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RELATIVISTIC EFFECTS ON RADIATIVE TRANSFER EQUATION: ORDER OF MAGNITUDE STUDY

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ABSTRACT

We have analyzed how the high gas velocities which are observed in the objects such as the atmospheres of supergiant stars, novae, supernovae, accretion discs around high density objects (like black holes, neutron stars etc.) would change the coefficients of different terms in the transfer equation which is solved in a comoving frame. Velocities as low as 5000 km s^{-1} are enough to show the relativity effects in different terms in the transfer equation. We have considered the terms of the order of V/C where V and C are the gas velocity and the velocity of light respectively. We have neglected the terms containing V^2/C^2 and higher powers of V/C. The comoving frame is adopted because it is convenient and natural to treat the photon transfer in a medium which is expanding at a very high radial velocity. The rest frame calculations would require a larger grid of angle-frequency-radius which is not practicable in our calculations. We have, in this investigation, considered the medium expanding in a steady state (time independent) conditions. Further more its velocity and density are set to change according to the equation of continuity given by $4\pi r^2 \rho V$ = constant.

We have fixed the variation of velocity by the following four cases :

$$V(r) = K$$
$$V(r) = Kr^{-1}$$
$$V(r) = Kr^{-2}$$
$$V(r) = Kr^{-3}$$

Where K is a constant and V(r) is the velocity at the radial point of r. In all the cases we found that the relativistic effects of the order of V/C are felt at a velocity of the gas as low as 5000 km s^{-1}. This suggests that the relativity effects must be taken into account in almost all the cases where the line profiles are studied in the expanding atmospheres.

Key Words: radiative transfer, comoving frame, relativistic effects, equation of continuity

1. Introduction

Mills (1983) reports objects moving with velocity 30000 km s^{-1} ($V/C \simeq 0.1$) and Kundt (1985) reports that gas in SS 433 moves with the velocity whose $V/C \simeq 0.26$ and he believes that this factor is nearly equal to 1. There are several objects in which the matter moves with extremely large velocities. In such cases we shall have to consider the relativity effects while calculating the line profiles formed in such objects. Before we really make an attempt to solve the transfer equation we should try to see at what range of velocities relativistic effects start becoming significant.

It is well known that it is convenient to calculate the radiation field in a comoving frame instead of rest frame There are several advantages in working with a comoving frame In the comoving frame the observer in the fluid frame need not worry about the Doppler shifts in the frequency dependent absorption and emission coefficients The scattering integral in a comoving frame corresponds to that given in a static medium In the case of rest frame the scattering integral is strongly dependent on the velocity field as the scattering integral involves the redistribution function which describes the scattered photons This function creates a strong angle-frequency-radius dependence and this changes continuously throughout the atmosphere because of the changing velocity field In the case of comoving frame such difficulty does not exist

The velocity gradients are well taken care of in a comoving frame rather than in a rest frame When the matter is moving with the constant velocity the transverse velocity gradients do exist although the radial velocity gradients do not exist. In a rest frame the transverse velocity gradients are totally neglected and therefore this effect is not very well taken care of However, in the comoving frame, these transverse velocity gradients are taken into consideration automatically In the next section, we shall demonstrate the relativity effects in radiative transfer equation

2 Discussion

The non-relativistic transfer equation in a comoving frame is written as

$$\mu \frac{\partial I(r,\mu,\nu)}{\partial r} + \frac{1-\mu^2}{r} \frac{\partial I(r,\mu,\nu)}{\partial \mu} = \eta(r,\mu,\nu) - \chi(r,\mu,\nu) I(r,\mu,\nu)$$

$$+ \frac{\nu}{r} \frac{V}{C} \{1-\mu^2 \ (1- \frac{d\ln V}{d\ln r})\} \frac{\partial I(r,\mu,\nu)}{\partial \nu} \tag{1}$$

Where the two terms on the left hand side of the above equation refer to the divergence of the transfer of the radiant flux and the combination of the first two terms on the right hand side represent the excess of emission over absorption and the third term of right hand side represent the comoving term One can see that if $V/C = 0$ then this term vanishes The comoving frame transfer equation to the order $0(V/C)$ in shperical geometry is given by (Mihalas 1978)

$$\{(\mu_0 + V/C) \frac{\partial}{\partial r_0} + \frac{1-\mu^2_0}{r_0} [1+ \frac{\mu_0 V}{C} (1- \frac{d\ln V}{d\ln r_0})] \frac{\partial}{\partial \mu_0}$$

$$- \frac{\nu_0 V}{C r_0} [1-\mu_0^2 \ (1- \frac{d\ln V}{d\ln r_0})] \frac{\partial}{\partial \nu_0}$$

$$+ \frac{3v}{Cr_0}\left[1-\mu_0^2\left(1-\frac{d\ln V}{d\ln r_0}\right)\right]\} \; I^0(r_0,\mu_0,\nu_0)$$
$$= \eta^0(\nu_0)-\chi^0(\nu_0)I^0(r_0,\mu_0,\nu_0) \qquad (2)$$

where

$$\mu_0 = \frac{\mu-\beta}{1-\beta\mu}\;, \quad \beta = V/C$$
$$\frac{\nu}{\nu_0} = 1+\mu_0\;, \quad \frac{\nu_0}{\nu} = 1-\beta\mu \qquad (3)$$

If we compare equations (1) and (2) we immediately notice that the coefficient of the term $\frac{\partial I}{\partial r}$ contains the angle influenced by the aberration effects. Similarly the curvature term $\frac{\partial I}{\partial \mu}$ is changed considerably and the effects of the velocity together with the aberration are involved in the relativistic curvature term. The frequency derivative term contains the aberration effects together with the change in the frequency because of transformation of coordinate system. However, the most important change is the term given by

$$\frac{3v}{Cr_0}\left[1-\mu_0^2\left(1-\frac{d\ln V}{d\ln r_0}\right)\right] \; I^0(r_0,\mu_0,\nu_0) \qquad (4)$$

This is an extra term in the relativistic radiation transfer equation. If we observe carefully this term seems to be approximately three times the coefficient of the frequency derivative term and therefore even when we consider similar velocities of the order of 1000 or 2000 km s^{-1} this term given in equation (4) becomes rather important.

We would like to study the changes that are introduced in the coefficients of various terms of equation (1) and (2). Let us assume that the matter is flowing according to the equation of continuity or mass conservation. Let

$$V(r) = K$$
$$V(r) = Kr^{-1}$$
$$V(r) = Kr^{-2}$$
$$V(r) = Kr^{-3}$$

where K is a constant and V is the velocity at the point r. Let

$$\beta = V/C$$
$$D = \mu_0 = \frac{\mu-\beta}{1-\beta\mu}$$
$$E = \mu_0+\beta$$

$$F = (1-\mu_0^2)\ (1+\alpha\beta\mu_0)$$
$$G = (1-\alpha\mu_0^2)\ \beta$$
$$H = 3\beta(1-\alpha\mu_0^2) \qquad (6)$$

where

$$\alpha = 1 - \frac{r}{V} \times \frac{dV}{dr} = 1 - \left(\frac{d\ln V}{d\ln r}\right)$$

Therefore

$$\alpha = 1 \quad \text{for} \quad V = K$$
$$\alpha = 2 \quad \text{for} \quad V = Kr^{-1}$$
$$\alpha = 3 \quad \text{for} \quad V = Kr^{-2}$$
$$\alpha = 4 \quad \text{for} \quad V = Kr^{-3} \qquad (7)$$

We shall now try to calculate the various terms given in the form of D,E,F,G and H in equation (6) We shall adopt four Gauss points for the cosine of the angle i e. μ, given by

$$\mu_1 = 0\ 06943$$
$$\mu_2 = 0.33001$$
$$\mu_3 = 0\ 66999$$
$$\mu_4 = 0.93057 \qquad (8)$$

We have presented the results in the following four tables with α = 1,2,3 and 4 respectively We have considered the velocities 100,1000,2000,3000,4000,5000,6000,7000,8000,9000, 10000,20000,30000,40000,50000,100000 km s^{-1}

Let us examine the results given in Table 1 for $\alpha = 1$ When matter is at rest ($\beta = 0$) then the quantities H and G are 0 which is expected. As we increase the velocity to 100 km s^{-1} D, E, F change slightly whereas G and H introduce changes which are considerable in magnitude. At 1000 km s^{-1} the quantities D, E and F do change but do not have much effect. However, the quantities G and H introduce the substantial modification from 2000 km s^{-1} onwards. We see that all quantities from D to H are effected substantially. A change in the second place in the quantity D is enough to completely destabilize the solution of radiative transfer equation. However, the quantity E which is nothing but summation of D and β increases substantially with the velocity and this will improve the stability of the solution of the equation of transfer considerably However the effect of the terms, F, G and H is yet to be investigated. These terms are representive of velocity fields. As the velocity increases to 30000 km s^{-1} and goes up to 100000 km s^{-1} the term E becomes strong and this is a happy sign of obtaining a stable solution.

Table 1. Quantities D, E, F, G and H for $\alpha = 1$

μ	V (km s^{-1})	β (=V/C)	D	E	F	G	H
0.0694		0.0000	0.0694	0.0694	0.9952	0.0000	0.0000
0.3300		0.0000	0.3300	0.3300	0.8911	0.0000	0.0000
0.6700		0.0000	0.6700	0.6700	0.5511	0.0000	0.0000
0.9306		0.0000	0.9306	0.9306	0.1340	0.0000	0.0000
0.0694	100	0.0003	0.0691	0.0694	0.9952	0.0003	0.0010
0.3300	100	0.0003	0.3297	0.3300	0.8914	0.0003	0.0009
0.6700	100	0.0003	0.6698	0.6701	0.5515	0.0002	0.0006
0.9306	100	0.0003	0.9305	0.9309	0.1342	0.0000	0.0001
0.0694	1000	0.0033	0.0661	0.0694	0.9958	0.0033	0.0100
0.3300	1000	0.0033	0.3270	0.3304	0.8940	0.0030	0.0089
0.6700	1000	0.0033	0.6681	0.6715	0.5548	0.0018	0.0055
0.9306	1000	0.0033	0.9301	0.9335	0.1353	0.0004	0.0013
0.0694	2000	0.0067	0.0628	0.0695	0.9965	0.0066	0.0199
0.3300	2000	0.0067	0.3241	0.3307	0.8969	0.0060	0.0179
0.6700	2000	0.0067	0.6663	0.6730	0.5585	0.0037	0.0111
0.9306	2000	0.0067	0.9297	0.9363	0.1366	0.0009	0.0027
0.0694	3000	0.0100	0.0595	0.0695	0.9971	0.0100	0.0299
0.3300	3000	0.0100	0.3211	0.3311	0.8998	0.0090	0.0269
0.6700	3000	0.0100	0.6644	0.6744	0.5622	0.0056	0.0168
0.9306	3000	0.0100	0.9292	0.9392	0.1378	0.0014	0.0041
0.0694	4000	0.0133	0.0561	0.0695	0.9976	0.0133	0.0399
0.3300	4000	0.0133	0.3181	0.3314	0.9026	0.0120	0.0360
0.6700	4000	0.0133	0.6626	0.6759	0.5660	0.0075	0.0225
0.9306	4000	0.0133	0.9288	0.9421	0.1391	0.0018	0.0055
0.0694	5000	0.0167	0.0528	0.0695	0.9981	0.0166	0.0499
0.3300	5000	0.0167	0.3151	0.3317	0.9055	0.0150	0.0451
0.6700	5000	0.0167	0.6607	0.6774	0.5697	0.0094	0.0282
0.9306	5000	0.0167	0.9283	0.9450	0.1404	0.0023	0.0069
0.0694	6000	0.0200	0.0495	0.0695	0.9985	0.0200	0.0599
0.3300	6000	0.0200	0.3121	0.3321	0.9083	0.0181	0.0542
0.6700	6000	0.0200	0.6588	0.6788	0.5734	0.0113	0.0340
0.9306	6000	0.0200	0.9278	0.9479	0.1417	0.0028	0.0084
0.0694	7000	0.0233	0.0462	0.0695	0.9989	0.0233	0.0699
0.3300	7000	0.0233	0.3090	0.3324	0.9110	0.0211	0.0634
0.6700	7000	0.0233	0.6569	0.6803	0.5772	0.0133	0.0398
0.9306	7000	0.0233	0.9274	0.9507	0.1430	0.0033	0.0098

Table 1 (Contd)

μ	V (km s^{-1})	β (=V/C)	D	E	F	G	H
0 0694	8000	0 0267	0 0428	0 0695	0.9993	0 0266	0 0799
0 3300	8000	0 0267	0 3060	0.3327	0 9138	0 0242	0 0726
0 6700	8000	0 0267	0 6550	0 6817	0.5809	0 0152	0 0457
0.9306	8000	0 0267	0 9269	0 9536	0 1443	0.0038	0 0113
0 0694	9000	0 0300	0 0395	0 0695	0.9996	0 0300	0.0899
0 3300	9000	0 0300	0 3030	0 3330	0 9165	0 0273	0.0818
0.6700	9000	0 0300	0 6531	0 6831	0 5847	0.0172	0 0516
0.9306	9000	0 0300	0 9264	0 9565	0 1457	0.0043	0.0128
0 0694	10000	0 0334	0 0362	0 0695	0 9999	0.0333	0.0999
0.3300	10000	0 0334	0 3000	0 3333	0 9191	0 0304	0.0911
0 6700	10000	0 0334	0 6512	0 6845	0.5885	0.0192	0 0576
0 9306	10000	0.0334	0 9260	0 9593	0.1470	0 0048	0.0143
0 0694	20000	0 0667	0 0027	0.0694	1.0002	0 0667	0.2001
0 3300	20000	0 0667	0 2692	0 3359	0 9442	0 0619	0.1856
0 6700	20000	0 0667	0 6315	0 6982	0 6265	0.0401	0.1203
0 9306	20000	0 0667	0 9210	0 9877	0.1610	0.0101	0.0304
0 0694	30000	0 1001	- 0 0309	0 0692	0 9960	0 1000	0 2999
0 3300	30000	0 1001	0 2378	0.3379	0.9659	0.0944	0.2832
0 6700	30000	0 1001	0 6109	0 7109	0.6651	0 0627	0.1882
0.9306	30000	0 1001	0 9158	1 0158	0 1761	0 0161	0.0484
0 0694	40000	0 1334	- 0 0646	0 0688	0 9872	0 1329	0.3986
0 3300	40000	0 1334	0 2056	0 3391	0 9840	0 1278	0 3834
0 6700	40000	0 1334	0 5892	0 7227	0 7041	0 0871	0 2613
0.9306	40000	0 1334	0 9102	1 0436	0 1925	0.0229	0 0687
0 0694	50000	0 1668	- 0 0985	0.0683	0 9740	0.1652	0.4955
0 3300	50000	0.1668	0 1727	0 3395	0.9981	0 1618	0.4854
0 6700	50000	0 1668	0.5665	0.7333	0.7432	0.1133	0.3398
0 9306	50000	0 1668	0 9041	1 0709	0.2101	0.0305	0 0914
0.0694	100000	0 3336	- 0 2704	0.0632	0.8433	0.3092	0.9275
0.3300	100000	0 3336	0.0040	0 3296	0.9987	0 3336	1.0007
0.6700	100000	0.3336	0 4332	0 7668	0 9297	0 2710	0.8129
0.9306	100000	0 3336	0 8657	1 1993	0 3228	0.0836	0 2507

Table 2. Quantities D, E, F, G and H for $\alpha = 2$

μ	V (km s^{-1})	β (=V/C)	D	E	F	G	H
0.0694		0.0000	0.0694	0.0694	0.9952	0.0000	0.0000
0.3300		0.0000	0.3300	0.3300	0.8911	0.0000	0.0000
0.6700		0.0000	0.6700	0.6700	0.5511	0.0000	0.0000
0.9306		0.0000	0.9306	0.9306	0.1340	0.0000	0.0000
0.0694	100	0.0003	0.0691	0.0694	0.9953	0.0003	0.0010
0.3300	100	0.0003	0.3297	0.3300	0.8915	0.0003	0.0008
0.6700	100	0.0003	0.6698	0.6701	0.5516	0.0000	0.0001
0.9306	100	0.0003	0.9305	0.9309	0.1342	- 0.0002	- 0.0007
0.0694	1000	0.0033	0.0661	0.0694	0.9961	0.0033	0.0099
0.3300	1000	0.0033	0.3270	0.3304	0.8950	0.0026	0.0079
0.6700	1000	0.0033	0.6681	0.6715	0.5560	0.0004	0.0011
0.9306	1000	0.0033	0.9301	0.9335	0.1357	- 0.0024	- 0.0073
0.0694	2000	0.0067	0.0628	0.0695	0.9969	0.0066	0.0199
0.3300	2000	0.0067	0.3241	0.3307	0.8989	0.0053	0.0158
0.6700	2000	0.0067	0.6663	0.6730	0.5610	0.0007	0.0022
0.9306	2000	0.0067	0.9297	0.9363	0.1374	- 0.0049	- 0.0146
0.0694	3000	0.0100	0.0595	0.0695	0.9976	0.0099	0.0298
0.3300	3000	0.0100	0.3211	0.3311	0.9027	0.0079	0.0238
0.6700	3000	0.0100	0.6644	0.6744	0.5659	0.0012	0.0035
0.9306	3000	0.0100	0.9292	0.9392	0.1391	- 0.0073	- 0.0218
0.0694	4000	0.0133	0.0561	0.0695	0.9983	0.0133	0.0398
0.3300	4000	0.0133	0.3181	0.3314	0.9065	0.0106	0.0319
0.6700	4000	0.0133	0.6626	0.6759	0.5709	0.0016	0.0049
0.9306	4000	0.0133	0.9288	0.9421	0.1408	- 0.0097	- 0.0290
0.0694	5000	0.0167	0.0528	0.0695	0.9990	0.0166	0.0498
0.3300	5000	0.0167	0.3151	0.3317	0.9102	0.0134	0.0401
0.6700	5000	0.0167	0.6607	0.6774	0.5759	0.0021	0.0064
0.9306	5000	0.0167	0.9283	0.9450	0.1425	- 0.0121	- 0.0362
0.0694	6000	0.0200	0.0495	0.0695	0.9995	0.0199	0.0597
0.3300	6000	0.0200	0.3121	0.3321	0.9139	0.0161	0.0483
0.6700	6000	0.0200	0.6588	0.6788	0.5809	0.0026	0.0079
0.9306	6000	0.0200	0.9278	0.9479	0.1443	- 0.0144	- 0.0433
0.0694	7000	0.0233	0.0462	0.0695	1.0000	0.0233	0.0698
0.3300	7000	0.0233	0.3090	0.3324	0.9175	0.0189	0.0567
0.6700	7000	0.0233	0.6569	0.6803	0.5859	0.0032	0.0096
0.9306	7000	0.0233	0.9274	0.9507	0.1460	- 0.0168	- 0.0504

Table 2 (Contd.)

μ	V (km s^{-1})	β = (V/C)	D	E	F	G	H
0.0694	8000	0.0267	0.0428	0.0695	1.0004	0.0266	0.0798
0.3300	8000	0.0267	0.3060	0.3327	0.9212	0.0217	0.0651
0.6700	8000	0.0267	0.6550	0.6817	0.5909	0.0038	0.0114
0.9306	8000	0.0267	0.9269	0.9536	0.1478	- 0.0192	- 0.0575
0.0694	9000	0.0300	0.0395	0.0695	1.0008	0.0299	0.0898
0.3300	9000	0.0300	0.3030	0.3330	0.9247	0.0245	0.0735
0.6700	9000	0.0300	0.6531	0.6831	0.5959	0.0044	0.0132
0.9306	9000	0.0300	0.9264	0.9565	0.1496	- 0.0215	- 0.0645
0.0694	10000	0.0334	0.0362	0.0695	1.0011	0.0333	0.0998
0.3300	10000	0.0334	0.3000	0.3333	0.9282	0.0274	0.0821
0.6700	10000	0.0334	0.6512	0.6845	0.6010	0.0051	0.0152
0.9306	10000	0.0334	0.9260	0.9593	0.1514	- 0.0258	- 0.0715
0.0694	20000	0.0667	0.0027	0.0694	1.0004	0.0667	0.2001
0.3300	20000	0.0667	0.2692	0.3359	0.9608	0.0570	0.1711
0.6700	20000	0.0667	0.6315	0.6982	0.6519	0.0135	0.0405
0.9306	20000	0.0667	0.9210	0.9877	0.1703	- 0.0465	- 0.1394
0.0694	30000	0.1001	- 0.0309	0.0692	0.9929	0.0999	0.2996
0.3300	30000	0.1001	0.2378	0.3379	0.9884	0.0888	0.2663
0.6700	30000	0.1001	0.6109	0.7109	0.7035	0.0254	0.0762
0.9306	30000	0.1001	0.9158	1.0158	0.1909	- 0.0678	- 0.2033
0.0694	40000	0.1334	- 0.0646	0.0688	0.9787	0.1323	0.3969
0.3300	40000	0.1334	0.2056	0.3391	1.0103	0.1221	0.3664
0.6700	40000	0.1334	0.5892	0.7227	0.7554	0.0408	0.1223
0.9306	40000	0.1334	0.9102	1.0436	0.2133	- 0.0876	- 0.2629
0.0694	50000	0.1668	- 0.0985	0.0683	0.9578	0.1635	0.4906
0.3300	50000	0.1668	0.1727	0.3395	1.0261	0.1568	0.4705
0.6700	50000	0.1668	0.5665	0.7335	0.8074	0.0597	0.1792
0.9306	50000	0.1668	0.9041	1.0709	0.2377	- 0.1059	- 0.3176
0.0694	100000	0.3336	- 0.2704	0.0632	0.7597	0.2848	0.8544
0.3300	100000	0.3336	- 0.0040	0.3296	0.9973	0.3336	1.0007
0.6700	100000	0.3336	0.4332	0.7668	1.0471	0.2083	0.6250
0.9306	100000	0.3336	0.8657	1.1993	0.3952	- 0.1664	- 0.4993

Table 3. Quantities D, E, F, G and H for $\alpha = 3$

μ	V (km s^{-1})	$\beta = (V/C)$	D	E	F	G	H
0.0694		0.0000	0.0694	0.0694	0.9952	0.0000	0.0000
0.3300		0.0000	0.3300	0.3300	0.8911	0.0000	0.0000
0.6700		0.0000	0.6700	0.6700	0.5511	0.0000	0.0000
0.9306		0.0000	0.9306	0.9306	0.1340	0.0000	0.0000
0.0694	100	0.0003	0.0691	0.0694	0.9953	0.0003	0.0010
0.3300	100	0.0003	0.3297	0.3300	0.8916	0.0002	0.0007
0.6700	100	0.0003	0.6698	0.6701	0.5517	- 0.0001	- 0.0003
0.9306	100	0.0003	0.9305	0.9309	0.1342	- 0.0005	- 0.0016
0.0694	1000	0.0033	0.0661	0.0694	0.9963	0.0033	0.0099
0.3300	1000	0.0033	0.3270	0.3304	0.8960	0.0023	0.0068
0.6700	1000	0.0033	0.6681	0.6715	0.5573	- 0.0011	- 0.0034
0.9306	1000	0.0033	0.9301	0.9335	0.1361	- 0.0053	- 0.0160
0.0694	2000	0.0067	0.0628	0.0695	0.9973	0.0066	0.0198
0.3300	2000	0.0067	0.3241	0.3307	0.9008	0.0046	0.0137
0.6700	2000	0.0067	0.6663	0.6730	0.5635	- 0.0022	- 0.0066
0.9306	2000	0.0067	0.9297	0.9363	0.1382	- 0.0106	- 0.0319
0.0694	3000	0.0100	0.0595	0.0695	0.9982	0.0099	0.0297
0.3300	3000	0.0100	0.3211	0.3311	0.9056	0.0069	0.0207
0.6700	3000	0.0100	0.6644	0.6744	0.5697	- 0.0032	- 0.0097
0.9306	3000	0.0100	0.9292	0.9392	0.1404	- 0.0159	- 0.0477
0.0694	4000	0.0133	0.0561	0.0695	0.9991	0.0132	0.0396
0.3300	4000	0.0133	0.3181	0.3314	0.9103	0.0093	0.0279
0.6700	4000	0.0133	0.6626	0.6759	0.5759	- 0.0042	- 0.0127
0.9306	4000	0.0133	0.9288	0.9421	0.1425	- 0.0212	- 0.0636
0.0694	5000	0.0167	0.0528	0.0695	0.9998	0.0165	0.0496
0.3300	5000	0.0167	0.3151	0.3317	0.9149	0.0117	0.0351
0.6700	5000	0.0167	0.6607	0.6774	0.5821	- 0.0052	- 0.0155
0.9306	5000	0.0167	0.9283	0.9450	0.1447	- 0.0264	- 0.0793
0.0694	6000	0.0200	0.0495	0.0695	1.0005	0.0199	0.0596
0.3300	6000	0.0200	0.3121	0.3321	0.9195	0.0142	0.0425
0.6700	6000	0.0200	0.6588	0.6788	0.5884	- 0.0060	- 0.0181
0.9306	6000	0.0200	0.9278	0.9479	0.1469	- 0.0317	- 0.0950
0.0694	7000	0.0233	0.0462	0.0695	1.0011	0.0232	0.0696
0.3300	7000	0.0233	0.3090	0.3324	0.9241	0.0167	0.0500
0.6700	7000	0.0233	0.6569	0.6803	0.5946	- 0.0069	- 0.0206
0.9306	7000	0.0233	0.9274	0.9507	0.1491	- 0.0369	- 0.1107

Table 3 (Contd)

μ	V (km s^{-1})	β = (V/C)	D	E	F	G	H
0.0694	8000	0.0267	0.0428	0.0695	1.0016	0.0265	0.0796
0.3300	8000	0.0267	0.3060	0.3327	0.9286	0.0192	0.0576
0.6700	8000	0.0267	0.6550	0.6817	0.6009	- 0.0077	- 0.0230
0.9306	8000	0.0267	0.9269	0.9536	0.1513	- 0.0421	- 0.1263
0.0694	9000	0.0300	0.0395	0.0695	1.0020	0.0299	0.0896
0.3300	9000	0.0300	0.3030	0.3330	0.9330	0.0218	0.0653
0.6700	9000	0.0300	0.6531	0.6831	0.6072	- 0.0084	- 0.0252
0.9306	9000	0.0300	0.9264	0.9565	0.1536	- 0.0473	- 0.1418
0.0694	10000	0.0334	0.0362	0.0695	1.0023	0.0332	0.0997
0.3300	10000	0.0334	0.3000	0.3333	0.9373	0.0244	0.0731
0.6700	10000	0.0334	0.6512	0.6845	0.6135	- 0.0091	- 0.0272
0.9306	10000	0.0334	0.9260	0.9593	0.1558	- 0.0524	- 0.1573
0.0694	20000	0.0667	0.0027	0.0694	1.0005	0.0667	0.2001
0.3300	20000	0.0667	0.2692	0.3359	0.9775	0.0522	0.1566
0.6700	20000	0.0667	0.6315	0.6982	0.6772	- 0.0131	- 0.0393
0.9306	20000	0.0667	0.9210	0.9877	0.1797	- 0.1031	- 0.3092
0.0694	30000	0.1001	- 0.0309	0.0692	0.9898	0.0998	0.2994
0.3300	30000	0.1001	0.2378	0.3379	1.0108	0.0831	0.2493
0.6700	30000	0.1001	0.6109	0.7109	0.7418	- 0.0120	- 0.0359
0.9306	30000	0.1001	0.9158	1.0158	0.2057	- 0.1517	- 0.4551
0.0694	40000	0.1334	- 0.0646	0.0688	0.9701	0.1318	0.3953
0.3300	40000	0.1334	0.2056	0.3391	1.0365	0.1165	0.3495
0.6700	40000	0.1334	0.5892	0.7227	0.8068	- 0.0056	- 0.0167
0.9306	40000	0.1334	0.9102	1.0436	0.2342	- 0.1982	- 0.5945
0.0694	50000	0.1668	- 0.0985	0.0683	0.9415	0.1619	0.4858
0.3300	50000	0.1668	0.1727	0.3395	1.0540	0.1519	0.4556
0.6700	50000	0.1668	0.5665	0.7333	0.8715	0.0062	0.0186
0.9306	50000	0.1668	0.9041	1.0709	0.2652	- 0.2422	- 0.7266
0.0694	100000	0.3336	-0.2704	0.0632	0.6761	0.2604	0.7812
0.3300	100000	0.3336	- 0.0040	0.3296	0.9960	0.3336	1.0007
0.6700	100000	0.3336	0.4332	0.7668	1.1645	0.1457	0.4372
0.9306	100000	0.3336	0.8657	1.1993	0.4675	- 0.4165	- 1.2494

Table 4. Quantities D, E, F, G and H for $\alpha = 4$

μ	V (km s^{-1})	$\beta = (V/C)$	D	E	F	G	H
0.0694		0.0000	0.0694	0.0694	0.9952	0.0000	0.0000
0.3300		0.0000	0.3300	0.3300	0.8911	0.0000	0.0000
0.6700		0.0000	0.6700	0.6700	0.5511	0.0000	0.0000
0.9306		0.0000	0.9306	0.9306	0.1340	0.0000	0.0000
0.0694	100	0.0003	0.0691	0.0694	0.9953	0.0003	0.0010
0.3300	100	0.0003	0.3297	0.3300	0.8917	0.0002	0.0006
0.6700	100	0.0003	0.6698	0.6701	0.5519	- 0.0003	- 0.0008
0.9306	100	0.0003	0.9305	0.9309	0.1343	- 0.0008	- 0.0025
0.0694	1000	0.0033	0.0661	0.0694	0.9965	0.0033	0.0098
0.3300	1000	0.0033	0.3270	0.3304	0.8969	0.0019	0.0057
0.6700	1000	0.0033	0.6681	0.6715	0.5585	- 0.0026	- 0.0079
0.9306	1000	0.0033	0.9301	0.9335	0.1365	- 0.0082	- 0.0246
0.0694	2000	0.0067	0.0628	0.0695	0.9977	0.0066	0.0197
0.3300	2000	0.0067	0.3241	0.3307	0.9027	0.0039	0.0116
0.6700	2000	0.0067	0.6663	0.6730	0.5659	- 0.0052	- 0.0155
0.9306	2000	0.0067	0.9297	0.9363	0.1391	- 0.0164	- 0.0492
0.0694	3000	0.0100	0.0595	0.0695	0.9988	0.0099	0.0296
0.3300	3000	0.0100	0.3211	0.3311	0.9084	0.0059	0.0176
0.6700	3000	0.0100	0.6644	0.6744	0.5734	- 0.0077	- 0.0230
0.9306	3000	0.0100	0.9292	0.9392	0.1416	- 0.0246	- 0.0737
0.0694	4000	0.0133	0.0561	0.0695	0.9998	0.0132	0.0395
0.3300	4000	0.0133	0.3181	0.3314	0.9141	0.0079	0.0238
0.6700	4000	0.0133	0.6626	0.6759	0.5808	- 0.0101	- 0.0303
0.9306	4000	0.0133	0.9288	0.9421	0.1442	- 0.0327	- 0.0981
0.0694	5000	0.0167	0.0528	0.0695	1.0007	0.0165	0.0495
0.3300	5000	0.0167	0.3151	0.3317	0.9197	0.0101	0.0302
0.6700	5000	0.0167	0.6607	0.6774	0.5883	- 0.0124	- 0.0373
0.9306	5000	0.0167	0.9283	0.9450	0.1468	- 0.0408	- 0.1224
0.0694	6000	0.0200	0.0495	0.0695	1.0015	0.0198	0.0595
0.3300	6000	0.0200	0.3121	0.3321	0.9252	0.0122	0.0367
0.6700	6000	0.0200	0.6588	0.6788	0.5958	- 0.0147	- 0.0442
0.9306	6000	0.0200	0.9278	0.9479	0.1495	- 0.0489	- 0.1467
0.0694	7000	0.0233	0.0462	0.0695	1.0022	0.0232	0.0695
0.3300	7000	0.0233	0.3090	0.3324	0.9306	0.0144	0.0433
0.6700	7000	0.0233	0.6569	0.6803	0.6033	- 0.0170	- 0.0509
0.9306	7000	0.0233	0.9274	0.9507	0.1521	- 0.0570	- 0.1709

Table 4 (Contd.)

μ	V (km s^{-1})	$\beta = (V/C)$	D	E	F	G	H
0.0694	8000	0.0267	0.0428	0.0695	1.0027	0.0265	0.0795
0.3300	8000	0.0267	0.3060	0.3327	0.9360	0.0167	0.0501
0.6700	8000	0.0267	0.6550	0.6817	0.6109	- 0.0191	- 0.0573
0.9306	8000	0.0267	0.9269	0.9536	0.1548	- 0.0650	- 0.1951
0.0694	9000	0.0300	0.0395	0.0695	1.0032	0.0298	0.0895
0.3300	9000	0.0300	0.3030	0.3330	0.9412	0.0190	0.0570
0.6700	9000	0.0300	0.6531	0.6831	0.6184	- 0.0212	- 0.0636
0.9306	9000	0.0300	0.9264	0.9565	0.1575	- 0.0730	- 0.2191
0.0694	10000	0.0334	0.0362	0.0695	1.0035	0.0332	0.0995
0.3300	10000	0.0334	0.3000	0.3333	0.9464	0.0214	0.0641
0.6700	10000	0.0334	0.6512	0.6845	0.6260	- 0.0232	- 0.0697
0.9306	10000	0.0334	0.9260	0.9593	0.1602	- 0.0810	- 0.2431
0.0694	20000	0.0667	0.0027	0.0694	1.0007	0.0667	0.2001
0.3300	20000	0.0667	0.2692	0.3359	0.9942	0.0474	0.1421
0.6700	20000	0.0667	0.6315	0.6982	0.7025	- 0.0397	- 0.1191
0.9306	20000	0.0667	0.9210	0.9877	0.1890	- 0.1597	- 0.4790
0.0694	30000	0.1001	- 0.0309	0.0692	0.9867	0.0997	0.2991
0.3300	30000	0.1001	0.2378	0.3379	1.0333	0.0774	0.2323
0.6700	30000	0.1001	0.6109	0.7109	0.7801	- 0.0493	- 0.1479
0.9306	30000	0.1001	0.9158	1.0158	0.2205	- 0.2356	- 0.7069
0.0694	40000	0.1334	- 0.0646	0.0688	0.9615	0.1312	0.3936
0.3300	40000	0.1334	0.2056	0.3391	1.0628	0.1109	0.3326
0.6700	40000	0.1334	0.5892	0.7227	0.8581	- 0.0519	- 0.1556
0.9306	40000	0.1334	0.9102	1.0436	0.2550	- 0.3087	- 0.9260
0.0694	50000	0.1668	- 0.0985	0.0683	0.9252	0.1603	0.4809
0.3300	50000	0.1668	0.1727	0.3395	1.0820	0.1469	0.4406
0.6700	50000	0.1668	0.5665	0.7333	0.9357	- 0.0473	- 0.1420
0.9306	50000	0.1668	0.9041	1.0709	0.2927	- 0.3785	- 1.1356
0.0694	100000	0.3336	- 0.2704	0.0632	0.5925	0.2360	0.7080
0.3300	100000	0.3336	- 0.0040	0.3296	0.9947	0.3335	1.0006
0.6700	100000	0.3336	0.4332	0.7668	1.2819	0.0831	0.2494
0.9306	100000	0.3336	0.8657	1.1993	0.5399	- 0.6665	- 1.9994

In Table 2 we have given the quantities D,E,F,G and H for $\alpha = 2$. Now here ofcourse there is no change in the value of E. However, the changes introduced by the values of F,G,H or H are of a different nature. Some of these terms when neglected would cause considerable amount of uncertainity in the process of obtaining solution. Some of these terms become even magnified for the angle whose cosine is equal to 0.9306 even for small velocities like 100 km $\bar{s}^1$. This trend of change of numbers with the velocity suggest that we shall have to modify our way of thinking of solving the transf

In Tables 3 and 4 we have listed the same quantities for $\alpha = 3$ and 4. numbers in the quantities in G and H are getting magnified in these case velocity these terms will have to be dealt with extreme caution.

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