TRANSMISSION OF ACOUSTIC WAVE ENERGY ACROSS A MAGNETIC FLUX SHEATH

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Abstract. The presence of finite current sheets at the boundaries of a magnetic flux sheath will lead to a somewhat reduced transmission of the energy of an incident acoustic wave.

In order to study the role played by magnetic structures in the energy transport in the solar atmosphere, Cram and Wilson (1975) considered the transmission of the energy of an acoustic wave incident on a magnetic flux sheath embedded in a homogeneous non-magnetic plasma. It is clear from their results that the transmission is affected substantially whenever the wavelength of the incident wave becomes comparable to the thickness of the flux sheath. This fact becomes very significant because the small scale magnetic structures in the chromosphere (e.g. Foukal, 1971) have their cross-sections comparable to the wavelengths of some of the more prominent waves (e.g. those corresponding to the five minutes oscillations) which may be incident on them*.

Cram and Wilson assumed a rectangular profile for the magnetic field intensity in the structure. Such a structure obviously implies infinite current densities at the boundaries. In this note, we consider a similar structure but with finite current sheets between the flux sheath and the non-magnetic region outside. The geometrical configuration is given in Figure 1. The magnetic field in the current sheet (of thickness 'a') has a constant spatial gradient along the x-direction.

It levels off at a constant value H_{III} in the flux sheath and continues for a length 'b' times 'a'. Thereafter it reduces symmetrically to zero at the edge of the field-free plasma.

The x-dependance of the field prevents all perturbations from being simple harmonic along that direction. We have therefore assumed perturbations of the form $X(x)\exp[i(k_y y-\omega t)]$. The linearised hydromagnetic equations for a compressible fluid are

$$\frac{\partial \mathbf{h}}{\partial t}=\nabla\times(\mathbf{v}\times\mathbf{H}_0),$$

$$\frac{\partial \rho_1}{\partial t}=-\nabla\cdot(\rho_0\mathbf{v}),$$

* For example, the thickness of the Hα-fibrils is ~2000 km (Foukal, 1971) and a vertically propagating acoustic wave of period ~5 min will have a wavelength ~3000 km in the upper chromosphere (this follows from the values of pressure and temperature in the HSRA model which correspond to a phase velocity of ~10 km s^{-1}).

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$$\rho_0 \frac{\partial \mathbf{v}}{\partial t} = -S^2 \nabla \rho_1 + \frac{\mu}{4\pi}\{(\nabla \times \mathbf{h}) \times \mathbf{H}_0 + (\nabla \times \mathbf{H}_0) \times \mathbf{h}\},$$

$$\nabla \cdot \mathbf{h} = 0\,, \tag{1}$$

where $\mathbf{h}$ is the perturbation over the unperturbed field $\mathbf{H}_0$,

ρ_1 is the perturbation over the unperturbed density ρ_0, $\mathbf{v} = u\hat{x} + v\hat{y}$ is the velocity perturbation and S is the velocity of sound propagation which is assumed to be constant with same value everywhere.

Eliminating ρ_1 and $\mathbf{h}$ from Equations (1), the equation of motion can be written as

$$\rho_0 \omega^2 u + \frac{\partial}{\partial x}\left\{\frac{\partial}{\partial x}(\rho_0 u) + \frac{\partial}{\partial y}(\rho_0 v)\right\} + MH_0\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)(uH_0) +$$

$$+ M \frac{\mathrm{d}H_0}{\mathrm{d}x}\frac{\partial}{\partial x}(uH_0) = 0 \tag{2}$$

and

$$\omega^2 v + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 u}{\partial x \partial y} = 0\,,$$

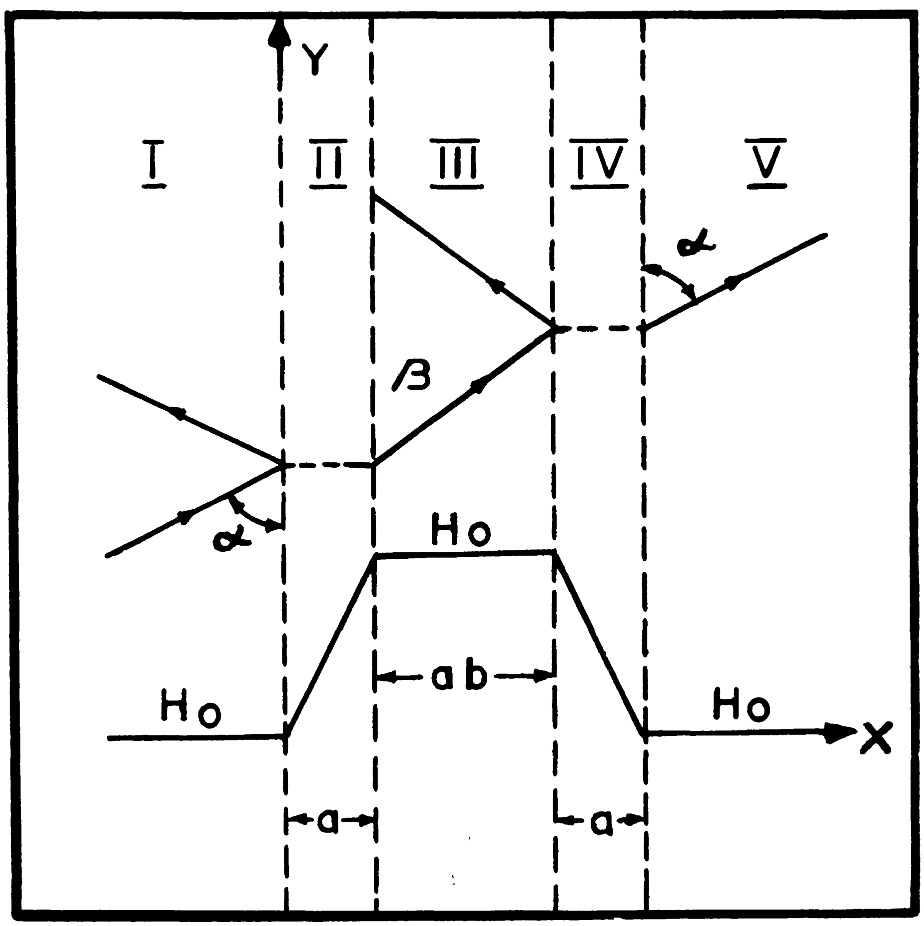

Fig. 1. Schematic diagram of equilibrium structure. H_0 takes the value 0, $H_{\mathrm{III}}x/a$, H_{III}, $H_{\mathrm{III}}(2+b-x/a)$ and 0 in regions I, II, III, IV and V respectively. The incident, reflected and refracted rays are also shown.

where the zero order equilibrium condition

$$\rho_0 S^2 + \frac{\mu H_0^2}{8\pi} = \text{constant}$$

has been used and all variables are expressed in dimensionless form by the following transformations:

$$x \to ax;\ y \to ay;\ k \to k/a;\ \omega \to \omega S/a;\ u \to Su;$$

$$v \to Sv;\ h \to H_{\rm III}h;\ H_0 \to H_{\rm III}H_0;\ \rho_1 \to \rho_{\rm III}\rho_1 \quad \text{and} \quad \rho_0 \to \rho_{\rm III}\rho_0\,.$$

We also define a parameter 'M' such that

$$M = \frac{\mu H_{\rm III}^2}{4\pi \rho_{\rm III} S^2}\,.$$

Eliminating between Equations (2) and recalling the form of our perturbations, we have

$$\frac{\mathrm{d}}{\mathrm{d}x}\left\{(\rho_0 + MH_0^2 \sin^2\alpha)\frac{\mathrm{d}X}{\mathrm{d}x}\right\} + \omega^2 \sin^2\alpha(\rho_0 - MH_0^2\cos^2\alpha)X = 0\,, \qquad (3)$$

where α is the angle between the incident wave vector and the boundary (cf. Figure 1).

We notice that in the field-free region, two linearly independent solutions of Equation (3) are sound waves, one with positive k_x and the other with negative k_x. In the constant field region, only the fast mode is allowed as shown by Cram and Wilson, and the two linearly independent solutions can be taken as fast modes, one with positive k_x, the other with negative k_x. In this region we have $k_x^2 < 0$ for $\alpha < \alpha_c$ where $\alpha_c = \cos^{-1}(1/M)^{1/2}$ is the critical 'cut-off' angle, which exists for $M > 1$. (Conversely $M_c = \sec^2\alpha$ is the critical 'cut-off' value of M for a given α.)

In the current sheets, each solution represents a wave propagating in the y-direction and having an amplitude $X(x)$ which varies along the x-direction according to Equation (3).

Since the boundaries are allowed to be perturbed, the following boundary conditions are necessary and sufficient:

(i) $[\mathbf{v}\cdot\mathbf{n}] = 0,$

(ii) $\left[p + \frac{\mu}{4\pi}(\mathbf{H}_0\cdot\mathbf{h})\right] = 0,$

which in our present notation reduce to

$$[X] = 0$$

and

$$[X'] = 0$$

across the boundaries. Here the rectangular brackets represent the jump across the boundary. Applying these boundary conditions with the additional condition that there is no wave with negative k_x in region V we get eight homogeneous equations in the nine unknown amplitudes of the solutions in the five regions. By normalising all these amplitudes in terms of the incident wave amplitude, we get eight linear inhomogeneous equations in the eight normalised amplitudes. After solving these, the energy fluxes of these waves are computed from the formula (Bray and Loughhead, 1974)

$$Q_i = \tfrac{1}{4}(pv_i^* + p^* v_i) + \frac{H_{0j}}{16\pi}(h_j v_i^* + h_j^* v_i) - \frac{H_{0i}}{16\pi}(h_j v_j^* + h_j^* v_j),$$

$$i = x, y, \quad j = x, y\,.$$

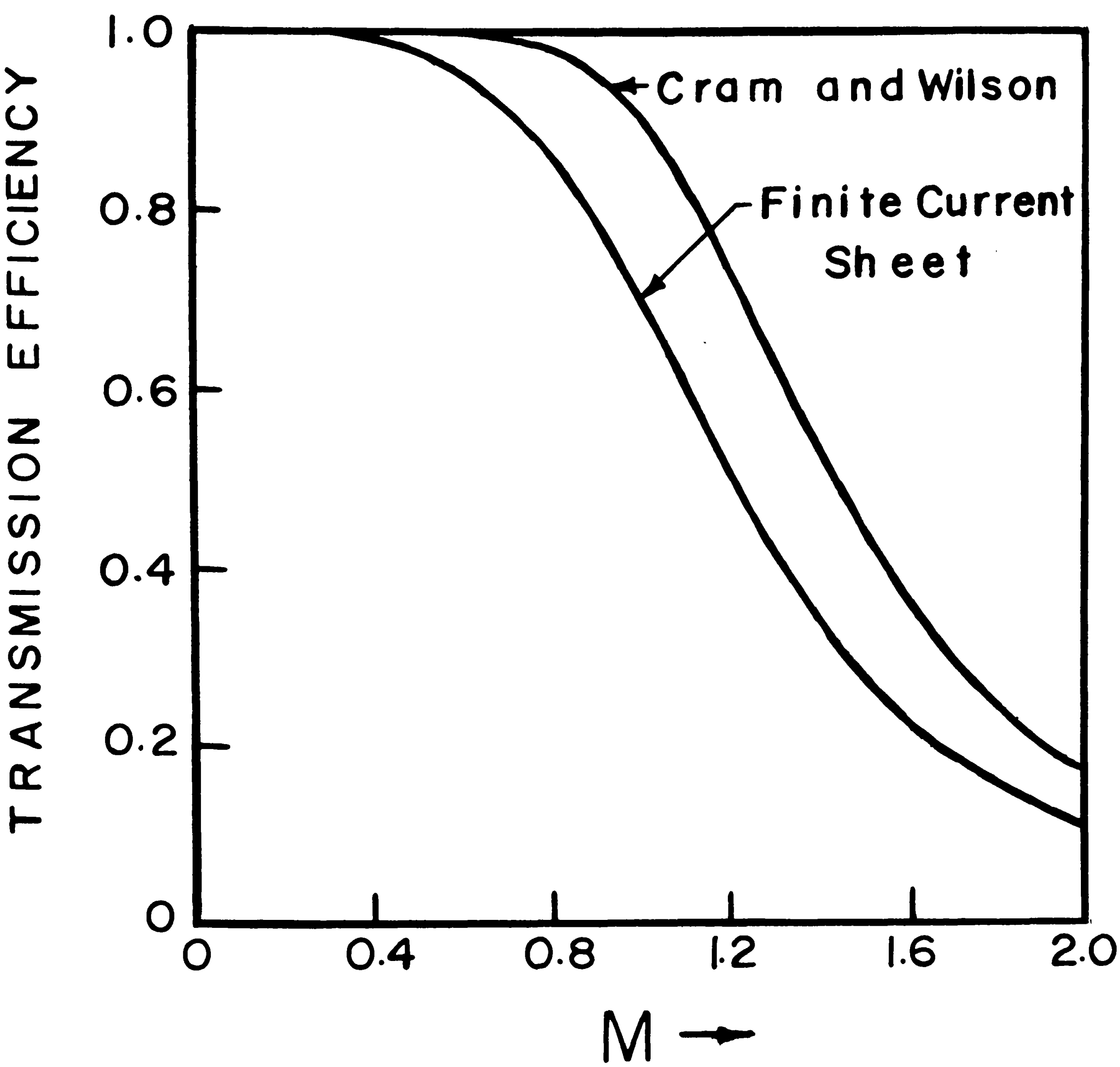

Fig. 2. Plot of transmission efficiency versus 'M' for $\alpha = 45°$, $\omega = 1$ and $b = 2\pi$.

In Figure 2, the transverse transmission efficiency (which is the ratio of the transverse component of the emergent flux to that of the incident flux) is plotted against M for $\alpha = 45°$, $\omega = 1$ and $b = 2\pi$. This choice of b implies a current sheet thickness which is $\frac{1}{2}\pi$ times the flux sheath thickness, and the value of ω implies that the wavelength λ is equal to the thickness 'ba' of the flux sheath. We have also calculated the efficiency of transmission of an acoustic wave of the same wavelength across a flux sheath of thickness ba with infinitesimally thin current sheets using the formulae of Cram and Wilson.

It is seen that for identical values of the relevant parameters, the structures with finite current sheets allow less transmission of acoustic waves.* However the transmission efficiency is of the same order of magnitude for both kinds of magnetic structures. Thus for the purpose of estimating the energy transmission, the infinitesimal current sheet approximation used by Cram and Wilson seems adequate.

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* We have also compared the transmission efficiencies of the two kinds of structures with equal amounts of magnetic flux and with the same value for the field intensity ($M = 1.2$) in the flux sheath. This sample calculation indicates that the main conclusion is valid even for such a comparison.