The cosmological constant as a unifying link

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1. Introduction

Einstein introduced the cosmological constant term into his field equations in order to get a static model of the universe [1]. This term introduced a repulsive force increasing with distance to balance the mutually attractive gravitational forces between the various bodies in the universe. This Λ term brought negative pressure, with an equation of state $p = -\rho c^2$, into play Later when Hubble discovered the universal expansion between galaxies and evidence for a dynamic evolving universe, Einstein abandoned the Λ term, calling it his biggest blunder [2].

However Lemaitre and others showed that even while retaining the cosmological constant, it was possible to get expanding cosmological model and indeed de-Sitter in 1917 had already obtained an exponentially expanding (with time) universe with no matter but only a Λ term Many cosmologists retained the cosmological constant term as it had the effect of increasing the age of the universe consistent with the age of the oldest stars in the globular clusters [3] Even quite recently when the Hubble space telescope (HST) team, first obtained a Hubble constant, $H_0 \geq 80$ km/s/Mpc, implying an age for a closed universe of only about ten Gyr, a Λ term was suggested as a panacea

In modern quantum field theory a vacuum zero point energy of virtual particle-antiparticle pairs is present and has to be considered. In calculating quantum transitions between states A and B, say, the vacuum energy is cancelled out, but once gravity is involved, vacuum energy also contributes to gravity A covariant quantum vacuum energy has an equation of state $p = -\rho c^2$ (or $T_{\alpha\beta} = \Lambda g_{\alpha\beta}$), precisely that of an effective cosmological constant Phase transitions in the early universe are also accompanied by large changes in the quantum vacuum energy [4]

2. Dark energy and an accelerating universe

The past few years have seen some exciting developments in observational cosmology [4,5] An impressive variety of recent observations, which include luminosity evolutions of high redshift supernovae strongly suggest that the universe is accelerating at the present epoch The deceleration parameter in cosmology is given by $q_0 = (1/2)\Omega_m - \Omega_\Lambda$, where Ω_m and Ω_Λ are the relative contributions of the matter and vacuum energy in terms of the critical density ρ_c Normally one would expect the expansion of the universe to slow down owing to the combined gravitational attraction of all its constituents This gives a deceleration parameter, $+q_0$, indicating the slow down in the expansion of the Universe However a dark energy like ω_Λ term causes a net repulsive gravity force (due to negative pressure causing the Universe to accelerate) An overall cosmic repulsion as implied by a Λ term would make q_0 negative and cause a cosmic acceleration The recent observations [6] independently corroborate this, the universe indeed appears to be in an acceleration phase with a negative q_0 of around $q_0 = -0\ 55$ Again analysis of the anisotropies in the cosmic microwave background (CMBR) as implied by the Wilkinson microwave anisotropy probe (WMAP) satellite and other projects is consistent with a flat universe having a total Ω, i e , $\Omega_m + \Omega_\Lambda$ of one, i e , a universe with a density equal to the critical density Taken with the supernova results this in turn implies a Ω_Λ of at least 0 7 and Ω_m of about 0 3 The early universe nucleosynthesis [7], when about one-fourth of the the hydrogen was converted to helium and trace amounts of deuterium (10^{-5}) and Lithium (10^{-9}) (which are fully consistent with observations of these abundances) imply a baryon fraction of around five percent The present observations are consistent with this, implying a baryon fraction of only four per cent This suggests an inventory of the universe of about 70% dark energy (DE), 26% dark matter (DM) and four per cent baryons [8] The dark energy has an equation of state like that for the cosmological constant, i e ,

$$P = -w\rho c^2$$

where P is the pressure and ρ the density, and $w \approx -1$, corresponds to the cosmological constant term The DE exerts a negative pressure causing a cosmic repulsion and since this dominates, the universe is in an accelerating phase (q_0 is negative) Current indications support $w = -1$ Thus ironically the Λ term which Einstein discarded now seems to overwhelmingly dominate the dynamic of the universe

3. What is the dark matter (DM)?

As mentioned in the previous section 2, about 26% of the universe in the form of non-baryonic dark matter (readers may see article by V Trimble in the present book for an introduction)

The deduction of DM has quite a tradition and goes back to over seventy years when Fritz Zwicky observed that several massive clusteres of galaxies (including the Coma cluster) have velocity dispersions $< v^2 >$ which imply a dynamical mass (given by $M_{dyn} \approx< v^2 > R_c/G$, R_c is cluster size) at least a hundred times more than the visible mass as deduced from their luminosity So the luminous mass in the form of stars, etc , is only a small fraction of the total mass gravitationally binding the cluster This means that most of the mass is non-luminous, i e , dark This soon became the paradigm for the other large structures as well, including large spiral galaxies binary galaxies etc [9] For individual spiral galaxies like our own it was observed from the rotation curves (i e , from the velocities of various stars, nebulae etc. at different distances from the galactic centre) that while one would expect the velocities of stars further away from the central region to show a Keplerian drop (i e., $\propto R^{-1/2}$), the velocities even at large distances ($> 20kpc$) tend to remain constant, i e , a flat rotation curve The light from the galaxy on the contrary comes mainly from the core and falls off exponentially in the outer regions(with increasing radius) This suggests that there is a progressive increase in DM in the galactic halo (keeping the orbital velocities constant, $M \propto R$, would keep the velocity of rotation of the galaxy at a distance R, V, constant as seen from Kepler's laws) So DM progressively increases as one goes to the outer regions implying that at least 90% of the total matter in individual galaxies is unknown DM. This is also true for binary galaxies, elliptical galaxies, dwarf galaxies, etc. It was thought that the DM could be in the form of low mass stars or compact evolved objects like white dwarfs, neutron stars or solar mass black holes But results from gravitational microlensing (like that of MACHOS, OGLE and EROS projects) suggest that only a small fraction of the galactic halo mass could be in the form of such objects [10] The bulk of the DM is expected to be of the non-baryonic form. An early suggestion made by several authors in the 1960s [11-13], was that massive neutrinos could constitute the bulk of the DM Once the case for the hot dense early phase for the universe was made by the discovery of the CMBR by Penzias and Wilson in 1965, it was well established even much earlier by the work of Alpher, Herman, Follin and others that the early universe would have left an equally profuse relic of primordial neutrinos (several hundred per cubic cm at the present epoch) So even if the neutrino had a small mass (a few electron volts) the combined mass of so many neutrinos would far exceed the baryon mass (see Trimble's article in the present book) Even formation of large structures with massive neutrinos was discussed as early as 1964 Neutrino oscillations [14] (involving three species of massive neutrinos) were discussed in 1961 while a degenerate massive neutrino Sea [13] dominating the mass was described in 1965. However the conjecture of several authors in the 1960's that massive neutrinos could dominate cosmic dynamics is now known not to be consistent with cosmological observations and current experiments on neutrino oscillations which yield too low a mass (implying that neutrinos perhaps contribute hardly one per cent to the universe mass) Again neutrinos being relativistic at decoupling constitute what is called hot dark matter which forms only very large scale structures at first. This is clearly inconsistent with observations which imply a bottom-up scenario, where the smallest structures formed first.

It is now believed that the DM could be either the axion (μeV) or the neutralino (a supersymmetry candidate weighing a few GeV) There are several other particle physics candidates which have been postulated ranging from cento PeV champs to glueballs, gluinos, wimpzillas and what not! There are several ongoing searches with many clever experimental techniques to look for such particles [15] Many of the experiments have been going on for years So far there is no confirming evidence Several possible candidates have been ruled out while, upper flux limits have been put on several others Unlike DE, DM is clustered in halos of galaxies or in galaxy clusters and exerts positive pressure $P_D = \rho_D v^2$, like ordinary matter. One has also to understand why at the present epoch DM and DE are within an order of magnitude of each other (the cosmic coincidence problem) and why the baryon density is only 0 04. In the succeeding sections we shall try to understand some of these aspects

4. Cosmological constant and interaction coupling strengths

The present author has long since been advocating a cosmological constants dominated universe Some of these arguments have to do with the evolution of quantum vacuum energy in an expanding universe [16-18]

Again it is remarkable that the coupling constant of various fundamental interactions and masses of elementary particles have remained invariant for the entire Hubble age Of course there were recent claims that spectral observations of distant quasars implied increase of the electromagnetic fine structure constant α, with epoch, [19]. However the latest results as well as recent sensitive laboratory experiments comparing measurements of hyperfine transitions in rubidium-87 and Cesium-133 over a four years period are still consistent with a zero-time variation in the fine structure constant [20] Thus the remarkable constancy of the fundamental constants and particle masses is a 'steady state feature' of the evolving universe So if the cosmological constant is again included as a fundamental cosmic parameter (it sets a cosmic scale!) it introduces a steady state feature (at least asymptotically) It may be natural to link the vacuum energy to local physical parameters [21]

We have already given several examples of this in earlier works [22, 23] One such intriguing relation arises if for instance we consider a wave packet of spread r Its gravitational self energy density is then given by

$$E_g \approx \frac{G(\hbar/rc)^2}{8\pi r^4} = \frac{G\hbar^2}{8\pi r^6 c^2}, \tag{1}$$

(from the uncertainty principles the energy of the packet is $\approx \hbar c/r$), in which G is the Newtonian gravitational constant, $\hbar$ the Planck's constant, and c the velocity of light

To be bound by its gravitational attraction this should be at least the same as the repulsive energy density of the cosmological vacuum given by

$$E_c = \frac{\Lambda c^4}{8\pi G}. \tag{2}$$

This gives

$$r^6 = \frac{L_{pl}^4}{\Lambda}, \tag{3}$$

where L_{pl} is Planck's length $\sqrt{G\hbar/c^3} = 1.6 \times 10^{-33}$ cm.

So the required size of the wave packet can be written:

$$r = \frac{L_{pl}^{2/3}}{\Lambda^{1/6}} = \frac{1}{\Lambda_{pl}^{1/3}\Lambda^{1/6}}, \tag{4}$$

in which $\Lambda_{pl} = L_{Pl}^{-2}$, is the Planck curvature

If this localized packet acquires an electron charge e, then its electrostatic self energy can be calculated This precisely works out to be the electron rest mass m_e. Thus we can write: (There was absolutely no apriori reason to expect this)

$$m_e = \frac{\alpha\hbar}{c}\Lambda_{pl}^{1/3}\Lambda^{1/6}, \tag{5}$$

(where α is the electromagnetic fine structure constant).

If we use the pion-nucleon strong interaction constant $\frac{g^2}{\hbar c} \approx 14$, instead of α, in the above relation, we would get the proton rest mass. In a recent work we had obtained the relation for the τ-lepton mass as [24]

$$m_\tau = \frac{\alpha\hbar}{c}\sqrt{\frac{\hbar c}{G_F}}, \tag{6}$$

where G_F is the universal Fermi-weak interaction constant $G_F = 1\ 5 \times 10^{-49}$ erg cm^3. We have $m_e/m_\gamma = \Lambda_{pl}^{1/3}\Lambda^{1/6}\Lambda_w^{-1/2}$ (Λ_w is the 'weak' curvature $(\hbar c/G_F)$ The muon mass was obtained as·

$$m_\mu = \frac{3}{2}\frac{\hbar}{c}\Lambda_{pl}^{1/3}\Lambda^{1/6}. \tag{7}$$

The masses of several other particles were obtained similarly in [25] Again the quantum chromodynamic (QCD) energy scale, Λ_{QCD}, and the quark-gluon dimensionless strong coupling constant [26] can be related to Λ and G through the relation

$$\alpha_s = \sqrt{\Lambda}\frac{c^6\hbar^2}{G\Lambda_{QCD}^2} = \sqrt{\Lambda}\frac{\hbar c^5}{G}\frac{\hbar c}{\Lambda_{QCD}}\frac{1}{\Lambda_{QCD}^2}. \tag{8}$$

Using $\Lambda \approx 10^{-56}$ cm^{-2} (as implied by recent observations), $\Lambda_{QCD} = 160$ MeV, α_s turns out to about 0.13. α is related to the weak interaction as,

$$\alpha = \Lambda_{Pl}^{2/3}\Lambda^{1/6}\sqrt{\frac{G_F}{\hbar c}}. \tag{9}$$

The proton-electron mass ratio, another fundamental ratio can be expressed as,

$$\frac{m_p}{m_e} = \frac{1}{2\Lambda_{pl}^{1/3}}\sqrt{\frac{\hbar c}{G_F}}\frac{1}{\Lambda^{1/6}} \tag{10}$$

All these intriguing relations show a subtle link between the underlying cosmic vacuum energy and the coupling constants (particle masses) of the fundamental interactions. This is a natural culmination of the pioneering attempts made by Zeldovich [43] in this direction. The above relations all imply a cosmological vacuum dark energy of $\approx 10^{-29}$ gm cm^{-3} precisely what is found in the latest cosmological observations.

5. Origin of dark energy

There have been recent attempts to understand the cosmological coincidence between ρ_Λ and ρ_m etc. [28] This framework assumes a mass scale around the electroweak scale m_w and together with the Planck scale M_{pl} one has

$$\rho_\Lambda^{1/4} \approx \frac{M_w^2}{M_{pl}} \tag{11}$$

This is almost similar to our earlier work [26, 29], where a cosmological constant of the observed magnitude (i.e , $\Lambda \approx 10^{-56}$ cm^{-2}) was obtained through the electroweak vacuum made up of the weak boson condensate which (for $\hbar = c = 1$) gives,

$$\Lambda \approx \frac{8\pi G M_w^7}{M_{pl}^3} \approx 10^{-56} cm^{-2} \tag{12}$$

The above arguments suggest that Λ may also be related to the QCD strong interaction scale, ($\Lambda_{QCD} \approx 160$ MeV, close to m_π), i.e

$$\Lambda \approx 8\pi G^2 \Lambda_{QCD}^6 \tag{13}$$

Eqs (10) and (11) and other equations along with similar results in the cited references [28, 29] above give rise to the intriguing relation connecting electroweak, strong and gravity mass scales as

$$M_w^7 = M_{pl} \Lambda_{QCD}^6 \tag{14}$$

Defining $L_w = \sqrt{G_F/\hbar c} = 7 \times 10^{-17}$ cm as the weak scale length, and L_{QCD} as the strong interaction (quark-gluon) length scale corresponding to Λ_{QCD} ($L_{QCD} \approx 1/\Lambda_{QCD} \approx 10^{-13}$ cm), then we get the following intriguingly beautiful relation between the strong QCD scale, Fermi Weak Constant (G_F), the Newtonian constant (G_N) and the cosmological constant (Λ), as ($\hbar = c = 1$)·

$$\frac{L_{QCD}^3}{L_w^2}\sqrt{\Lambda} \approx \frac{G}{G_F} \approx 10^{-33}. \tag{15}$$

As G_F is related to α, the electromagnetic coupling through eqn (8), eqn (13) connects the interaction strengths of all four fundamental interactions to the cosmological vacuum energy A similar earlier hypothesis by the present author published in 1982 (much before there was talk of a dominating cosmic vacuum energy!), gave the following [30], relation for Λ in terms of G, electric charge e, pion-nucleon charge, g, Fermi constant G_F, $\hbar$, c as

$$\Lambda = \frac{G^2 e^{16}}{g^8} \frac{\hbar}{c^7 G_F^3} \tag{16}$$

This gives exactly the vacuum energy $\rho_\Lambda = \Lambda c^4/8\pi G$, deduced from the recent high-z supernovae observations. The value for the Hubble constant, H_0 obtained was,

$$H_0 = \frac{Gc^8}{g^4}\sqrt{\frac{\hbar}{c^5 G_F^3}} = 70km/s/Mpc \quad (17)$$

The recent WMAP results have fixed H_0 as 71 km/s/Mpc Eqs (15-17) contain the actual measured values of the coupling constants as occurring in the low energy universe It was also interpreted in terms of a residual dominating cosmological constant, Λ related to H_0 as $\Lambda = H_0^2/c^2$, $\rho_c = 10^{-29}$ gm cm^{-3}.

6. Vacuum energy and large scale structure

Given a Λ dominated universe, the requirement that for the various large scale structures (held together by self gravity) to form on a variety of length scales their gravitational self energy density should at least match the ambient vacuum energy repulsion, was shown to imply, especially in ref. [30, 32] (and in earlier works cited therein), a scale invariant mass-radius relationship of the form (for the various structures):

$$\frac{M}{R^2} \approx \frac{\sqrt{\Lambda}}{G}c^2, \quad (18)$$

(essentially $GM^2/R^4 \approx \Lambda c^4/8\pi G$) This can be easily shown to imply rough equality of ρ_Λ, ρ_m, etc

Eq (18) predicts a universality of M/R^2 for a large variety of structures For a typical spiral galaxy, $M_{gal} \approx 10^{12} M_\odot$, $R = 30kpc$ and for globular cluster $R \approx 100pc$, $M = 10^6 M_\odot$, and for clusters $M_c \approx 10^{16} M_\odot$ and $R_c = 3Mpc$ For these and other structures, M/R^2 is the same and equals $\sqrt{\Lambda}c^2/G$ (as given by Eq (18))

7. The six numbers of Rees

Martin Rees has enumerated the six numbers needed to fix the universe [33] Two of these are the vacuum energy term (Λ) and E the strength of the force that binds atomic nuclei together It would appear as if all the six numbers of Rees are independent entities The above arguments in the previous sections would imply Λ as a connecting link Let us see if Λ and E, two of Rees' numbers could be linked This has to do with the nuclear binding energy For a nucleus of mass number A and radius r, the binding surface energy can be written as $4\pi r^2(A^{2/3} - 1)T$, where T is the 'surface tension' of the nuclear force, i e , energy per unit area (the nucleus behaves like a liquid drop) For the helium nucleus $A = 4$, so that $A^{2/3} \approx 16^{1/3} \approx 2\ 5$ So the nuclear binding energy now becomes: (for the helium nucleus) $\Delta E_n = 6\pi r^2 T$ Now for T, which is essentially the energy per unit area, we use the same value as in Eq (18), i e,

$$T = \frac{\sqrt{\Lambda}}{G}c^4 \approx 10^{20} ergs/cm^2 \quad (19)$$

This gives (when substituted into ΔE_n) for the binding energy of the helium nucleus as $\Delta E_n = 4\ 5 \times 10^{-5} ergs$, i e ,

$$4\pi(\hbar/m_\pi c)^2(A^{2/3} - 1)\frac{c^4}{G}\sqrt{\Lambda} = \Delta E_n = 4\ 5 \times 10^{-5} ergs \quad (20)$$

This is precisely the binding energy released in the conversion of hydrogen to helium, the ΔE_n which is 0.007 fraction of the rest Energy So Eq. (18), not only gives the surface energy (energy per unit area) of the large scale structures (galaxies, globular clusters, galactic clusters, etc.) but also the nuclear surface tension T of the atomic nucleus In fact r in Eq. (3) corresponds to the nuclear radius. Thus the cosmological vacuum energy also seems to fix T for the atomic nucleus, providing a connecting link Many similar relations exist [34].

8. Why is the baryon fraction only 4 percent?

As noted above, $\rho_B = 0.04\rho_c$, how do understand this small ratio? The total baryon mass is $m_B < 6 \times 10^{54}$ gm. In an earlier paper [35], published in 1982, this upper limit was fixed as follows: In general relativity, the limiting luminosity is $c^5/G = 3 \times 10^{59}$ ergs/sec. So baryons (like H, etc.) undergoing nuclear reactions in all kinds of objects cannot exceed the Eddington luminosity $L_E = 4\pi GMm_pc/\sigma_T$ So the combined luminosity ΣL_E should be limited by $\Sigma L_E \leq c^5/G$. This fixes,

$$\Sigma M_B < \frac{\sigma_T c^4}{4\pi G^2 m_p} = 6 \times 10^{54} gm, \tag{21}$$

in which σ_T is the Thompson cross-section, and m_p the proton mass.

This gives the baryonic density as,

$$\rho_B = \frac{\sigma_T c H_0^3}{8\pi^3 G^2 m_p} = 3 \times 10^{-31} gm.cm^{-3}, \tag{22}$$

(23)

and

$$\frac{\rho_B}{\rho_c} = \frac{8\pi}{3}\frac{Ge^{12}}{g^4 m_e^2 m_p}\sqrt{\frac{\hbar}{c^1 1 G_F^3}} \approx 4\ 1\% \tag{24}$$

This estimate would be valid whatever be the conditions under which the nuclear reactions take place, either in the early universe or inside supermassive objects. It was shown that this gives the photon-baryon ration as $m_\gamma/m_B \approx (\hbar c/Gm_p^2)^{1/4} \approx 2.5 \times 10^9$ [35]

9. The ratio of $\rho_{DM}/\rho_{vac} \approx 0.3$

Consider the DM to consist of collisionless particles just bound by their self gravity [36, 37] All of them have the same kinetic energy given by,

$$T = (1/2)m_d v_d^2 = GMm_d/2r, \tag{25}$$

$M = \Sigma m_d$, where m_d is the mass of the DM particle, M is a function of the radius, i e., $M = M(r)$

We thus have· $M(r) = (2Tr/Gm_d)$, $(dM(r)/dr) = 4\pi r^2 \rho(r)$, so

$$\begin{aligned} \rho(r) &= \frac{T}{2\pi r^2 G m_d} \\ &= \frac{v_d^2}{4\pi G r^2} = \rho_d(r), \end{aligned} \tag{26}$$

Now the vacuum energy is $\rho_\Lambda = (\Lambda c^2)/(8\pi G)$ (over large scales). Thus

$$\frac{\rho_d}{\rho_\Lambda} = \frac{2V_d^2}{c^2}\frac{1}{\Lambda r^2}. \tag{27}$$

Now the largest structures have size $r \approx 200 Mpc$, with $\Lambda \approx 10^{-57}$ cm^{-2}, $V \approx 2000 km/s$ (largest dispersion velocities), this gives.

$$\frac{\rho_d}{\rho_\Lambda} \approx \frac{1}{3}. \tag{28}$$

So $\rho_d \approx 0,3\rho_\Lambda$.

10. Conclusion

Recent observations strongly suggest that the universe is dominated by dark energy, i.e., vacuum energy given by the cosmological constant. Dark matter of an unknown type constitutes about one-fourth of the cosmic matter while baryons account for hardly four percent. Again the latest evidence suggests that the coupling constants of various fundamental interactions have remained remarkably constant from the earliest epochs. It is suggested that the cosmic vacuum energy plays a basic role in fixing these constants and an attempt has been made to understand how the various cosmological parameters acquire their present values and the seminal role of the cosmological constant in fixing the coupling strengths of the various interactions and particle masses.

Appendix A: Detection of dark matter

There have been several ongoing experiments for more than two decades to search for dark matter. As we have stated precise measurements of the CMBR have shown that DM constitutes a fourth of the energy budget of the universe. However, the nature of this DM remains a mystery. It is only clear that massive neutrinos perhaps account for less than a percent. For general arguments as to why neutrinos cannot account for most of the DM see ref. [37] and reference therein. Present experiments are trying to search for exotic particles (predicted in many particle physics models such as supersymmetry, etc.) like neutralinos, wimps, axions etc. They are expected to weigh a GeV. The scattering of nucleons by these particles in a crystal, or in superfluid vortices, etc. are some of the techniques suggested to detect them. To date there are no detections. All strong annihilation decay (γ) radiation from the galactic center might be a signal for DM annihilation. May be CERNs LHC may produce neutralinos after 2007. Several axion detectors are also trying to look for these cold DM particles from the sun, from the galactic halo etc. For a recent summary of all such current experiments see ref. [38]. (For example, and the references cited therein). Recently the Chandra Observatory has made x ray observations of hot gas in about 26 clusters of galaxies. The results suggest that the dark energy density may even be a constant consistent with the cosmological constant first introduced by Einstein [39]. See also in this context the references [40-42]. In this context, the predictions made through Eqs. (14) and (15) appear explicitly in refs [18, 30, and 40].

Appendix B: Holography

The relations between Λ, $\Lambda - pl$, and Λ_{QCD} as given in section (4), i.e, Eqs. (3), and (11) etc. are consistent with the holographic principle which has emerged as a novel idea in discussing black hole entropy, which is proportioned to the horizon surface

In the case of a universe dominated by Λ the entropy is proportional to the horizon surface (given by $\sim 1/\Lambda$) divided by, L_{pl}^2, i e , $\sim 1/\Lambda L_{pl}^2$ This is the upper limit imposed by the holographic principle. In Eq. (3), r is $\sim 1/\Lambda_{QCD}$ So the volume entropy is proportional to $\sim 1/\Lambda^{3/2}\Lambda_{QCD}^3$ As this should not exceed the holographic bound we arrive at Eq (3), i e , $\Lambda = \Lambda_{QCD}^6/\Lambda_{pl}^2$, thus relating the strong interaction, gravitational and cosmological scales .

References

1 Einstein A, Sitzungsber *IT Akad* , 6 (1917), 142

2 Pais A, *Subtle is the Lord*, Clarendon Press, Oxford, (1982), 286

3 Bondi H, *Cosmology*, Cambridge University Press, (1961)

4 Perlmutter S, Aldering G, Goldhaber G, *Astrophys J* , 483 (1997), 565

5 Perlmutter S, Aldering G, Goldhaber G, *Nature*, 391 (1998), 51.

6 de Bernardis, Ade P, Bock J, *Nature*, 404 (2001), 955

7 Kolb E W, Turner M S, *The Early Universe*, Addison Wesley, (1990)

8 Cohn J , Astro-ph/9807128 ,
Carroll, S Astro-ph/0004075

9 Reiss A G et al , Astro-ph/0001384

10 Garnavich P M et al , *Astrophys J* , 509 (2001), 74

11 Markov M A, *Phys Lett* , 10 (1964), 122

12 Gershtein G, Zeldovich Ya B, *J Expt Theo Phys* , 4 (1966), 120

13 Hayakawa, Y, *Prog Theor Phys Suppl* , 33 (1965), 538

14 Maki Z, *Prog Theo Phys* , 28 (1961), ,
Pontercorvo B M, *Sov Phys JETP*, 7 (1962), 172

15 Bahcall et al , *Dark Matter Universe*, Worldscientific, (1991), 176

16 Sivaram C et al , *Found Phys* , 6 (1976), 65

17 Sivaram C et al , *Found Phys* , 6 (1976), 616

18 Sivaram C, *Int J Theor Phys.*, 25 (1986), 825;
Sivaram C, *Astrophys Spc. Sci* , 24 (1986), 133

19 Webb K K et al , *Phys Rev Lett.*, 87 (2001), 091301

20 Asenfelter T P et al , *Phys Rev. Lett* , 92 (2004), 041102

21 Sivaram C, *Astrophys Spc Sci* , 100 (1984), 467 , 124, 195

22 Sivaram C, *Am J Phys* , 50 (1982), 279

23 Sivaram C, *Int J Theor. Phys* , 26 (1987), 1425

24 Sivaram C, *Astrophys Spc Sci* , 271 (2000), 321

25 Sivaram C, *Astrophys Spc Sci* , 207 (1993), 317

26 Sivaram C, *Mod Phys Lett* , 34 (1999), 2463

27 Sivaram C, *Astrophys J* , 520 (1999), 317

28 Arkani-Hamed N et al , *Phys Rev Lett* , 85 (2000), 4434

29 Sivaram C, *Bull Astron Soc Ind* , 27 (1999), 377

30 Sivaram C, *Astrophys Spc Sci* , 82 (1982), 507

31 Sivaram C, *Astrophys Spc Sci* , 219 (1994), 135

32 Sivaram C, *Int J Theor Phys.*, 33 (1994), 2407

33 Rees M, *Just Six Numbers*, Cambridge University Press, (2000)

34 Sivaram C, *Astrophys Spc Sci* , 124 (1986), 195

35 Sivaram C, *Astrophys Spc Sci* , 86 (1982), 501,
Sivaram C, *Astrophys Spc Sci* , 206(1992), 313

36 Sivaram C, *Astron Astrophys* , 275 (1993), 37

37 Sivaram C, *Acta Astronomica*, 37 (1987), 201
38 Zioutask et al , *Science*, 306 (2004), 1485
39 Allen S et al , *Mon Not. Roy Astron Soc* , (2005) (in press)
40 Sivaram C, *Proc Symp Foundation of Mod. Phys* , Univ Joensuu, 1987, Finland, 105
41 Krauss L M, *Astrophys. J* , 604 (2004), 481
42 Spergel D N et al , *Astrophys J Suppl* , 148 (2003), 175