tional lengthening in the stick takes on the form

$$\Delta l^0 = (v/c^2) l \Delta v/(1 - v^2/c^2)^{3/2}. \quad (8)$$

Consequently the total lengthening resulting from the acceleration from v_1 to v_2 is

$$l^0(v_2) - l^0(v_1) = \int_{v_1}^{v_2} (v/c^2) l /(1 - v^2/c^2)^{3/2}\, dv. \quad (9)$$

Remembering l is a constant in the present case, we obtain finally

$$l^0(v_2) - l^0(v_1) = l(1 - v_2^2/c^2)^{-1/2} - l(1 - v_1^2/c^2)^{-1/2}, \quad (10)$$

which is identical to Eq. (3).

Finally, it should be mentioned that the lengthening in the proper length of the stick is, of course, due to the force acting along the stick, which produces the acceleration.

[1]See, e.g., Panofsky and Phillips, *Classical Electricity and Magnetism* (Addison-Wesley, Reading, MA, 2nd ed., 1962), p. 291.
[2]R. d'E. Atkinson, Am. J. Phys. **48**, 581 (1980).
[3]A. A. Evett, Am. J. Phys. **40**, 1170 (1972).
[4]The equation is directly obtained from Lorentz transformation by setting $t_A = t_B$ in the OXY system.

Cosmological and quantum constraint on particle masses

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In his well-known book *Gravitation and Cosmology*, Steven Weinberg[1] has drawn attention to a curious relation involving the Hubble constant H_0, the gravitational constant G, Planck's constant $\hbar$, the velocity of light c, and the mass of a typical elementary particle m. The relation is[2] [Eq. (16.4.2) of Weinberg]

$$m = (\hbar^2 H_0/Gc)^{1/3} \approx m_\pi. \quad (1)$$

He considers this as a clue pointing to the fact that parameters pertinent to particle physics are not determined solely by considerations of microphysics, but in part by the influence of the whole universe. He also suggests that in considering the possible interpretation of Eq. (1), one must note the remarkable fact that it relates a single cosmological parameter, H_0, to the fundamental constants $\hbar$, G, C, and m_π. He also points out that Eq. (1) is so far unexplained. In the following discussion we shall attempt to understand the hitherto unexplained relation (1), as a simple constraint imposed on the mass of an elementary particle by combination of the uncertainty principle with H_0. We first ask the question whether the gravitational self-energy of a single particle has any meaning in the quantum sense of measurability. Is it a measurable quantity? Consider an elementary particle of mass m. By quantum mechanics we have to localize the wave packet representing it over a region of dimension $(\hbar/mc)$. The gravitational self-energy of the particle corresponding to this localization would be

$$E_p = \frac{Gm^2}{\hbar/mc} = \frac{Gm^3c}{\hbar}. \quad (2)$$

This has to be measurable at least over the Hubble age of the universe given by $(1/H_0)$. The uncertainty principle would then constrain E_p and therefore m through the relation

$$(Gm^3c/\hbar)(1/H_0) \approx \hbar, \quad (3)$$

giving

$$m \approx (\hbar^2 H_0/Gc)^{1/3}, \quad (4)$$

which is the same as Eq. (1). Weinberg's relation may then be understood as the operational requirement that the mass of an elementary particle be such that its gravitational self-energy be at least measurable over a Hubble period. The notion of the gravitational self-energy of a single particle and its measurability is usually ignored in discussions on quantum gravity.

[1]S. Weinberg, *Gravitation and Cosmology* (Wiley, New York, 1972).
[2]Reference 1, Chap. 16, p. 619.