4. This shows change in the internal energy of the galaxy for various velocities of ejection as a result of tidal influence of the ejected object.

calculations suggest that a recoil must take place at a velocity V_0 slightly exceeding $V_{esc}(0)$ in order to be a successful one. For the sake of convenience, we shall call it V_{crit}. A recoil taking place at $V_0 < V_{crit}$ is arrested. This is due to the effect of dynamical friction (however small) which is more clearly depicted by the behaviour of the $V_0 = 0.60$ curves in the Figure 3 for the velocity function $F(z)$ vs z where upward bending of the curve above the $F(z) = 1$ line implies reversal of motion to ensue. Dotted lines are $F'(z)$ curves. For $V_0 = 0.80$, the curves overlap here also. For the parameters we have chosen, $V_{crit} = 1.01\ V_{esc}(0)$.

Starting from zero at the center, ΔU_1 increases in general as $V(z)$ gets smaller and smaller and tends to blow up for $V_0 = V_{crit}$ when $F(z) \gtrsim 1$. This is shown in Figure 4. In Figure 5, we have plotted the force of dynamical friction vs z for various velocities of ejection. It is zero to start with ($\Delta U_1(0) = 0$), increasing fast initially, and then it decreases for $V_0 > V_{crit}$. In fact, the larger the velocity of ejection, the smaller is the value of $-f_D(z)$. Only for $V_0 \sim V_{crit}$, does it blow up substantially as $F(z) \to 1$. It is to be noted that choosing a smaller value for α upscales $V_{esc}(0)$ and $W(z)$. Still the condition $V_0 \geq V_{crit}$ for a successful recoil will hold. On the other hand, if we increase M_2 slightly, the value of V_{crit} will get larger than 1.01 $V_{esc}(0)$, since $\Delta U_1 \sim \langle (\Delta V)^2 \rangle$. The value of V_{crit} increases also when one takes into account the extended nature of the object, however compact from galactic standards. The object, for the sake of illustration, is regarded as an isothermal sphere of 10^8 stars (mass = $10^8\ M_\odot$) around the black hole (mass = $10^9\ M_\odot$), i.e., a loaded polytrope of Huntley and Saslaw (1975), which we truncate at $\lesssim r_c$. In such a case a quantity $\Delta U_2(z)$ for the increase in the internal energy of this system due to tidal influence of the galaxy is computed and used to correct Equation (20). The value of ΔU_2 at a given value of z turns out to be comparable to ΔU_1 and, together they increase dynamical friction and V_{crit} to 1.1 $V_{esc}(0)$.

For recoil velocities $< V_{crit}$, the black hole falls back and should execute oscillatory motion which may last for a fairly large number of oscillations before it could eventually settle in the center of the galaxy due to dynamical friction. However, the motion would not be simple rectilinear; the rotation of the galaxy and the coherent scattering off the fluctuations in the stellar density produced by the massive object in its wake can progressively bend its trajectory (Saslaw, 1975) when $V(z)$ falls below $\langle V^2 \rangle$ and eventually place it into a circular orbit. This aspect of the problem is somewhat beyond the scope of the present study and requires numerical simulations to get a correct picture.

In the case of $M_2 \gtrsim 10^{10} M_\odot$ and $V_0 \sim V_{crit}$, tidal influence of the ejected object on the galaxy would be noticeable. But the question is whether a galaxy would eject 10% of its mass in condensed form. On the other hand, while ejection of a black hole of mass $10^8 M_\odot$ is quite likely, the force of dynamical friction would be lesser by two orders of magnitude in comparison with that on a $10^9 M_\odot$ object. It is interesting to note that the velocity increments for stars in the galaxy ($10^{11} M_\odot$) are so small that even an object of mass $10^9 M_\odot$ ($V_0 \gtrsim V_{crit}$) is unable to pull substantial amount of galactic material in its wake. Instead its track would be delineated by a very faint perturbation in the distribution of stars and gas in the wake, ionizing effects on the insterstellar medium,

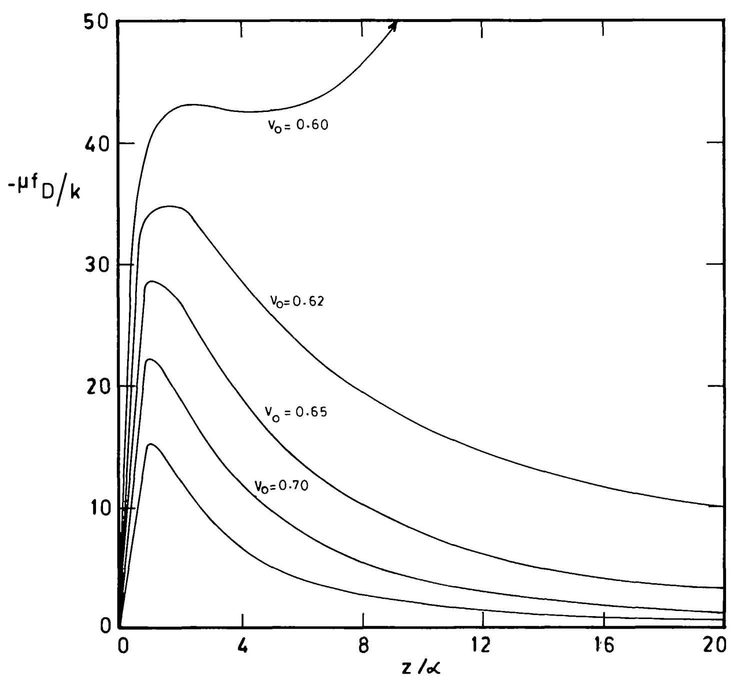

Fig. 5. The general effect of the force of dynamical friction on the motion of the object is shown as its distance varies from the center of the galaxy, for various velocities of ejection; the quantity $k = 1.41 \times 10^{31}$ dynes.

bursts of star formation, and hot spots (plasma spewed out at large velocities from the object) giving rise to a blue gradient steepening towards the ejected object.

To sum up, in this paper we have made a preliminary study of the tidal interaction of a supermassive black hole with a galaxy which has been ejected from the center of the latter. Our numerical study is general in the sense that it would apply to a supermassive object as long as it can be treated as a point mass. The calculations are based on the impulsive approximation which can be taken to be valid as long as $F(z) \lesssim 1$. We find that for velocities of ejection exceeding a certain critical velocity V_{crit}, which is slightly larger than that of escape at the center of the galaxy $V_{esc}(0)$, the black hole escapes. The force of dynamical friction has virtually no effect on the motion and whatever is there decreases as ejection velocity is increased. The effect of dynamical friction increases mildly if one replaces the point mass by an extended configuration of the same mass although highly compact from galactic standards. The quantity $\Delta U_1/U_1$ in any case remains $\ll 1$ implying virtually no damage to the galaxy. The ejection phenomenon may be spectacular since the black hole would carry enough fuel to be 'visible' provided a high density cusp in the galactic nucleus has developed prior to the ejection.

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References

Ahmed, F. and Alladin, S. M.: 1981, *Bull. Astron. Soc. India* **9**, 40.
Alladin, S. M.: 1965, *Astrophys. J.* **141**, 768.
Arp, H. C., Pratt, N. M., and Sulentic, J. W.: 1975, *Astrophys. J.* **199**, 565.
Begelman, M. C., Blandford, R. D., and Rees, M. J.: 1980, *Nature* **287**, 307.
Bekenstein, J. D.: 1973, *Astrophys. J.* **183**, 657.
Blandford, R. D.: 1979, in L. Smarr (ed.), *Sources of Gravitational Radiation*, Cambridge Univ. Press, Cambridge, p. 191.
Cooperstock, F. I.: 1977, *Astrophys. J.* **213**, 250.
Fitchett, M. J.: 1982, *Monthly Notices Roy. Astron. Soc.* **203**, 1049.
Fricke, K. J., Kollatschny, W., and Schleicher, H.: 1983, in A. Ferrari and A. G. Pacholczyk (eds.), *Astrophysical Jets*, D. Reidel Publ. Co., Dordrecht, Holland, p. 149.
Gurzadyan, V. G.: 1982, *Astron. Astrophys.* **114**, 71.
Harrison, E. R.: 1977, *Astrophys. J.* **213**, 827.
Huntley, J. M. and Saslaw, W. C.: 1975, *Astrophys. J.* **199**, 328.
Kapoor, R. C.: 1976, *Pramana* **7**, 334.
Lipunov, V. M. and Sazin, M. V.: 1982, *Soviet Astron. Letters* **8**, 350.
Narlikar, J. V.: 1984, *Bull. Astron. Soc. India* **12**, 1.
Oohara, K. and Nakamura, T.: 1983, *Prog. Theor. Phys.* **70**, 757.
Rees, M. J.: 1982, in D. S. Heeschen (ed.), 'Extragalactic Radio Sources', *IAU Symp.* **97**, 211.

Rees, M. J. and Saslaw, W. C.: 1975, *Monthly Notices Roy. Astron. Soc.* **171**, 53.
Saslaw, W. C.: 1975, *Astrophys. J.* **195**, 773.
Sastry, K. S. and Alladin, S. M.: 1970, *Astrophys. Space Sci.* **7**, 261.
Shapiro, S. L. and Teukolsky, S. A.: 1983, *Black Holes, White Dwarfs, and Neutron Stars*, John Wiley and Sons Inc., New York, pp. 349, 420.
Shklovsky, I. S.: 1972, in D. S. Evans (ed.), 'External Galaxies and Quasi-Stellar Objects', *IAU Symp.* **44**, 272.
Shklovsky, I. S.: 1982, in D. S. Heeschen (ed.), 'Extragalactic Radio Sources', *IAU Symp.* **97**, 475.
Smarr, L., Vessot, R. F. C., Lundquist, C. A., Decher, R., and Piran, T.: 1983, *Gen. Rel. Grav.* **15**, 129.
Sulentic, J. W.: 1983, *Astrophys. J.* **265**, L49.
Toomre, A.: 1977, in H. M. Tinsley and R. B. Larson (eds.), *The Evolution of Galaxies and Stellar Populations*, Yale Univ. Obs., Connecticut, p. 401.
Tremaine, S. J.: 1981, in S. M. Fall and D. Lynden-Bell (eds.), *The Structure and Evolution of Normal Galaxies*, p. 67.
Tyson, J. A., Baum, W. A., and Kreidl, T.: 1982, *Astrophys. J.* **257**, L1.
Valtonen, M. J.: 1977, *Astrophys. J.* **213**, 356.
Valtonen, M. J.: 1979, *Astrophys. J.* **227**, L79.
Wilson, A. S., Baldwin, J. A., Ward, M. J., and Ulvestad, J. S.: 1983, *Bull. Am. Astron. Soc.* **15**, 988.