A COSMOLOGICAL PREDICTION FROM THE COUPLING CONSTANTS OF FUNDAMENTAL INTERACTIONS

(Letter to the Editor)

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Abstract. The notion of gravitational charge is used to suggest a link in the Machian sense between fundamental interactions and cosmology. The observed values of the coupling constants of these interactions then give a definite value for the total mass of the Universe.

In a recent paper (Sivaram, 1982a), it was pointed out that the gross parameters characterising the Universe, such as the overall size, can be arrived at from microphysical considerations involving the fundamental interactions of elementary particle physics. For example, the Hubble radius of the Universe was obtained in terms of the coupling constants of the four fundamental interactions as

$$R_H = \frac{g^4}{Ge^8}\,(c^7 G_\mathrm{F}^3/\hbar)^{1/2} \simeq 10^{28}\ \mathrm{cm\ s^{-1}}\,, \tag{1}$$

where G is the gravitational constant, $\hbar$ is the Planck's constant, c is the velocity of light, and G_F is the universal Fermi weak interaction constant ($= 1.5 \times 10^{-49}$ ergs cm^3). The electromagnetic and strong interaction are characterized by dimensionless coupling constants, $e^2/\hbar c = \frac{1}{137}$ and $g^2/\hbar c \simeq 14$, the strong interaction pion–nucleon coupling. Several other interesting relationships connecting the parameters of cosmology and elementary particle physics were also given in other papers (Sivaram, 1984a, b). Again in another paper (Sivaram, 1982b) it was pointed out that general relativistic considerations combined with electromagnetic laws of electron photon scattering would enable Eddington's cloud-bound observer to deduce quantities like the total mass of the Universe, entropy per baryon, etc. Sans observations. The mass of the Universe from such considerations was obtained as $M_U \simeq 4 \times 10^{78}\, m_p \simeq 10^{21} \times 4\, M_\odot$; m_p being the proton mass, thus giving $N \approx 4 \times 10^{78}$ for the number of particles. A link between N (or M_U) and the values of the coupling constants of the weak, electromagnetic, and strong interactions may be suggested if these interactions are pictured to arise in a Machian sense, wherein local physical parameters are determined by the Universe as a whole. For instance as explored in the work of Hayakawa (1965) the electric charge is interpreted as arising from $1/\sqrt{N}$ fluctuations of the electron number of the Universe. It was suggested by Sivaram (1984a) that one can similarly picture $1/\sqrt{N}$ fluctuations of the total gravitational charge of all the electrons as giving rise to the weak interaction

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force between electrons, accounting for dimensionless relations between weak and gravitational interactions. In the Machian picture, as has often been suggested, the inertial mass of a particle arises from its gravitational interactions with the rest of the particles in the Universe (i.e., involving the quantity $N \cdot Gm^2$, i.e., the product $N\sqrt{G}\,m \times \sqrt{G}\,m$, a total gravitational charge with the local charge).

The concept of the gravitational charge in understanding the discrete nature of elementary particle masses was explored by Motz (1972) and Sivaram (1974). Unlike the electric charge which has a universal value characterized by e^2, gravitational charges could take different values depending on the masses of the particles being considered, being defined as Gm^2. However, if we consider that the proton and the electron are the only stable conserved particles with rest mass, constituting the present (low-energy!) Universe as observed, we can construct three and only three possible gravitational charges, whose interactions would be characterized by the dimensionless coupling constants: i.e.,

$$Gm_p^2/\hbar c\,, \qquad Gm_p m_e/\hbar c \quad \text{and} \quad Gm_e^2/\hbar c\,;$$

where m_p and m_e are the proton and electron rest masses. In the spirit of the discussion above and in the earlier papers, if N be the total number of particles, one can express the local fluctuations in these couplings arising from the interaction of one of the gravitational charges with all the others as

$$\sqrt{N} \times Gm_p^2/\hbar c\,, \qquad \sqrt{N} \times Gm_p m_e/\hbar c \quad \text{and} \quad \sqrt{N} \times Gm_e^2/\hbar c$$

(i.e., $1/\sqrt{N} \times \sqrt{G}m \times N\sqrt{G}m \sim \sqrt{N} \times Gm^2$).

If we use for N, the figure mentioned above, i.e., 4×10^{78}, we have $\sqrt{N} = 2 \times 10^{39}$, we we thus get for the values of these three possible dimensionless coupling constants

$$\sqrt{N}\, Gm_p^2/\hbar c \simeq 13\,, \tag{2a}$$

suggesting identification with the strong interaction pion–nucleon coupling $g^2/\hbar c \simeq 14$ which characterizes all low-energy nuclear reactions and particle interactions.

$$\sqrt{N} \times Gm_p m_e/\hbar c \simeq 7.2 \times 10^{-3} \simeq \tfrac{1}{137}\,; \tag{2b}$$

which would of course be identified with the electromagnetic coupling constant $e^2/\hbar c \simeq \frac{1}{137}$, and

$$N \times Gm_e^2/\hbar c \simeq 4 \times 10^{-6}\,; \tag{2c}$$

to be identified with the weak interaction decay coupling constant which is $\approx 9 \times 10^{-6}$ or less, as determined empirically from experiments.

A priori, there was no reason to expect the dimensionless combinations (2a), (2b), and (2c) to agree so remarkably well with the actually measured empirical values of the dimensionless strong, electromagnetic, and weak interactions. Since the only physical quantity in Equations (2a)–(2c), that cannot be locally measured or determined is N, it follows that N must indeed be near about 4×10^{78}, to be consistent with the observed coupling constants of the fundamental interactions. Since N is a conserved quantity and is time-independent and if G, m_p, m_e, $\hbar$, and c are all unvarying constants it follows that in this picture despite the Machian manner of the description of the interactions, none of the coupling constants change with time. Thus the remarkable coincidences implied in Equations (2a)–(2c) do not imply any time-variation in the coupling constants of the strong, electromagnetic, and weak interactions which as was pointed out in Sivaram (1984a) is consistent with the stringent limits imposed on their variation by recent data on isotopic abundances.

Again it must be pointed out that despite remarkable progress in our understanding of elementary particle interactions and several elegant recent attempts to unify the fundamental interactions we are nowhere near explaining why the low-energy values of the coupling constants of these interactions have the values they have. The grand unified theories predict the behaviour of the couplings at very high energies which are inaccessible to present experiments and offer no insight as to why at low energies they have the observed strengths. These elegantly formulated theories, moreover, involve a plethora of particles with as many different coupling (SU(5) for instance has 24 generators) which do not shed any light as to why the low-energy interactions have the typically observed values for their coupling constants. Almost all the predictions of these theories are at near Planck energies, by far inaccessible! Considering all this, it must be again seen as remarkable that Equations (2a)–(c), do give the observed low-energy values (i.e., as they actually occur in the Universe!) of the coupling constants of the strong, electromagnetic, and weak interactions. Suggesting that one must have at least a partially open mind to Machian interpretations pointing to the fact that parameters pertinent to particle physics are not solely determined by microphysics alone but in part by influence of the whole Universe (also Sivaram, 1982c, 1983). Moreover, we are able to say now with some confidence (to account for the remarkable agreement) that N should be centered around 4×10^{78}. Again it may be noted that the slope of Regge trajectories in this Machian picture would be given by

$$S \approx \frac{1}{2\pi}\,(G\sqrt{N}/\hbar c)\,,$$

which for the above value of N, gives $\approx (1\ \text{GeV})^{-2}$, consistent with the observed mass spectroscopy of hadronic resonance states lying on rising Regge trajectories appearing in a Chew–Frautschi plot as straight lines with a universal slope of $\simeq (1\ \text{GeV})^{-2}$. Further support for this value of N can come if we try to obtain consistency with the uncertainty principle. For this it is to be noted that in typical Robertson–Walker cosmological models the position and velocity of particles are not quite independent but related by

$$V = HR \quad \text{so} \quad \Delta V = \mathrm{H}\Delta\mathrm{R}\,.$$

Then $m \times \Delta V \times \Delta R = m(\Delta V)^2/H$. The kinetic energy fluctuation $\frac{1}{2}m(\Delta V)^2$ can be equated to the fluctuation $mc^2/\sqrt{N}$ in the rest energy of the particle due to interactions with all other particles. Thus, we have:

$$m \times \Delta V \times \Delta R = 2mc^2/\sqrt{N}\,H\,. \tag{3}$$

Considering electrons ($m = m_e$) for instance, we should have the right-hand side of Equation (3), equal $\hbar/2$ to obtain consistency with the uncertainty principle $\Delta p \times \Delta R \geq \hbar/2$. Thus $2m_e c^2/\sqrt{N}\, H = \hbar/2$.

Substituting $2m_e c^2 \simeq 10^{-6}$ *ergs*, $\sqrt{N} = 2 \times 10^{39}$, $H = 10^{-18}\,\mathrm{s}^{-1}$, gives $\hbar/2 = 10^{-27}/2$ or $\hbar = 10^{-27}$ erg s. This further justifies N to be near to 4×10^{78} particles (four quinvigintillion) giving a mass for the Universe of $M_\mathrm{U} \simeq 4 \times 10^{78} \times 1.7 \times 10^{-24}\,\mathrm{g} \simeq 7 \times 10^{54}$ g, i.e., a mass of seven septendecillion grams. This predicted mass combined with the observed Hubble radius as given by Equation (1), would imply a mean density for the Universe of $\rho_\mathrm{U} \simeq 5 \times 10^{-31}\,\mathrm{g\,cc}^{-1}$. So this is another observational consequence of this prediction. Based on the measurement of deuterium and lithium abundances Mathews and Viola (1979) were able to give a Hubble constant independent observational estimate of the density of the Universe as ('best guess' estimate according to these authors) $\sim 7 \times 10^{-31}\,\mathrm{g\,cc}^{-1}$ which agrees well with the value given above. For a Hubble constant of $H = 50\,\mathrm{km\,s}^{-1}\,\mathrm{Mpc}^{-1}$, this would imply a $\Omega = \rho/\rho_c \simeq 0.1$, i.e., consistent with primordial light element abundances. Dynamics of galactic clusters would give a somewhat higher Ω of about 0.1–0.2. On the contrary, if the above value of ρ is to be regarded as a closure density as would be required by inflationary models (which imply $\Omega = 1$) the corresponding H would be $< 20\,\mathrm{km\,s}^{-1}\,\mathrm{Mpc}^{-1}$. Considering the present uncertainties in distance measurements even this value of H cannot be ruled out!

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