A GENERAL UPPER LIMIT ON THE MASS AND ENTROPY PRODUCTION OF A CLUSTER OF SUPERMASSIVE OBJECTS

(*Letter to the Editor*)

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Abstract. The recent discovery of a distortion in the 3 K spectrum of the black-body background cosmic radiation has led to suggestions that a part or even all of the radiation was generated by pregalactic supermassive stars. A general upper limit on the mass of a cluster of these objects and the entropy of the radiation produced by them is obtained.

Recently a distortion of the 3 K spectrum of the cosmic background radiation shortward of its black-body peak was reported (Woody and Richards, 1979). This raised the possible question as to whether pre-galactic stars could have generated the microwave background as the distortion could be explained if some 25% of the background was radiation generated by these stars and thermalized by grains also produced by these objects (Carr, 1981; Wright, 1981). It has also been suggested by several investigators that the whole background could have been produced by pre-galactic supermassive stars which may have formed in the period between decoupling and galaxy formation (Layzer and Hively, 1973, Rees, 1978). These objects are expected to be considerably more massive than the stars forming today which makes it interesting to study the evolution of a cluster or clusters of these massive ($\sim 10^7 M_\odot$) objects which might have formed after recombination. Most of these massive objects are also expected to have culminated their evolution as black holes these remnants then generating even more radiation. As remarked earlier clusters of these objects can form well before galaxies form. The individual stars of these clusters would be much larger than 10^2–$10^3 M_\odot$ and would hence be radiation-dominated. They would emit at an Eddington luminosity (Hoyle and Fowler, 1963). For an object of mass M this is given by

$$L_E = \frac{4\pi GMc}{\kappa}, \tag{1}$$

where κ is the opacity which for these objects is given by

$$\kappa = \sigma_T/m_p, \tag{2}$$

where σ_T is the Thompson cross-section for electron photon scattering, m_p is the

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proton mass. σ_T is given by

$$\sigma_T = \frac{\delta\pi}{3}\left(\frac{e^2}{m_e c^2}\right)^2, \tag{3}$$

where e and m_e are the electron's charge and the rest mass, respectively. If there are a large number of these objects in a cluster then their total luminosity is given by

$$L_{ET} = \sum \frac{4\pi GMc}{\kappa} = \frac{4\pi Gc}{\kappa}\Sigma M, \tag{4}$$

where $\Sigma M = M_T$ = total mass of the cluster. If M_T is the total cluster mass, then general relativity imposes as is well known a lower limit on its size or spatial localization which is given by

$$R_m \sim \frac{GM_T}{c^2}. \tag{5}$$

The shortest time-scale that can be associated with the cluster is then

$$t_m \sim \frac{GM_T}{c^3}. \tag{6}$$

If during this time-scale a substantial portion $\eta(\sim 1)$ of the mass is converted into energy, then the maximum possible luminosity is given by

$$L_M \sim \frac{\eta M_T c^2}{GM_T/c^3} \sim \frac{\eta c^5}{G} \sim \eta \times 4 \times 10^{59}\ \text{ergs s}^{-1}. \tag{7}$$

With $\eta = 1$, Equation (7) becomes the so called 'Gunn luminosity' given by $c^5/G = 4 \times 10^{59}$ ergs s^{-1}. This gives the upper limit to the power that can be radiated by the cluster. As all the individual objects are radiating at their Eddington luminosity the total luminosity L_{ET} being given by Equation (4) we can equate L_{ET} to the maximum possible luminositiy given by equation (7). Thus with $L_{ET} = L_M$ and using Equations (2) and (3) for κ and σ_T, the upper limit to the cluster mass then turns out to be

$$\Sigma M = M_T \simeq \frac{e^4}{G^2 m_p m_e^2}, \tag{8}$$

or

$$M_T \sim \left(\frac{e^2}{Gm_e^2}\right)\left(\frac{e^2}{Gm_p^2}\right) m_p \sim 4 \times 10^{78}\ m_p \sim 10^{21}\ M_\odot. \tag{9}$$

The effective cluster size corresponding to the maximal luminosity would be given by Equation (5) and is: (using Equation (8) for M_T)

$$R_m \sim \left(\frac{e^2}{m_e c^2}\right)\left(\frac{e^2}{G m_p m_e}\right). \tag{10}$$

The effective temperature of the radiation produced (assuming thermalization has taken place) would be given by

$$T_{\text{eff}} \sim \left(\frac{L_T}{4\pi\sigma_{\text{SB}} R_m^2}\right)^{1/4}, \tag{11}$$

where σ_{SB} is the Stefan–Boltzmann constant given as

$$\sigma_{\text{SB}} = \frac{\pi^2 K_{\text{B}}^4}{45\hbar^3 c^2}, \tag{12}$$

K_{B} being the Boltzmann constant.

Use of Equation (7) for L_T (with $\eta \sim 1$) and Equation (10) for R_m together with Equation (12), Equation (11) then gives for T_{eff} the expression

$$T_{\text{eff}} = \left(\frac{G m_p^2 \hbar^3}{4\pi^3}\right)^{1/4}\left(\frac{m_e}{K_{\text{B}} e^2}\right) c^{11/4} \simeq 10\ \text{K}. \tag{13}$$

Although a substantial portion of the matter has been converted into energy the baryon number within the system is still conserved. So we can estimate the photon to baryon ratio or alternately the entropy per baryon as

$$S = \frac{4aT^3}{3nK_{\text{B}}}; \tag{14}$$

n being the particle density which can be estimated from the mass and radius. By use of T_{eff} as given by Equation (13), Equation (14) then gives the expression for the maximum entropy per baryon that can be produced by the cluster of supermassive stars as

$$S_{\text{max}} = \left(\frac{e^2}{m_e c^2}\right)\left(\frac{1}{L_{\text{P}}\gamma_p}\right)^{1/2}, \tag{15}$$

where L_{P} is the Planck length given by $L_{\text{P}} = (\hbar G/c^3)^{1/2}$ and $\hbar/m_{p\text{C}} = \gamma_p$ is the proton Compton wavelength. Substituting numerical values gives $S_{\text{max}} \simeq 10^9$ photons/baryon. A larger value of the photon/baryon ratio (which would be implied in a low-density Universe) thus cannot be produced by such superstellar clusters.

Equation (15) for the entropy per baryon should be compared with the expression by Rees (1978) based on a somewhat different approach – i.e., on the characteristic nuclear burning time-scale (so-called Salpeter time) for radiation dominated massive objects, i.e. $t \sim (c\sigma_{\text{T}}/4\pi G m_p)$.

Rees expression is:

$$S \sim \left(\frac{e^2}{Gm_p^2}\right)^{1/4} \left(\frac{m_p}{m_e}\right)\left(\frac{e^2}{\hbar c}\right)^{3/4} \sim 10^8 \text{ photons/baryon.}$$

which is similar to Equation (15).

Finally we note as very interesting that Equations (8) and (10) for the critical mass and radius of the cluster lead to results which compare well with the mass and radius of the Universe, and quite naturally expresses Dirac's large numbers hypothesis – i.e., that the ratio of the mass of the Universe to the proton mass is the square of the ratio of the strengths of the gravitational and electromagnetic forces, and the radius of the Universe is the classical electron radius times this ratio of these strengths (Dirac, 1938). It thus appears that Eddington's cloud-bound observer can get a very good idea of not only the masses and luminosities of stars but also about the overall mass and size and background temperature of the Universe.

References

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