INFLATION WITH MASSIVE SPIN-2 FIELD IN CURVED

SPACE-TIME

C. Sivaram and Venzo de Sabbata

World Laboratory, Lausanne - Switzerland
Institute of Astrophysics, Bangalore - India
Dipartimento di Fisica, Università di Ferrara-Italy

1. Introduction

The hot big bang model of Cosmology is generally accepted as providing a correct description of the evolution of the universe. It naturally accounts for the isotropic cosmic microwave black-body background $3^{o}K$ radiation. Moreover it gives a good quantitative estimate of the amounts of light elements such as helium and deuterium synthesized several seconds after the expansion started. These estimates involve very little input of physics with no additional assumptions and agree quantitatively with the actually observed abundances of these light elements in stars and in interstellar matter. However when one extrapolate the model to early epochs one encounters rather puzzling aspects regarding the initial conditions. For instance we have to do with the expansion proceeding at a critical rate ($H^2_{crit} = 8\pi G\rho/3$) to a very high degree of precision at the early epochs implying that the universe was close to critical density (ρ_c) to very high degree of precision (i.e. to within one part in 10^{16} at epoch of nucleosynthesis and to one part in 10^{60} at the Planck epoch of $t \approx 10^{-43}$s! To make this more precise, we note that the observations indicate that the present value of $\Omega \equiv \rho/\rho_c$ (which measures ratio of energy density of universe to the critical energy density) though not known with great precision lies in the range $0.01 \leq \Omega \leq$ few units The luminous matter in the universe would indicate $\Omega \approx 0.1 \div 0.3$ Again from the uncertainties in the deceleration parameter defined as $q_o \equiv -(R/R)H^2 = \Omega/2$, one could restrict Ω to at most a few times unity. From the Robertson-Walker equation:

$$K/R^2H^2 = \rho/(3\pi^2/8\pi G) - 1 \qquad (1)$$

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one can write Ω in a time dependent form:

$$\Omega = 1/(1 - y(t)) \qquad (2)$$

where $y(t) = (K/R^2)/(8\pi G\rho/3)$.
Ω is not constant but varies with time since $y(t) \propto R(t)^n$ ($n = 1$ for matter dominated universe and $n = 2$ for radiation domination). Equation (2) implies that at epoch of nucleosynthesis, value of y ($=y_N$) was $y_N \le 10^{-16}$ which means $\Omega_N \simeq 1 + 0(\le 10^{-16})$ and at Planck epoch $y_{Pl} \le 10^{-60}$ so that consequently $\Omega_{Pl} = 1 + 0(10^{-60})$. If this ratio was not infinitesimally small at early epochs, the universe would have recollapsed long ago (for $K > 0$) or began a coasting phase ($K < 0$) with $R \propto t$. This extreme smallness of the ratio y if required as an initial condition is very strange, as in other words it would mean that the kinetic term $(\dot{R}/R)^2$ and the potential term $(8\pi G\rho/3)$ in the R-W equation balanced each other to arbitrarily high degree of precision (one part in 10^{60} at Planck epoch!) at early epochs. It is as if from very early epochs on, the ratio of curvature term to density term was extremely small (see eq.(2)), that is the universe began as extremely flat (with Ω arbitrarily close to one) which is a very special initial condition.

Another problem is the horizon problem. As is evident from the microwave background the universe on the largest scales is extremely homogeneous and isotropic (to better than one part in 10^4). However, as is known, standard cosmology has particle horizons. When matter and radiation last interacted vigourously (at $t \simeq 10^{13}$s, and Temperature $\simeq 1/3$ ev), what was to become the presently observable universe was comprised of $\simeq 10^6$ causally distinct regions. The particle horizon at decoupling only subtends an angle of about $(1/2)^\circ$ on the sky today; then how is that the microwave background temperature is so uniform on angular scales $\gg (1/2)^\circ$? At early epochs the number of causally distinct regions keeps increasing. For instance one second after the big bang the size of the universe currently observable was $\approx 10^{10}$cm. So there were about $(10^{14}/3\cdot 10^{10})^3 \sim 10^{27}$ causally distinct regions not communicating with each other. As the universe expanded at earliest epochs as $t^{1/2}$ whereas the horizon expands with light velocity as ct, the number of incommunicable regions $\sim t^{1/2}/ct \to \infty$ as $t \to 0$. With so many causally distinct regions in the early universe why is the present universe so homogeneous and isotropic all over?

Then we have the magnetic monopole problem. There should have been a glut of monopoles produced with densities several orders larger than the critical density at the Guts spontaneous symmetry breaking (GSSB) phase transition. Then why don't we see any monopoles?

The so called inflationary universe paradigm [1],[2] was invented to take care of the above problems confronting big bang cosmology at its earliest epochs. This invokes a vacuum dominated exponential expansion rate for the universe at an early phase with $H_{Infl} \simeq (8\pi V(0)/3M_{Pl}^2)^{1/2} \approx M_G^2/M_{Pl}$, $V(0)$ assumed as $\approx M_G^4$, where M_G is the mass scale of scalar field which drives the expansion. While H_I is constant, R grows as exp(Ht). So a typical homogeneous region can expand physically by a factor of e^{100}, to encompass the whole of the observed universe, therefore taking care of the homogeneous or horizon problem, i.e. a single causally connected region can expand exponentially to give rise to the observed universe. Such an expansion also accounts for the curvature term becoming vanishingly small after inflation, i.e. the y term as defined in eq.(2) tends to zero after inflation and the inflationary scenario predicts a $\Omega \approx 1.0 \mp 0(10^{-BIG})$. So that $\Omega \approx 1$ to a very high degree of precision. Again the inflationary expansion would have exponentially diluted away any large relic monopole density thus removing the monopole problem. The additional bonus is that quantum fluctuations of the scalar field [2] would give rise to scale-invariant perturbations which seem to be required to account for the formation of large scale hierarchy of structure in the universe. However the amplitude $(\delta\rho/\rho)_I$ of the fluctuations seems too large $\approx 10^2$ in most scenarios.

Of course one could have alternatives to the conventional inflationary scenarios requiring massive scalar fields with very 'flat' potential wells. One such alternative could be modification of general relativity at the Planck scale. The monopole and flatness problems can be solved by producing large amounts of entropy. Again if during an early epoch ($t \approx 10^{-43}$s), R, the scale factor, increased as rapidly as or more rapidly than t (for eg. $t^{1.2}$ or more) then d_H (horizon distance) $\rightarrow \infty$, eliminating horizon problem. One such possibility of modification is to consider the Weyl type lagrangian for high energy gravity at the Planck scale [3]. This would be of type $L_W \sim \alpha C^2 + \beta R^2$, i.e. quadratic in the curvatures C and R with dimensionless constants α and β (appropriate for a renormalizable theory of gravity in contrast to the dimensional Newtonian constant for the Einstein non-renormalizable gravity). The field equations would be of fourth order i.e. of form $\alpha \nabla^4\Phi = km\delta^3(r)$ with a solution for the potential rising with r as Φ = const r = ar. The corresponding solution for the scale factor would be of type $R = at^2$ rather than the usual $R = \text{const}\ t^{1/2}$ type of solution. As R now increases faster than t, the horizon problem is eliminated. Again the flatness problem is also solved in this theory as quadratic curvature lagrangians of above type are known to have classical solutions with zero total energy, which means a $K = 0$ cosmological model, i.e. complete equality of kinetic and potential energy terms in the R-W expansion [3]. Similar situation holds for lagrangians with quadratic torsion terms, so it is possible to solve the flatness and horizon problems in the framework of such models [4]. Moreover if we consider lagrangians of the type

$$L \approx \gamma_o k^{-2} R + \alpha R_{\mu\nu} R^{\mu\nu} - \beta R^2 ,$$

their solutions are of type [5]

$$V = -\frac{k^2 M}{8\pi\gamma r} + \frac{k^2 M}{6\pi\gamma}\frac{e^{-m_2 r}}{r} - \frac{k^2 M}{42\pi\gamma}\frac{e^{-m_o r}}{r} \quad ;$$

with $m_2 = \gamma^{1/2}(\alpha k^2)^{-1/2}$; $m_o = \gamma^{1/2}\left[2\ (3\beta - \alpha)k^2\right]^{-1/2}$,

i.e. their particle spectrum also contains massive tensor particles with mass m_2 and massive scalar particles with mass m_o. Massive scalar particles are contained in theories with lagrangians of type $L = k^{-2}R - \beta R^2$. These are precisely of the type used by Starobinsky [6] for inflationary models. In general these models are equivalent to those using massive scalar fields, there being a general transformation due to Whit [7] linking the two types of theories. Also the massive spin-2 field in the above relation for V, enters with an opposite sign (for energy) to that for the massive scalar field. This raises the possibility of having a zero energy momentum tensor for appropriate choice of constants with such a tensor naturally giving rise to a de Sitter solution. This will be elaborated in the next section. Again a lagrangian with non-minimal coupling $e^{\phi}R^2$ can be transformed to Einstein's theory with two scalar fields. We can also consider a general lagrangian L(R) with an arbitrary function of R [8]. Then scale invariant solutions g_{ij} give rise to a one-parameter family ($e^{2\alpha}g_{ij}$, α = const) of homothetically equivalent solutions. For lagrangian of type $L = R^m$, $m \neq 0$, the expanding solution $R(t) = t^2$ is an attractor solution for $L = R^{3/2}$ in the set of spatially flat Friedmann models. For arbitrary m we analogously have an attractor $R(t) \simeq t^n$, with $n = -(m-1)(2m-1)/(m-2)$; when $n \to \infty$ and $m \to 2$, this gives the usual attractor property of de Sitter space-time.

2. Massless and massive spin-2 fields in curved space-time

The massless spin-2 field can be described by a rank-4 tensor $\Phi_{\mu\nu\rho\sigma}$ which being a Weyl tensor [9], satisfies

$$\partial_\mu \Phi^{\mu\nu\rho\sigma} = 0 \qquad (1)$$

and

$$\partial_\lambda \Phi_{\mu\nu\rho\sigma} + \partial_\mu \Phi_{\nu\lambda\rho\sigma}\ \partial_\nu \Phi_{\lambda\mu\rho\sigma} = 0 \qquad (2)$$

If the potential $\psi_{\mu\nu} = \psi_{\nu\mu}$ is defined as

$$\Phi_{\rho\mu\nu} = \partial_\mu \psi_{\nu\rho} - \partial_\nu \psi_{\mu\rho} \qquad (3)$$

we have

$$\Box\psi_{\mu\nu} + \partial_{\mu\nu}\psi - \partial_{\nu\rho}\psi^{\rho}_{\mu} - \partial_{\mu\rho}\psi^{\rho}_{\nu} = 0 \quad (4)$$

$(\psi = \psi^{\mu}_{\mu})$.

The gauge transformations

$$\psi_{\mu\nu} \Rightarrow \psi_{\mu\nu} - \partial_{\mu}\vartheta_{\nu} - \partial_{\nu}\vartheta_{\mu}$$

leaves $\Phi_{\mu\nu\rho\sigma}$ unchanged. The Lorentz gauge $\Box\vartheta_{\mu} = 0$, reduces to

$$\Box\psi_{\mu\nu} = 0 \; ; \qquad \partial_{\mu}\psi^{\mu\nu} - (1/2)\partial^{\nu}\psi = 0 \quad (5)$$

The gauge conditions eliminate the spin-one and spin-zero components.

The appropriate lagrangianfor a massive spin-2 field in flat space-time is (mass m_2):

$$L(f_{\mu\nu}) = (1/2)f_{\mu\nu}\left[P^{\mu\nu\alpha\beta} + m_2^2(\eta^{\mu\nu}\eta^{\alpha\beta} \quad \eta^{\mu(\alpha}\eta^{\beta)\nu}\right]f_{\alpha\beta} \quad (6)$$

where $P^{\mu\nu\alpha\beta}$ is the massless spin-2 inverse propagator [10]

$$P^{\mu\nu\alpha\beta} = (g^{\mu\nu}g^{\alpha\beta} - g^{\mu(\alpha}g^{\beta)\nu})\Box - g^{\mu\nu}\partial^{(\alpha}\partial^{\beta)}$$
$$- g^{\alpha\beta}\partial^{(\mu}\partial^{\nu)} + g^{\alpha(\mu}\partial^{\nu)}\partial^{\beta} + g^{\beta(\mu}\partial^{\nu)}\partial^{\alpha} . \quad (7)$$

The field equation for $f_{\mu\nu}$ is

$$\left[P^{\mu\nu\alpha\beta} + m_2^2\,(\eta^{\mu\nu}\eta^{\alpha\beta} - \eta^{\mu(\alpha}\eta^{\beta)\nu})\right]f_{\alpha\beta} = 0 \quad (8)$$

The trace gives

$$\left[2(\eta^{\alpha\beta}\Box - \partial^{\alpha}\partial^{\beta}) + 3\,m_2^2\,\eta^{\alpha\beta})\right]f_{\alpha\beta} = 0$$

The divergence is

$$m_2^2\,(\eta^{\alpha\beta}\partial^{\mu} - \eta^{\mu(\alpha}\partial^{\beta)})f_{\alpha\beta} = 0 \quad (9)$$

An alternative form is (massive spin-2 field) [9]:

$$\Box f_{\mu\nu} - \partial_{\mu}\partial_{\gamma}f^{\gamma}_{\nu} - \partial_{\nu}\partial_{\gamma}f^{\gamma}_{\mu} + \partial_{\mu}\partial_{\nu}f + g_{\mu\nu}(\partial_{\gamma}\partial_{\rho}f^{\gamma\rho} - \Box f)$$
$$- m_2^2\,(f_{\mu\nu} - g_{\mu\nu}f) = 0 \quad (10)$$

the divergence condition is: $\partial^{\gamma}f_{\gamma\mu} = \partial_{\mu}f$

and the trace conditions on f: $f = 0$; $\partial^{\gamma}f_{\gamma\mu} = 0$

then (10) becomes

$$\Box f_{\mu\nu} - m_2^2\,f_{\mu\nu} = 0 \quad (12)$$

These constraints can be generalized to curved space time as:

$$f_{\gamma\mu;\gamma} = 0\ , \qquad f \quad 0 \qquad (13)$$

which reduce to $\partial^{\gamma} f_{\gamma\mu} = 0$ and $f = 0$ in flat space-time.

The most general covariant field equation for $f_{\mu\nu}$ in curved space-time can be written:

$$\Box f_{\mu\nu} - R_{\mu\gamma} f^{\gamma}_{\nu} - R_{\nu\gamma} f^{\gamma}_{\mu} + 2R_{\mu\gamma\nu\rho} f^{\gamma\rho} + H_{\mu\nu\gamma\rho} f^{\gamma\rho} = 0 \quad (14)$$

We can try to write in the form

$$\Box f_{\mu\nu} + J_{\mu\nu\gamma\rho} f^{\gamma\rho} = 0 \qquad (15)$$

(as an appropriate generalization of eq.(12)), where we define (analogous to Weyl tensor for massless case above):

$$J_{\mu\nu\gamma\rho} = H_{\mu\nu\gamma\rho} - (1/2)(g_{\mu\gamma} R_{\nu\rho} + g_{\mu\rho} R_{\nu\gamma} + g_{\nu\gamma} R_{\mu\rho} + g_{\nu\rho} R_{\mu\gamma})$$

$$+ 2R_{\mu\nu\gamma\rho} \qquad (16)$$

Tensors J and H have the same symmetries. In fact:

$$\nabla_{\gamma}\nabla_{\mu} f^{\gamma}_{\nu} + \nabla_{\gamma}\nabla_{\nu} f^{\gamma}_{\mu} = R_{\mu\gamma} f^{\gamma}_{\nu} + R_{\nu\gamma} f^{\gamma}_{\mu} - 2R_{\mu\gamma\nu\rho} f^{\gamma\rho} \qquad (17)$$

imposed with corresponding constraints (in curved space):

$$\nabla^{\gamma} f_{\gamma\mu} = 0 \quad \text{or} \quad f_{\gamma\mu;\gamma} = 0 \qquad (18)$$

The divergence equation now read (in curved space, analogue to eq.(9)):

$$\Box\nabla^{\gamma} f_{\gamma\mu} - R_{\gamma\mu}\nabla^{\rho} f^{\gamma}_{\rho} + (\nabla_{\mu} R_{\gamma\rho} - \nabla_{\gamma} R_{\rho\mu} - \nabla_{\rho} R_{\gamma\mu} + \nabla^{\alpha} H_{\alpha\mu\gamma\rho}) f^{\gamma\rho}$$

$$+ H_{\mu\nu\gamma\rho}\nabla^{\nu} f^{\gamma\rho} \qquad (19)$$

Again we have the following postulated relations:

$$(\nabla_{\mu} R_{\gamma\rho} - \nabla_{\gamma} R_{\rho\mu} - \nabla_{\rho} R_{\gamma\mu} + \nabla^{\alpha} H_{\alpha\mu\gamma\rho}) f^{\gamma\rho} = G_{\mu} f \qquad (20)$$

$$H_{\mu\nu\gamma\rho}\nabla^{\nu} f^{\gamma\rho} = K_{\mu\varepsilon}\partial^{\varepsilon} f + 4I_{\mu\varepsilon}\nabla_{\gamma} f^{\gamma\varepsilon} \qquad (21)$$

where G_{μ} , $K_{\mu\varepsilon}$ and $I_{\mu\varepsilon}$ are functions of the background metric and derivatives of the metric. The symmetry of $H_{\mu\nu\gamma\rho}$ (i.e. $\gamma \Leftrightarrow \rho$ and also $\mu \Leftrightarrow \nu$) enables us to write (with (21)):

$$H_{\mu\nu\gamma\rho} = 2(I_{\mu\gamma} g_{\nu\rho} + I_{\mu\rho} g_{\nu\gamma}) + K_{\mu\nu} g_{\gamma\rho} \qquad (22)$$

$$I_{\mu\gamma} = (1/4) g_{\mu\gamma} I \quad \text{with} \quad I = I^{\alpha}_{\alpha} \qquad (23)$$

The trace should vaanish with respect to γ,ρ of bracketed expressions. Combinations of the above relations leads to:

$$\nabla_\mu R_{\gamma\rho} - \nabla_\gamma R_{\rho\mu} - \nabla_\rho R_{\gamma\mu} + \nabla^\alpha H_{\alpha\mu\gamma\rho} - (1/4) g_{\gamma\rho} g^{\beta\varepsilon} \nabla^\alpha H_{\alpha\mu\beta\varepsilon} = 0$$

and (24)

$$\nabla_\mu R_{\gamma\rho} - \nabla_\gamma R_{\rho\mu} - \nabla_\rho R_{\gamma\mu} + (1/2)[g_{\mu\gamma}\partial_\rho + g_{\mu\rho}\partial_\gamma - (1/2) g_{\gamma\rho}\partial_\mu) I = 0$$

Taking trace with respect to ρ and μ:

$$\partial_\gamma R = (9/4)\, \partial_\gamma I \qquad (25)$$

where

$$I \;\; (4/9)\, R - A^2 \;,\; A^2 = \text{constant} \qquad (26)$$

and solving for $\nabla_\mu R_{\gamma\rho}$, from eq. (24), using eqs.(25),(26) we finally get:

$$\nabla_\mu R_{\gamma\rho} = (1/9)\left[(1/2)(g_{\mu\rho}\partial_\gamma + g_{\mu\gamma}\partial_\rho) + 2 g_{\gamma\rho}\partial_\mu\right] R \qquad (27)$$

The equation (14) for $f_{\mu\nu}$ is now in any arbitrary curved space:

$$\Box f_{\mu\nu} - R_{\mu\gamma} f^\gamma_\nu - R_{\nu\gamma} f^\gamma_\mu + 2R_{\mu\nu\gamma\rho} f^{\gamma\rho} + \left[(4/9)R - A^2\right] f_{\mu\nu} = 0 \qquad (28)$$

with the trace equation

$$\Box f + \left[(4/9)R - A^2\right] f = 0 \qquad (29)$$

For a space of constant curvature (i.e. a de Sitter background) with:

$$R_{\mu\gamma\nu\rho} = \Lambda/3 (g_{\mu\nu} g_{\gamma\rho} - g_{\mu\rho} g_{\nu\gamma}) \qquad (30)$$

equation (28) becomes:

$$\Box f_{\mu\nu} - (2/3)\Lambda f_{\mu\nu} = 0 \quad \text{with} \quad A^2 = -2\Lambda/9 \qquad (31)$$

with trace equation $\qquad (\Box - 2\Lambda) f = 0 \qquad (32)$

The above is a generalization of the procedure in ref.[9] which was specialized for space of constant curvature, particularly for case of de Sitter space. In fact the equations for massive spin-2 field (with coupling K_f) when linearized on de Sitter background become the de Sitter covariant theory of massive spin-2 and spin-0 particles with

$$(\Box - 2\Lambda/3) f_{\mu\nu} \sim -2K_f (T_{\mu\nu} - \eta_{\mu\nu} T/4) \quad \text{and} \quad (\Box - 2\Lambda f) \sim 2K_f T.$$

The opposite signs of the source terms in case of massive spin-2 and spin-0 [5],[9], enables the possibility of having a net $T_{\mu\nu}$ of zero, i.e. a vacuum dominated phase.

3. Inflationary solutions of above equations

We can generalize tha above Klein-Gordon equations with source terms by considering a perturbation $f'_{\mu\nu}$ of $f_{\mu\nu}$ and writing the equation of motion for $f'_{\mu\nu}$ as [10],[11]:

$$G^{\mu\nu\alpha\beta} f'_{\alpha\beta} = 0 \qquad (33)$$

Following ref.[10],[11] we can write:

$$G^{\mu\nu\alpha\beta} = P^{\mu\nu\alpha\beta} + M^{\mu\nu\alpha\beta} \qquad (34)$$

where $P^{\mu\nu\alpha\beta}$ is the massless spin-2 propagator (eq.7) and $M^{\mu\nu\alpha\beta}$ is an effective mass tensor defined as [10]:

$$M^{\mu\nu\alpha\beta} = 2K_f \Big[(g^{\mu\nu}T^{\alpha\beta} + g^{\alpha\beta}T^{\mu\nu})/4 - g^{\mu(\alpha}T^{\beta)\nu} - g^{\nu(\alpha}T^{\beta)\mu} + T^{\lambda}_{\lambda} g^{\mu(\alpha}g^{\beta)\nu}/2 - T^{\lambda}_{\lambda} g^{\mu\nu}g^{\alpha\beta}/4 - (1/2)\left\{\delta T^{\mu\nu}/\delta g_{\alpha\beta} + \delta T^{\alpha\beta}/\delta g_{\mu\nu}\right\}\Big]$$

so that the Jacobi equation for $f'_{\alpha\beta}$ can be written:

$$\int d^4x\sqrt{-g}\; f'_{\mu\nu}(P^{\mu\nu\alpha\beta} + M^{\mu\nu\alpha\beta}) f'_{\alpha\beta} = 0 \qquad (35)$$

with the usual conditions [11]:

$$\left.\begin{aligned} f_{\mu\nu}G^{\mu\nu\alpha\beta}f'_{\alpha\beta} &= 0 \\ \nabla_{\nu}G^{\mu\nu\alpha\beta}f'_{\alpha\beta} &= 0 \end{aligned}\right\} \qquad (36)$$

and

$$u_{\nu}G^{\mu\nu\alpha\beta}f'_{\alpha\beta} = 0$$

where u_{ν} is an arbitrary four vector;
with the harmonic coordinate condition [9],[11]:

$$\nabla_{\mu} f'^{\mu\nu} = (1/2)\nabla^{\nu} f'^{\mu} \qquad (37)$$

the constraints becoming

$$f^{\alpha\beta}f'_{\alpha\beta} = 0, \quad \nabla^{\alpha}f'_{\alpha\beta} = 0\ ; \quad u^{\alpha}f'_{\alpha\beta} = 0 \qquad (38)$$

we have

$$P^{\mu\nu\alpha\beta}f'_{\alpha\beta} = - g^{\mu(\alpha}g^{\beta)\nu}\ (\Box - 2\Lambda)f'_{\alpha\beta} \qquad (39)$$

$$M^{\mu\nu\alpha\beta}f'_{\alpha\beta} = - g^{\mu(\alpha}g^{\beta)\nu}\ K_f(\rho + p)f'_{\alpha\beta} \qquad (40)$$

giving the effective Klein-Gordon equation for $f'_{\alpha\beta}$ as:

$$\left\{\Box - 2\Lambda + K_f(\rho + p) + \ldots\right\} f'_{\alpha\beta} = 0 \qquad (41)$$

with an effective mass $M_f^2 = - K_f(\rho + p) - 2\Lambda$.

For a vanishing mass tensor (possible for appropriate combination of spin-2 and spin-0 massive fields), the equation (40) implies a vanishing $(\rho + p)$ or a negative pressure corresponding to a de Sitter type situation with exponential expansion.

An explicit exponentially expanding solution of the massive spin-2 field equations in a R-W background can also be obtained, Consider the metric:

$$ds^2 = - dt^2 + R^2(t)(dx^2 + dy^2 + dz^2) \qquad (42)$$

For this metric we have:

$$R^{o}_{\alpha o \beta} = R \ddot{R} \delta_{\alpha\beta} \qquad (43)$$

$$R^{\alpha}_{\beta\gamma\delta} = \dot{R}^2 (\delta^{\alpha}_{\gamma} \delta_{\beta\delta} - \delta^{\alpha}_{\beta} \delta_{\delta\gamma}) \qquad (44)$$

$$R_{oo} = -3 \ddot{R}/R \qquad (45)$$

$$R_{\alpha\beta} = (R \ddot{R} + 2 \dot{R}^2)\delta_{\alpha\beta} \qquad (46)$$

$$R = 6\left[(\dot{R}/R)^2 + \ddot{R}/R\right]$$

Substituting above relations in eq.(27) we have:

$$\frac{d}{dt}\left[2(\dot{R}/R)^2 - \ddot{R}/R\right] = 0 \qquad (47)$$

using D = lnR

$$\frac{d}{dt}(\ddot{D} - \dot{D}^2) = 0 \qquad (48)$$

$$\ddot{D} - \dot{D}^2 = - m_f^2 a^2 \qquad (49)$$

$$D = \pm m_f \quad ; \quad m_f > 0 \quad \text{(positive mass)}$$

or

$$R = R_o \exp(a\, m_f t) \qquad (50)$$

a = const. and $a\, m_f$ has dimension of inverse time.

The duration of the inflation corresponds to the decay of the massive spin-2 particles, or the oscillation time to massless spin-2 particle, i.e., as was pointed out in ref.[12], every spin-2 particle produced in interaction is to be regarded as a combination of massive and massless states corresponding to the eigenvalues of the mass matrix. The mass-mixing term in curved space can be written [13]:

$$L_m = - g^{1/2}(m_2^2/4k_1^2)(f^{\mu\nu} - g^{\mu\nu})(f^{\alpha\beta} - g^{\alpha\beta})(g_{\mu\alpha}g_{\nu\beta} - g_{\mu\nu} g_{\alpha\beta}) \qquad (51)$$

where as before $f^{\mu\nu} = \eta^{\mu\nu} + k_1 f'^{\mu\nu}$ (k_1 being the coupling constant for massive field) and $g_{\mu\nu} = k_2 h_{\mu\nu}$.

Up to quadratic terms, L_m becomes:

$$L_m = (m_2^2/4k_1^2)(k_1 f'^{\mu\nu} - k_2 h^{\mu\nu})(k_1 f'^{\alpha\beta} - k_2 h^{\alpha\beta}) \cdot \cdot (\eta_{\mu\alpha}\eta_{\nu\beta} - \eta_{\mu\nu}\eta_{\alpha\beta}) \qquad (52)$$

Introducing 2-component vectors [12]:

$$\psi_{\mu\nu} = \begin{bmatrix} f'_{\mu\nu} \\ h_{\mu\nu} \end{bmatrix}, \quad \psi^T_{\mu\nu} = \begin{bmatrix} f'_{\mu\nu}, h_{\mu\nu} \end{bmatrix},$$

the mixing term may be written as:

$$L_m = \psi^T_{\mu\nu} H \psi^{\mu\nu} - \psi^T H \psi \quad , \quad \psi = \psi^\mu_\mu$$

$$H = H^T \quad \text{is} \quad H = \frac{m_2^2}{4k_1^2} \cdot \begin{bmatrix} k_1^2 & -k_1 k_2 \\ -k_1 k_2 & k_2^2 \end{bmatrix} \tag{53}$$

which is not diagonal with respect to $f'_{\mu\nu}$ and$h_{\mu\nu}$ i.e. eigenvalue of H are $m_2^2/4 \ (1 + k_2^2/k_1^2)$ and 0 with eigenstates $\bar{f}'_{\mu\nu}$, $\bar{h}_{\mu\nu}$ related to $f'_{\mu\nu}$, $h_{\mu\nu}$ by a rotation angle θ (mixing angle)

$$\bar{f}_{\mu\nu} = \cos\theta \ f'_{\mu\nu} - \sin\theta \ h_{\mu\nu}$$

$$h'_{\mu\nu} = \sin\theta \ f_{\mu\nu} + \cos\theta \ h_{\mu\nu}$$

$$\cos\theta = k_1/\sqrt{k_1^2+k_2^2} \quad , \quad \sin\theta = k_2/\sqrt{k_1^2+k_2^2}$$

Thus the time evolution of massive spin-2 particles in the early universe is described by:

$$|f'_{\mu\nu}(t)\rangle = \cos\theta \exp(-iE_1 t)\exp[-(\Gamma/2)t](\cos\theta|f'_{\mu\nu}\rangle \quad \sin\theta|h_{\mu\nu}\rangle$$

$$+ \sin\theta \exp(-iE_2 t)(\sin\theta|f'_{\mu\nu}\rangle + \cos\theta|h_{\mu\nu}\rangle \tag{54}$$

$$E_1 = (|p_1|^2 + m_2^2)^{1/2} \quad , \quad E_2 = |p_2|$$

p_1 , p_2 are momenta of the massive and massless spin-2 particles respectively. The exponential damping of the state $|f'_{\mu\nu}\rangle$ in time is due to decay of massive tensor field with mean life $\tau = \Gamma^{-1}$. Massless gravitons have infinite lifetime and its amplitude does not decay. If at t = 0 we have purely massive spin-2 particles at any later time the number of massless spin-2 particles would be:

$$|\langle h_{\mu\nu}|f'_{\mu\nu}(t)\rangle|^2 = \frac{k_1^2 k_2^2}{(k_1^2+k_2^2)^2}[1+ \exp(-\Gamma t) \cdot 2\exp(-\Gamma t/2)\cos\Delta E t \tag{55}$$

$\Delta E = E_1 - E_2$. For $t >> \tau$, we have only massless spin-2 particles with a total intensity reduced by factor $k_1^2 k_2^2/(k_1^2+k_2^2)^2$ with respect to initial intensity of massive spin-2 particles.

We did not consider the possibility of decay to a massive scalar, i.e. we assumed the spin-2 massive field to have only five degrees of freedom. For the above type of mass lagrangian we can introduce Steuckelberg [14] fields to constrain the spin-2 field to have only five degrees of freedom. If we make the substitution:

$$f_{\mu\nu} \Rightarrow f_{\mu\nu} - \partial_\mu A_\nu - \partial_\nu A_\mu = K_{\mu\nu}$$

the mass term is replaced by

$$f_{\mu\nu} \Rightarrow -(1/2)m_2^2[(f_{\mu\nu} - \partial_\mu A_\nu - \partial_\nu A_\mu)(h^{\mu\nu} + \partial^\mu A^\nu + \partial^\nu A^\mu) - (h^\mu_\mu - 2\partial_\mu A^\mu)^2]$$

This is now gauge invariant under $f_{\mu\nu} \Rightarrow f_{\mu\nu} + \partial_\mu \zeta_\nu + \partial_\nu \zeta_\mu$ provided A_μ transforms as : $A_\mu \Rightarrow A_\mu - \zeta_\mu$ giving the field equations $\partial^\mu K_{\mu\nu} - \partial_\nu K^\mu_\mu = 0$, which gives four of the required conditions on $f_{\mu\nu}$ when going to the gauge $A_\mu = 0$. By requiring a new invariance under which only A_μ transforms as $A_\mu \Rightarrow A_\mu - \partial_\mu \zeta$, the variation of the mass term above is then $\partial L_m = 2m_2^2(\partial^\mu\partial^\nu f_{\mu\nu} - \partial^\mu\partial_\mu f^\nu_\nu)\zeta$ + surface terms. Terms that are quadratic in A_μ (in L_m) take the form $-(m_2^2/2)F^{\mu\nu}F_{\mu\nu}$, $F_{\mu\nu}$ is the usual spin-1 field strength. The right side of δL_m equals $2m_2^2\ R^{(L)}\zeta$, where $R^{(L)}$ is the linearized Ricci scalar. The variation δL_m may therefore be cancelled by adding the term $m^2\phi\ R^{(L)}$, where ϕ transforms as $\delta\phi = -2\zeta$. Field equation for ϕ is $R^{(L)} = 0$ and this together with trace of the $f_{\mu\nu}$ equation gives the required constraint $f^\mu_\mu = 0$ which eliminates the massive scalar.

4. Conclusions

What is the initial source of the massive spin-2 particles in the early universe? To answer this, it is to be noted that in most models of space compactification in higher dimensional unified theories such as Klein-Kaluza theories, one obtains consistent lower-dimensional theories with infinite towers of massive spin-2 particles interacting with gravity. For instance the five-dimensional metric $g_{AB}(x.y)$ may be expanded in Fourier modes [15]:

$$g_{AB}(x,y) = \sum_n g_{(n)AB}(x)\ e^{iny},$$

x denotes the co-ordinates of four-dimensional space-time and y is a co-ordinate on the circle with period 2π. The five dimensional general co-ordinate transformation parameters $\zeta^A(x,y)$ may similarly be expanded in Fourier series giving rise to an infinite number of four dimensional gauge symmetries $\zeta^A(x,y)=\sum_n \zeta^A_{(n)}(x)\ e^{iny}$ The n = 0 term in above sum, describes gravity and a massless spin-one field. Each term with $n \neq 0$, describes a massive spin-2 field with mass $\sim n/R_c$, R_c = compactification radius. The vector and scalar parts of $g_{(n)AB}(x)$, when $n \neq 0$ will be absorbed by Higgs mechanism, as all but the $n = 0$ symmetries are spontaneously broken. These are the A and ϕ fields discussed before. Again in superstrings theories one also obtains infinite towers of massive spin-2 fields and also higher spins and masses that increase indefinitely. So in the early universe when all the fundamental forces were unified one had a description in terms of superstring or Klein-Kaluza type of framework. As the universe expanded, the internal space became compactified generating the infinite towers of massive spin-2 particles. So if one had inflation induced by the massive spin-2 fields in the curved R-W space as described above, one gets a natural way of diluting away all the indefinitely large spectrum of higher spins and masses to very low values, so that they do not contribute much to the present background density. Without such a mechanism, all these indefinitely large remnants (of masses and spins) of compactification would have created very serious problems for cosmological observations! Many of the massive spin-2 particles could have larger masses than M_{Pl}, so that they would have formed miniblackholes of spin 2. The evaporation time of a 10 M_{Pl} miniblackhole would be $\sim 10^3\, t_{Pl}$ so that their decay over this time scale would give an expansion factor of $\sim e^{1000}$ (cf.eq.50), which is more than sufficient inflation.

Again the evaporation of the several miniblackholes, would generate sufficient amount of entropy during the inflationary phase. As shown in the other paper of these proceedings [16] the evaporation of $\sim 10^{60}$ blackholes of masses $\sim M_{Pl}$ would generate an entropy comparable to that seen in the microwave background. The generation of such a large amount of entropy in a time scale of a few times t_{Pl} would naturally resolve the flatness and horizon problems.

Moreover the evaporation of these blackholes would be most likely to violate CP invariance and also baryon number, as we know in any case that baryon number is not conserved in black hole decay or collapse. A small violation of CP of $\sim 10^{-9}$ in such decays is sufficient to produce the observed baryon asymmetry. In other words the evaporation of these miniblackholes in the early universe is capable not only of producing the observed entropy but also the observed net baryon number.

The present zero value of the cosmological constant can also be understood in the above picture. The effective cosmological constant driving the inflation was in this model related to the mass of the massive spin-2 particles generated in the early universe, i.e. $\Lambda \approx m_2^2 c^2/\hbar^2$ (see eqs.31,32,41 and 50). So when the massive particles decay to massless spin-2 particles (eqs.51-55), Λ tends to drop to zero ($m_2 \rightarrow 0$) and the inflation stops. So the end of the inflationary phase and the vanishing of the effective cosmological constant are both smoothly connected in this picture. In the more conventional models these are difficult questions to resolve. In short the inflationary phase that can be induced by the coupling of massive spin-2 particles to curved space-time may resolve several difficulties associated with early universe cosmology and particle physics.

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