DYNAMICAL FRICTION AND THE ESCAPE OF A COMPACT SUPERMASSIVE OBJECT SHOT FROM THE CENTER OF A GALAXY

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Abstract. A numerical study has been made of the motion of a compact object consisting of a supermassive black hole with a dense cluster of stars around through a galaxy which has recoiled from the center of the latter as a result of anisotropic emission of gravitational radiation or asymmetrical plasma emission. We find that the effect of dynamical friction on its motion through the galaxy (mass $\simeq 10^{11}\, M_\odot$) estimated using the impulsive approximation technique, is minimal for an object mass $\simeq 10^9\, M_\odot$ and for recoil taking place at a velocity larger than that of escape. A velocity $\simeq 1.1$ times the escape velocity is needed for the object to escape from the galaxy, whereas for velocities of recoil less than this critical velocity, damped oscillatory motion ensures. The energy exchange of the object with the galaxy is not large enough to cause appreciable change in the internal energy of the latter.

1. Introduction

Various arguments suggest that supermassive black holes should exist in the nuclei of galaxies that have undergone a violent active phase in their life (see Rees, 1984, for a review). The possibility of displacement or ejection of such a black hole from the galaxy's centre has been suggested by many workers (see Kapoor, 1985a, for references). Such a possibility is relevant to models of quasars which are thought to be extreme examples of active galactic nuclei, and jets, because there are several ways in which a supermassive black hole could acquire a high velocity (via gravitational wave recoil, one-sided jets, etc.). One then would like to know whether they then escape from the galaxy, or undergo damped oscillatory motion even if ejected at velocities large compared to that of escape, or are placed in a spiralling orbit, to eventually settle down in the center of the galaxy.

In previous papers (Kapoor, 1985a, b) we have dealt with one aspect of the problem, namely the ejection of a supermassive black hole ($\simeq 10^9\, M_\odot$) at velocities comparable to that of escape at the center of the ejecting galaxy (mass $\simeq 10^{11}\, M_\odot$) and its tidal interaction with the latter using the impulsive approximation technique as developed by Alladin (1965) for the study of interpenetrating collisions of galaxies. In this study, the system was idealized as a point mass interacting with the galaxy. It is, however, likely that the black hole would be surrounded by a compact cluster of a large number of stars, provided a high density cusp in the galactic nucleus has developed before the ejecting mechanism could become operative to displace the object from the center of the galaxy. In this paper, we present results of a numerical study of the tidal interaction which incorporates the structural details of the stellar system around the black hole. These calculations might have bearing on a number of observations where movement or

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displacement of the central engine is suggested, such as those relating to 3C 84 (Readhead *et al.*, 1983), Mkn 335 (Fricke *et al.*, 1983) and Mkn 205–NGC 4319 (Sulentic, 1983).

2. Tidal Interaction Between the Object and the Galaxy

Most of the hypotheses proposing ejection of the supermassive central component of the galaxy achieve this by requiring conservation of linear momentum. Given this, the tidal interaction between the ejecting galaxy and the ejected object can be studied using the impulsive approximation. Though reasonable, the limitation of the approximation restricts us to velocities of ejection (V_0) of the order of or more than the velocity of escape at the centre of the galaxy $V_{esc}(0)$. Large velocities of interest here would be possible only in an extreme situation.

The galaxy is represented by a Plummer sphere with a density distribution of the form

$$n_1(r) = n_0\left[1 + \frac{r^2}{\alpha_1^2}\right]^{-5/2}, \tag{1}$$

where $\alpha_1 = (3M_1/4\pi m n_0)$ is the scale length of the system. The potential function for the galaxy is

$$\phi_1(r) = -\frac{GM_1}{\alpha_1}\left(1 + \frac{r^2}{\alpha_1^2}\right)^{-1/2}, \tag{2}$$

with an internal energy

$$U_1 = -\frac{3\pi}{32}\frac{GM_1^2}{\alpha_1}. \tag{3}$$

It is necessary to specify the density distribution in the object, i.e., in the star system around the black hole. A reasonable representation for the density distribution in the system is an isothermal distribution of stars, each of mass m, which is loaded with a Newtonian singularity of mass M_2 (Huntley and Saslaw, 1975) and which we truncate at the accretion radius r_a of the black hole

$$\rho(r) = \lambda \exp -\left[\frac{\beta\alpha_2(r - r_{min})}{r r_{min}}\right],$$
$$r_{min} \le r \le r_a = \frac{2GM_2}{\langle V^2 \rangle}. \tag{4}$$

In Equation (4), λ is the central density; α_2, the scale length; and r_{min} is a certain inner radius of the stellar distribution where for $r < r_{min}$, the distribution of stellar velocities is highly anisotropic and not represented by a scalar pressure. For $r > r_{min}$ the velocities become more nearly isotropic and are closer to virial equilibrium. Following Huntley and Saslaw (1975), $r_{\min}$ is taken as a radius that encloses 1000 stars – i.e.,

$$r_{\min} = \left(\frac{1000\ \mathrm{m}}{(4\pi/3)\lambda}\right)^{1/3} . \tag{5}$$

The parameter β is given by

$$\beta = \frac{M_2}{4\pi\lambda\alpha_2^3} , \tag{6}$$

such that $\beta\alpha_2/r_{\min} = 3$ for loaded isothermal spheres. The mass distribution of the system is then

$$M_t(r) = M_2 + 4\pi \int_{r_{\min}}^{r} r^2\rho(r)\,\mathrm{d}r . \tag{7}$$

The potential function corresponding to this distribution is given

$$\phi_2(r) = -\left[\frac{GM_2}{r} + \frac{AG}{\alpha_2^3 r}\int_{r_{\min}}^{r} r^2\, e^{\beta\alpha_2/r}\,\mathrm{d}r + \frac{AG}{\alpha_3^2}\int_{r_{\min}}^{r} r\, e^{\beta\alpha_2/r}\,\mathrm{d}r\right] \tag{8}$$

if $r < r_a$, and

$$\phi_2(r) = -\frac{GM_t}{r} \tag{9}$$

if $r \geq r_a$. The quantity $A = 4\pi\lambda\alpha_2^3\, e^{-\beta\alpha_2/r_{\min}}$. The internal energy of the object can be written as

$$-U_2 = U_{\mathrm{BH}} + U_{\mathrm{star}} = M_2c^2 + \frac{1}{2}\int_{r_{\min}}^{r_a} \phi_2(r)\,\mathrm{d}M_t \simeq M_2c^2 . \tag{10}$$

In what follows, rotation of the system is neglected and motion of the object along only the z axis is assumed. When the object is at a distance z from the center of the galaxy, their mutual interaction energy is given by

$$W(z) = \int \phi_1(r')\,\mathrm{d}M_t(r'') , \tag{11}$$

where $r' = (r''^2 + z^2 - 2zr''\cos\theta'')^{1/2}$ and $\phi_1(r')$ represents the potential due to the galaxy at a star at P, a distance r' from its center, and $\mathrm{d}M_t$ is the mass element of the object at the point (Figure 1). In spherical polar coordinates r'', θ'', φ'' referring to the object, we have

$$W(z) = -GM_1M_2\chi(z) , \tag{12}$$

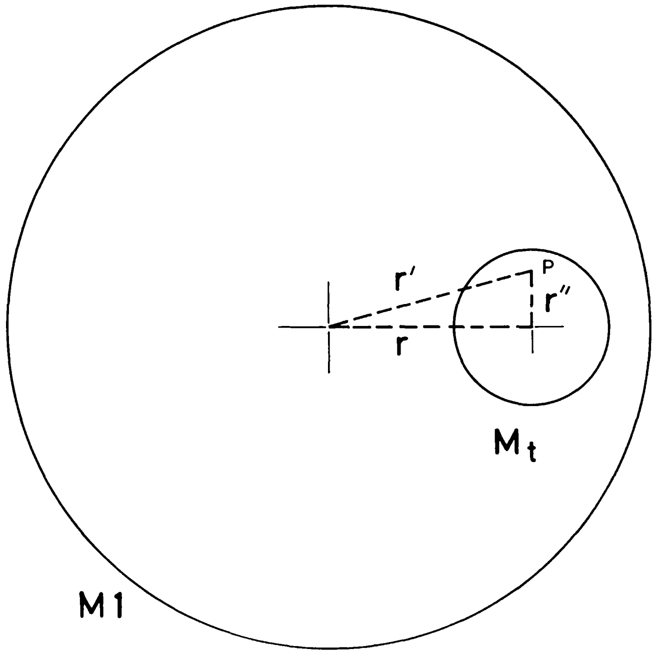

Fig. 1. This shows the instantaneous position $r = z$ of the object with respect to the center of the galaxy, and that of the star P.

where the function $\chi(z)$ is written as

$$\chi(z) = \frac{1}{(z^2 + \alpha_1^2)^{1/2}} + \frac{A}{\alpha_2^3 M_2 z} \int r'' \, e^{\beta\alpha_2/r''} \, \Lambda(r'', z) \, \mathrm{d}r'' \tag{13}$$

and

$$\Lambda(r'', z) = (r''^2 + z^2 + 2r''z + \alpha_1^2)^{1/2} - (r''^2 + z^2 - 2r''z + \alpha_1^2)^{1/2} \, . \tag{14}$$

When $z = 0$, we have $r' = r''$ and the form of the function in Equation (12) is

$$\chi(0) = \frac{1}{\alpha_1} + \frac{A}{\alpha_2^3 M_2} \int_{r_{\min}}^{r_a} \frac{r''^2 \, e^{\beta\alpha_2/r''}}{(r''^2 + \alpha_1^2)^{1/2}} \, \mathrm{d}r'' \, . \tag{15}$$

In Figure 2, we have plotted the interaction energy vs separation which just describes the behaviour of the function in Equation (13).

If we refer to Figure 1 where we have $\mathbf{r}'' = \mathbf{r}' - \mathbf{r}$, $\mathbf{r} = z$, and $r'' = (x'^2 + y'^2 + (z' - z)^2)^{1/2}$, the force per unit mass on P in the galaxy due to the attraction of the object can be written as

$$\mathbf{f}_s = \nabla'' \phi_2(r'') \, . \tag{16}$$

Hence,

$$f_{s,\, x-x'} = - \frac{GM_t(r'')}{r''^3} (x - x') \bigg|_{r'' < r_a} \tag{17a}$$

$$= - \frac{GM_t}{r''^3} \bigg|_{r'' \geq r_a} ; \tag{17b}$$

etc.

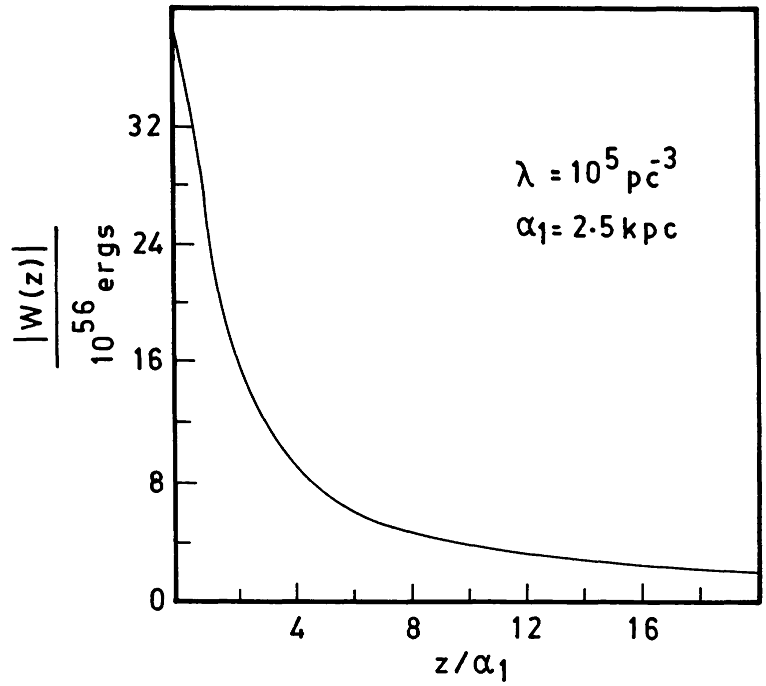

Fig. 2. This depicts the variation in the interaction energy with separation z.

The force per unit mass f_G on the galaxy due to the attraction of the object is

$$\mathbf{f}_G = \frac{1}{M_1} \nabla W(z) . \tag{18}$$

The tidal force per unit mass of the star is, therefore,

$$\mathbf{f}_T = \mathbf{f}_S - \mathbf{f}_G ; \tag{19}$$

so that, in the impulsive approximation, the velocity increment of a galactic star will be

$$\Delta \mathbf{V}(t) = \int \mathbf{f}_T \, \mathrm{d}t . \tag{20}$$

If the stars in the galaxy (as well as those in the system around the black hole) have random motion, the quantity $(\Delta \mathbf{V})^2$ can be taken to be a measure of the mean change in the kinetic energy $\langle \Delta T_1(r') \rangle$ of a specified number of galactic stars located on a shell of radius r'. In the impulsive approximation, the change in the kinetic energy equals that in the binding energy U, i.e. $\Delta T = \Delta U$. Therefore, a summation over the mass of the galaxy yields

$$\Delta U_1(z) = \int_0^{R_g} \langle \Delta T_1(r') \rangle \frac{\mathrm{d}M_1}{\mathrm{d}r'} \, \mathrm{d}r' ; \tag{21}$$

R_g being the dimension of the galaxy.

Next we must set up a similar formalism for the object in motion. Here

$$\mathbf{f}_T = \nabla' \phi_1(r') - \frac{1}{M_t} \nabla W(z), \tag{22}$$

so that $(\Delta \mathbf{V})^2$ can be estimated in this case too. Referring to Equation (7), we have the change in the binding energy of the star system around the black hole as

$$\Delta U_2(z) = \frac{A}{\alpha_2^3} \int \langle \Delta T_2(r'') \rangle \, r''^2 \, e^{\beta\alpha_2/r''} \, \mathrm{d}r'' , \tag{23}$$

where $\langle \Delta T_2(r'') \rangle$ refers to the mean change in the kinetic energy of a specified number of stars located on a shell of radius r'' around the black hole. The first term in Equation (7) is absent in the last equation here since the black hole is not tidally affected by the galaxy. The velocity of the object is given as

$$V(r) = V(z) = \left[\frac{2}{\mu} \{E(z) - W(z)\}\right]^{1/2}, \qquad \mu = \frac{M_1 M_t}{M_1 + M_t}, \tag{24}$$

where $E(z)$, the translational energy is given as

$$E(z) = E_i - (\Delta U_1(z) + \Delta U_2(z)), \qquad E_i = \tfrac{1}{2}\mu V_0^2 + W(0). \tag{25}$$

In the impulsive approximation the space derivative of the translational energy is related to deceleration due to the dynamical friction $f_{D,z}$ in the following manner

$$-f_{D,z} = \frac{1}{\mu} \frac{\mathrm{d}E}{\mathrm{d}z}. \tag{26}$$

We define the escape velocity as

$$\tfrac{1}{2}\mu V_{\mathrm{esc}}^2(z) + W(z) = 0. \tag{27}$$

A convenient way of demonstrating the dynamical frictional effect on the velocity of the object is through a velocity function

$$F(z) = \frac{V_{\mathrm{esc}}(z)}{V(z)} = \left(1 - \frac{E(z)}{W(z)}\right)^{-1/2}. \tag{28a}$$

If $F(z) < 1$ for $0 < z \leq R_g$, the object escapes (successful recoil), otherwise it falls back; reversal in the motion occurs when $F(z) \to \infty$ (failed recoil). For comparison, we use Equation (24) to define the velocity function with dynamical friction neglected – i.e., $E(z) = E_i$:

$$F'_{(z)} = \frac{V_{\mathrm{esc}}}{V'(z)}. \tag{28b}$$

3. Results and Discussion

We have carried out computations to estimate the changes in the internal energies of the galaxy and the star system around the black hole ΔU_1 and ΔU_2 as a function of their separation for various velocities of ejection. The main aim is to see the effect of the force of dynamical friction on the motion of the object ejected from the center of the galaxy at a velocity close to that of escape. The results show a minor quantitative difference from the case of a point mass-galaxy interaction for similar parameters (Kapoor, 1985a, b).

The galaxy as well as the star system around the black hole are divided into twenty shells characterized by radii a' and a'', respectively. For the galaxy $\alpha_1 \leq a' \leq 20\alpha_1 = R_g$. For the object $r_{\min} \leq a'' \leq r_a$. Each shell is defined by fourteen stars at specified locations. Taking 280 stars for each system, we believe that the galaxy and the object are fairly well approximated. The masses taken are $M_1 = 10^{11} M_\odot$, $M_2 = 10^9 M_\odot$, $M_t = 1.1 \times 10^9 M_\odot$. Measuring the velocity in units of 1000 km s^{-1}, the calculations for $V(z)$, $\Delta U_1(z)$, and $\Delta U_2(z)$ have been performed for $\lambda = 10^5$ pc^{-3}, $\alpha_1 = 2.5$ kpc, and $V_0 = 0.62$, 0.65, 0.70, and 0.80. Note that $V_{\text{esc}}(0) = 0.59$.

Starting with $z = 0.05\alpha_1$, we set $\Delta U_1 = \Delta U_2 = 0$ and calculate $\Delta V_{x'}$, $\Delta V_{y'}$, and $\Delta V_{z'}$ for all the stars on a shell for a given value of V_0. This gives us $\langle \Delta T_1(a') \rangle$ in general. On integration over the mass of the galaxy, Equation (21) then evaluates $\Delta U_1(z)$. In a similar manner, we compute $\Delta U_2(z)$. With these values on hand, velocity $V(z)$ of the object as given by Equation (24) is corrected. An iteration is performed till converged values of $\Delta U_1(z)$ and $\Delta U_2(z)$ within a specified tolerance factor ($= 10^{-3}$) are obtained. The process is repeated by changing z to trace out the trajectory of the object through the galaxy for various velocities of ejection.

In Figure 3, velocity of the object, corrected for dynamical friction is plotted against z. For the sake of comparison we have plotted $V'(z)$ also (dotted lines). The effect of dynamical friction is found to be slightly larger than in the case of a point mass-galaxy interaction although, in both the cases, it decreases as the ejection velocity is increased.

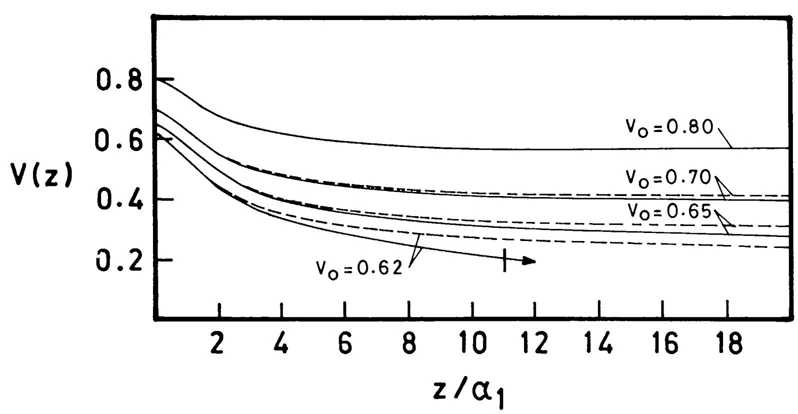

Fig. 3. This shows the decrease in velocity of the object with distance for various velocities of ejection. Solid lines take into account the dynamical friction while dotted lines do not.

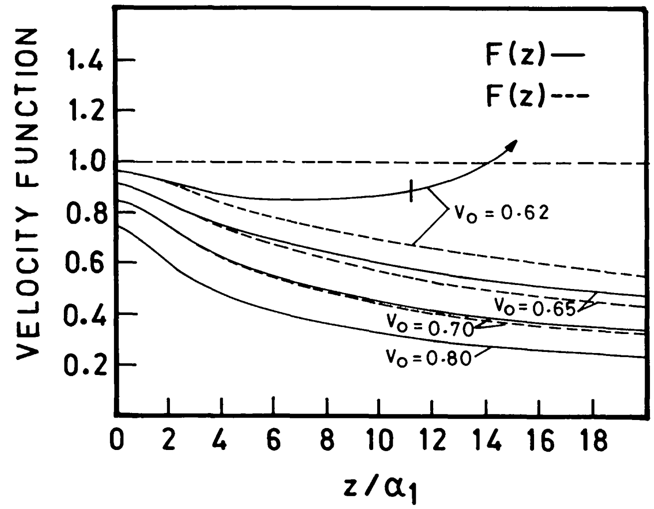

Fig. 4. This shows the behaviour of the velocity function as a function of separation z for various velocities of ejection. Upward bend in the $V_0 = 0.62$ curve implies a failed recoil. Solid lines take into account the dynamical friction but dotted lines do not.

For $V_0 = 0.80$, the dotted and the solid lines overlap. The bar on the $V_0 = 0.62$ curves implies the iterations becoming crude beyond this point. A general inference from the calculations is that only for ejections with a velocity $V_0 \geq V_{crit} \simeq 1.1\ V_{esc}(0)$ do we have a successful recoil. A recoil taking place at $V_0 = V_{esc}(0)$ is arrested. This is due to the effect of dynamical friction which is more clearly depicted by the behaviour of the $V_0 = 0.62$ curves in Figure 4 for the velocity function vs z. Here, the upward bending of the curve above the $F(z) = 1$ line implies reversal of motion to ensue. Dotted lines are $F'(z)$ curves. For $V_0 = 0.80$, the curves overlap here also.

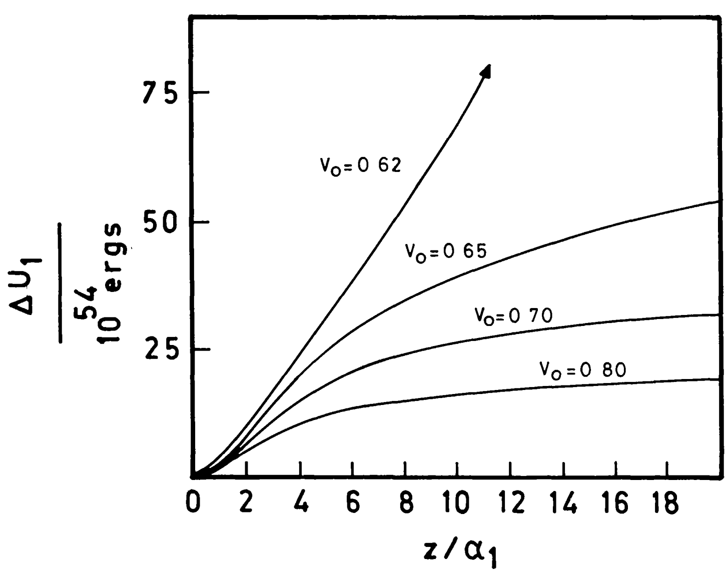

Fig. 5. This shows change in the internal energy of the galaxy for various velocities of ejection as a result of tidal interaction with the ejected object.

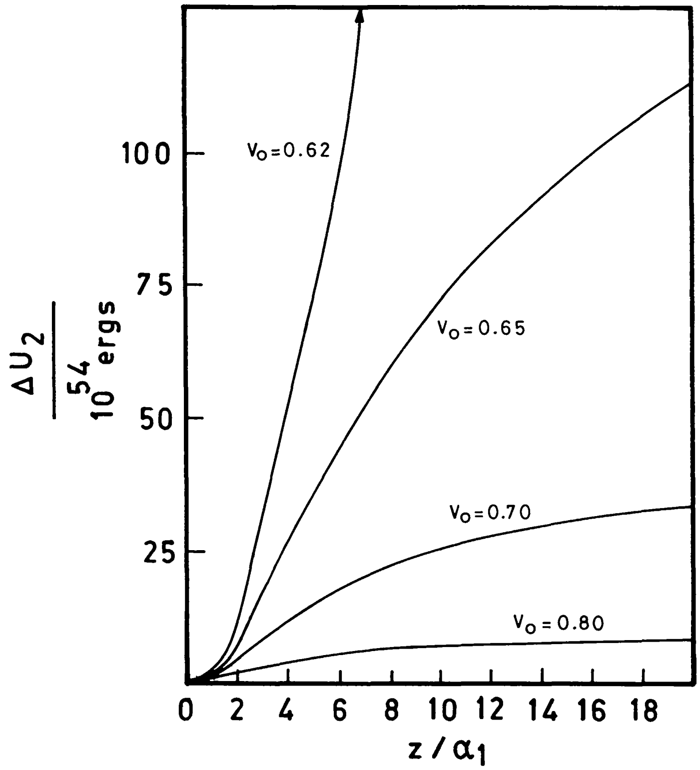

Fig. 6. This shows the change in the internal energy of the object for various velocities of ejection as a result of tidal interaction.

Starting from zero at the center, the small values of ΔU_1 and ΔU_2 which increase in general as $V(z)$ gets smaller and smaller and tend to blow up for $V_0 \lesssim V_{\text{crit}}$ are an important feature of these calculations. This is shown in Figures 5 and 6. It is to be noted

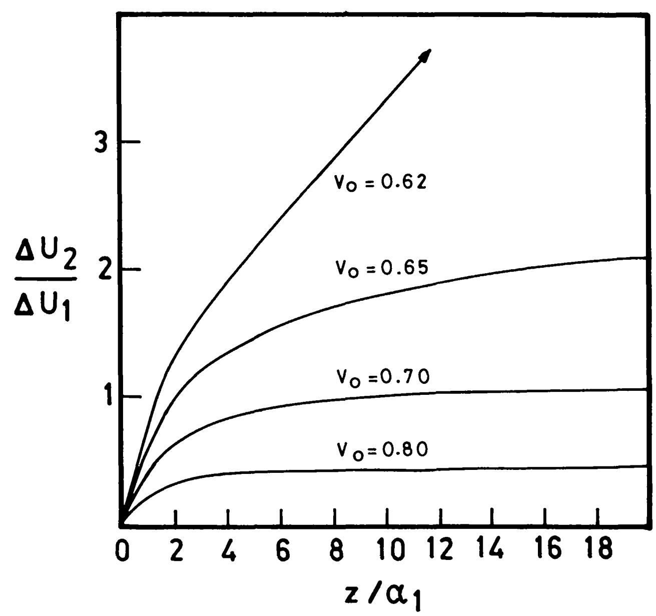

Fig. 7. This shows the behaviour of ratio of internal energy increments for various velocities of ejection with the increasing separation.

that $\Delta U_1/U_1 \ll 1$ and $\Delta U_2/U_2 \ll 1$ for all the velocities of ejection considered here. The energy exchange with the galaxy thus is not large enough to disrupt the galaxy. In contrast, the star system around the black hole confined within $\sim r_a$ will always escape pruning. The general behaviour of $\Delta U_2/\Delta U_1$ with z is interesting and is depicted in Figure 7 for different velocities of ejection. The curves tend to exceed unity for $V_0 \lesssim 0.65$. The general inference is that, only for $V_0 \lesssim V_{\rm crit}$, it is less massive of the systems that is tidally effected more. For $V_0 > V_{\rm crit}$ the reverse holds.

In Figure 8, we have plotted the force of dynamical friction vs z for various values of V_0. It is zero to start with $[\Delta U_1(0) = \Delta U_2(0) = 0]$, increasing fast initially and then it dwindles if $V_0 > V_{\rm crit}$. In fact, the larger the velocity V_0 of ejection, the smaller is $-f_D$.

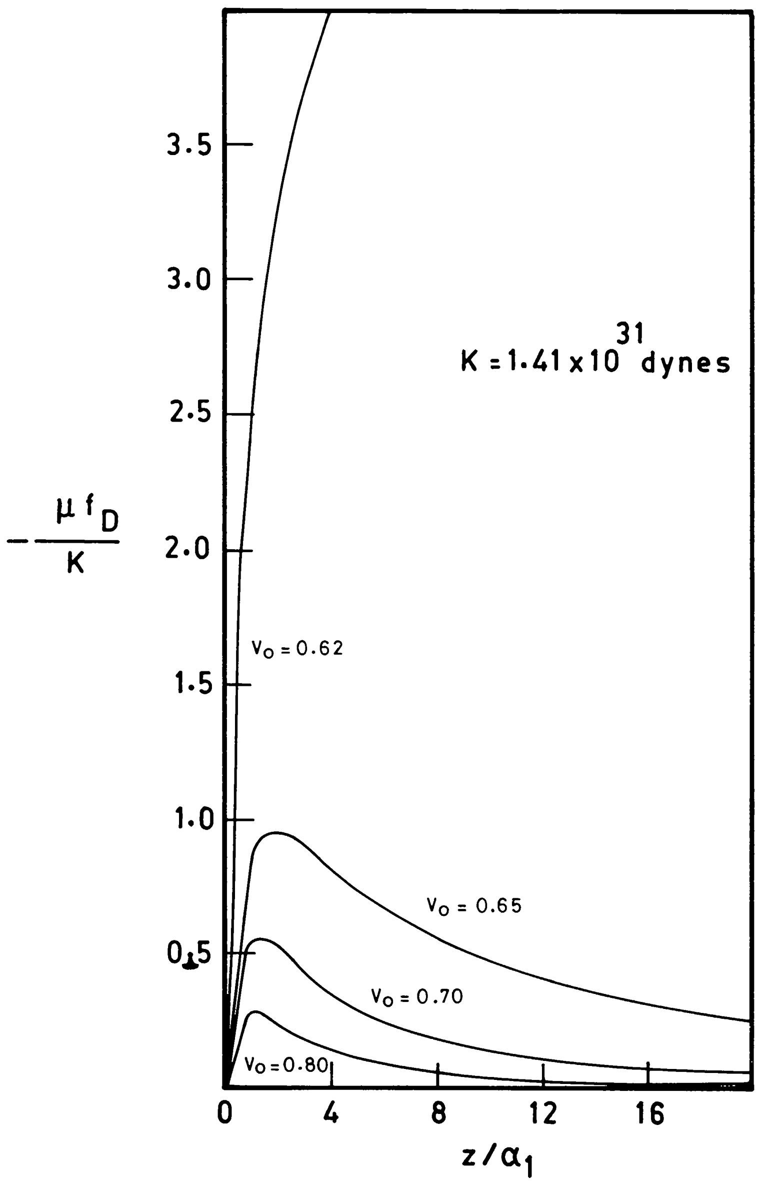

Fig. 8. The general effect of the force of dynamical friction on the motion of the object is shown as its distance varies from the center of the galaxy, for various velocities of ejection.

Only when $V_0 \lesssim V_{crit}$ does it blow up substantially. Here we wish to point out that choosing a smaller value for α_1 upscales $V_{esc}(0)$ and $W(z)$. Still the condition $V_0 \gtrsim V_{crit}$ holds for the recoil to be successful. The quantities $\Delta U_1/U_1$ and $\Delta U_2/U_2$ likewise remain small.

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