THE ANGULAR MOMENTUM OF THE ASTEROIDS

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(Received 24 January, 1985)

Abstract. Consideration of the basic physics involved in the structure of the object are used to obtain relationships for the radius, period, angular momentum, etc. of a typical asteroid. The mass-angular momentum relation for asteroids would tend to favour the fragmentation hypothesis.

In the case of small cold bodies like asteroids there is equilibrium between the negative pressure produced by attractive electrostatic forces between electrons and nearest neighbour ions arranged in a kind of lattice and the opposing degeneracy pressure of the free electrons. For a number density n of electrons and ions, the attractive electrostatic forces produce a negative pressure $\sim \frac{1}{2}e^2 n^{4/3}$ and the corresponding degeneracy pressure is from quantum mechanics $\sim \hbar^2 n^{5/3} m_e^{-1}$, where e, $\hbar$ and m_e are the electronic charge, Planck's constant and the electron rest mass, respectively. Thus approximate balance between these opposing pressures yields

$$n^{1/3} = \tfrac{1}{2}e^2 m_e/\hbar^2 = \frac{1}{2}\frac{e^2}{\hbar^2} m_e,$$

or for the typical density of an asteroid the relation

$$\varrho_A = n m_p = \frac{1}{8}\left(\frac{e}{\hbar}\right)^6 m_e^3 m_p \simeq 2.0\,\mathrm{g\,cm^{-3}}. \quad (1)$$

(m_p being the proton rest mass).

As the object is small and rigid we can assume the density to be uniform throughout and if it is rotating we have the criterion for stability (Ω is the rotational angular velocity)

$$\Omega^2/\pi G \varrho_A \simeq 0.32, \quad (2)$$

where G is Newton's gravitational constant. Substituting for ϱ_A from Equation (1), we get for the typical rotational period $P(=2\pi/\Omega)$ for an asteroid the value of

$$P = \frac{2\sqrt{\pi}}{0.075}\left(\frac{\hbar}{e}\right)^3 \frac{1}{m_e^{3/2} m_p^{1/2}} \frac{1}{G^{1/2}} \simeq 3 \times 10^4\,\mathrm{s} \quad (\simeq 10\,h). \quad (3)$$

Earth, Moon, and Planets **34** (1986) 231–233.

This would explain a fact previously noted by Alfvén (1964) that the spin periods of most of the asteroids are approximately equal and around 3×10^4 s; a prominently noticeable feature of many asteroids being their isochronous rotation period (~ 10 h) which is largely independent of their mass and radius (Golubeva, 1983).

An upper limit for the size of an asteroid can be estimated by defining it to be as the size where gravitational forces are able to counteract the shear modulus of the material thus tending to make the object spherical rather than an arbitrarily-shaped piece of rock. For a solid substance which is neither fully metallic nor single-crystalline the molecular bonds are responsible for shear rigidity holding the material together against elastic strain. From the theory of such materials the typical tensile strength or shear modulus is given as

$$\mu_{\text{shear}} \approx [Ry/(2\gamma_B)^3](m_e/m_p)^{1/2} \text{ dyne cm}^{-2}, \qquad (4)$$

where Ry is the Rydberg energy given by $Ry = e^4 m_e/2\hbar^2$, γ_B is the Bohr radius ($\gamma_B = \hbar^2/m_e e^2$). The factor $(m_e/m_p)^{1/2}$ takes care of the fact that vibrational energies of molecular bonds are smaller than the corresponding electronic energies as the entire mass of the molecule takes part in producing such modes. The typical sound speech in such a material would be

$$V_s \approx (\mu_{\text{shear}}/\varrho_A)^{1/2}. \qquad (5)$$

Thus hydrodynamic equilibrium would imply for the size of the object when gravitational forces tend to balance the shear forces tending to make it spherical as

$$R_A = V_s/(\pi G \varrho_A)^{1/2}. \qquad (6)$$

Substitution for V_s from Equation (5) and for μ_{shear} from Equation (4) and for ϱ_A from Equation (1), finally yields for the limiting size of the asteroid the value of

$$R_A \approx \frac{\hbar^2}{4e(\pi G)^{1/2} m_p^{5/4} m_e^{3/4}} \simeq 300\,\text{km} \quad (3 \times 10^7\,\text{cm}). \qquad (7)$$

The value of R_A as given by Equation (7) would denote the dividing line between the sizes of objects which are essentially spherical, because of their own gravity and smaller bodies which are irregular because the shear forces tending to deform them are larger. The larger asteroids are of just this size.

Equation (7) and (1) would enable the mass of the asteroid (M_A) to be deduced. The mass along with the rotation period as given by Equation (3) and the radius as given by Equation (7) would together enable the relation for angular momentum $J_A = M_A \Omega_A R_A^2$ to be written as

$$J_A \simeq \frac{0.6}{2^{29/2}\pi^2} \frac{e^4 m_e^{3/4}}{G^2 m_p^{19/4}} \hbar \simeq 10^{28}\,\text{kg}\,\text{m}^2\,\text{s}^{-1}. \qquad (8)$$

As ϱ_A and Ω_A are the same for most of the asteroids it is seen that J_A varies as

$$J_A \propto \Omega_A R_A^5 \propto R_A^5 \quad \text{(as } \Omega_A \text{ is same)}$$
$$\propto M_A^{5/3} \qquad \text{(as } \varrho_A \text{ is same).} \tag{9}$$

Thus, we have a $J_A \propto M_A^{5/3}$ as the mass-angular momentum relation for most of the asteroids from the above analysis. Empirically Hartmann and Larson (1967) showed that a $(\text{mass})^{5/3}$ power law fitted most of the asteroid data and Burns (1975) has confirmed this result drawing on a sample of about 70 asteroids and a more recent tabulation of Harris and Young (1983) would also fit the $J \propto m^{5/3}$ law for about 250 asteroids. Such a power law would tend to favour the fragmentation hypothesis for asteroid formation as for one thing, Equations (2) and (3) would imply that almost all asteroids spin close to the instability limit; and, secondly, we have the well-known result from the dynamics of rotating bodies (see, e.g., Landau and Lifshitz, 1975) asserting that the stability of the rotating body is lost (i.e., the body fragments) when the angular momentum satisfies the equation

$$J = 2.9 G^{1/2} M^{5/3} \varrho^{-1/6};$$

and if ϱ is constant, $J = \text{const. } M^{5/3}$, thus suggesting a fragmentation origin for the asteroids rather than an accretion origin. As the tidal interactions are negligible for most asteroids, they would in all probability still retain their primeval isochronous rotation and a $J \propto M^{5/3}$ law for the angular momentum.

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