# TOWARDS A UNIFICATION OF THE PARAMETERS UNDERLYING ELEMENTARY PARTICLES AND COSMOLOGY

*(Letter to the Editor)*

C. SIVARAM
*Indian Institute of Astrophysics, Bangalore, India*

(Received 16 August, 1982)

**Abstract.** It is pointed out that the gross parameters characterizing the Universe such as the overall size and mass can be arrived at from microphysical considerations involving the fundamental interactions of elementary particle physics. Interesting relations for the Hubble radius and closure density are obtained in terms of the coupling constants underlying these interactions.

In a recent paper (Sivaram, 1982a), general relativistic considerations combined with the electromagnetic laws of electron photon scattering were used to arrive at a general upper limit on parameters such as mass, size, and other quantities pertaining to a cluster of supermassive objects. It was also pointed out that Dirac's large numbers hypothesis (Dirac, 1937) emerged naturally from the treatment and one could relate the dimensions and mass of the Universe to those of a typical elementary particle. It also appeared that from a manipulation of the laws of physics, discovered locally, Eddington's cloud bound observer can get a very good idea of not only the masses and luminosities of stars but also about the overall mass, size, and background temperature of the Universe. Again in recent papers (Sivaram, 1982b, c) Weinberg's curious empirical relation (as mentioned in this well known book *Gravitation and Cosmology* (Weinberg, 1972) involving the Hubble constant $H_0$, the gravitational constant $G$, Planck's constant $\hbar$, velocity of light $c$, and the mass of a typical elementary particle $m_\pi$ – i.e.,

$$m_\pi \simeq \left(\frac{\hbar^2 H_0}{Gc}\right)^{1/3}, \tag{1}$$

was sought to be explained as a constraint on the gravitational self energy of the particle imposed by quantum gravity and cosmological considerations. In the latter paper (Sivaram, 1982c), Equation (1) was seen to arise naturally as a cosmological constraint on black-hole evaporation (a typically quantum gravitational process) giving rise to a characteristic or fundamental length, i.e.

$$l_0 = \left(\frac{3G\hbar}{32\pi c^2 H_0}\right)^{1/3} = \frac{e^2}{2m_e c^2} = \frac{\hbar}{m_\pi c}, \tag{2}$$

where $e$ and $m_e$ are the electron's charge and mass, respectively. $l_0$ is about

$1.4 \times 10^{-13}$ cms. In earlier work, while discussing the interesting interrelationships among the coupling constants of the fundamental interactions that are seen to arise in unified theories of weak, electromagnetic and strong interactions, the following relations were noted (Sivaram *et al.*, 1974a, b):

$$\frac{e^2}{2m_p c^2} = \left(\frac{G_F}{\hbar c}\right)^{1/2}, \tag{3}$$

$$\frac{g^2}{2m_p c^2} = \frac{e^2}{2m_e c^2} = \frac{\hbar}{m_\pi c} = l_0, \tag{4}$$

where $G_F$ is the universal Fermi weak interaction constant ($= 1.5 \times 10^{-49}$ ergs cm$^3$), $m_p$ is the proton mass, and $g^2/\hbar c \approx 14$ is the strong interaction pion-nucleon coupling constant analogous to the fine structure constant $\alpha = e^2/hc = 1/137$. It was noted in Sivaram (1974a, b) that $l_0$ can be taken as a fundamental length characterizing all the fundamental interactions. Also identified with the length occuring in Heisenberg's unified field theory, in Sivaram *et al.* (1975). Briefly, Equation (3) is explained by observing that according to the unified theory of weak and electromagnetic interactions, both the interactions become of the same strength at energies of $\sim 100$ GeV or equivalently at length scales $\sim (G_F/hc)^{1/2} \sim 10^{-16}$ cm. Equation (4) can be accounted for by postulating that the proton is much heavier than the electron because it takes part in the strong interactions as well, i.e. the bare masses are the same for both particles and the proton is 'dressed' in addition by the strong interaction, the ratio of their masses being the ratio $g^2/e^2$ (Sivaram *et al.*, 1974b). In fact, given the fundamental length $l_0$, one can write for the mass of a particle which takes part in interactions with coupling strengths $g(i)$, $m = \Sigma g(i)^2/2l_0c^2$, the bare mass of the particles (i.e., in the absence of any interaction) being taken as zero. The idea behind this approach was that in the absence of all interactions all particles would have identically zero mass, and all masses of the elementary particle are due to 'dressing up' by different interactions. On this basis the neutrino (taking part in only weak and gravitational interactions) would have a very small rest mass $\sim 10^{-3}$ eV, as the equivalent weak charge would be given by (Sivaram *et al.*, 1974a)

$$g_W^2/\hbar c = G_F/\hbar c \left(\frac{m_e c}{\hbar}\right)^2.$$

This is well within experimental limits! In the spirit of unification, one can define a gravitational charge as $g_g^2 = Gm_p m_e$ (Sivaram *et al.*, 1974b; Motz, 1972) which would give the smallest conceivable proper mass (as the gravitational interaction is the weakest) as (using $g_g^2$ and $l_0 = \hbar/m_\pi c$ in the above expression for $m$, i.e., $m_g = g_g^2/l_0c^2$)

$$m_g = \frac{Gm_p m_e m_\pi}{\hbar c}; \tag{5}$$

and from the uncertainty principle the maximal range associated with such fluctuations of energy due to the gravitational interaction is given by

$$R_{\mathrm{H}} = \frac{\hbar^2}{G m_p m_e m_\pi} . \tag{6}$$

Now Equations (3) and (4) enable us to write

$$\frac{e^2}{2m_e c^2} = \frac{g^2}{2m_p c^2} = \frac{\hbar}{m_\pi c} = \left(\frac{G_{\mathrm{F}}}{\hbar c}\right)^{1/2} \frac{g^2}{e^2} , \tag{7}$$

as a relation connecting strong, weak and electromagnetic interactions.

We can use Equation (7), to successively eliminate, from Equation (6), the masses $m_p$, $m_e$, and $m_\pi$. Thus with a little algebraic manipulation using Equation (7), Equation (6) for $R$ can be entirely expressed in terms of the coupling constants of the four fundamental interactions, as

$$R_{\mathrm{H}} = \frac{g^4}{Ge^8}\left(\frac{c^7 G_{\mathrm{F}}^3}{\hbar}\right)^{1/2} \simeq 10^{28}\ \mathrm{cms} . \tag{8}$$

Substituting the values of the coupling constants as given earlier (and which are known from the local physics) $R_{\mathrm{H}}$ turns out to be $10^{28}$ cm – i.e., the Hubble radius.

As explained in the earlier paper (Sivaram, 1982a), general relativity would impose a maximal mass (closure mass) corresponding to $R_{\mathrm{H}}$ as

$$M = \frac{c^2 R_{\mathrm{H}}}{G} , \tag{9}$$

which using Equations (8) and (9) would correspond to a closure density $\rho = M/\frac{4}{3}\pi R_{\mathrm{H}}^3$ to be

$$\rho_c = \frac{3Ge^{16}\hbar}{4\pi c^5 g^8 G_{\mathrm{F}}^3} \simeq 3 \times 10^{-29}\ \mathrm{g\ cc}^{-1} , \tag{10}$$

which agrees well with the value of the closure density as estimated by observations of Hubble's constant and the deceleration parameter. We have thus arrived at estimates of the overall size and density of the Universe based on the local laws of the micrphysics governed by the four fundamental interactions. Dirac's large numbers hypothesis involves gravitation and electromagnetic interactions. Here we have relations involving all the four fundamental interactions and cosmological parameters like $R_{\mathrm{H}}$ and $\rho_c$. We can use Equations (2), (3), (4), and (7) to summarize these relations neatly as

$$\left(\frac{3G\hbar}{32\pi c^2 H_0}\right)^{1/3} = \frac{e^2}{2m_e c^2} = \frac{g^2}{2m_p c^2} = \frac{\hbar}{m_\pi c} = \left(\frac{G_{\mathrm{F}}}{\hbar c}\right)^{1/2} \frac{g^2}{e^2} . \tag{11}$$

Further relations have been given in Sivaram (1981). We remark further that Equation (2) may be understood also as follows. Quantum gravitational fluctuations have a density of $\sim c^5/G^2\hbar$. On the other hand, general relativity would give a maximal mass for a Hubble radius of $c/H_0$ as $c^3/GH_0$. Therefore, if this entire mass had the density of a quantum gravitational fluctuation, its size would be just $(3G\hbar/4\pi c^2 H_0)^{1/3}$. Again using the quantum uncertainty principle a fluctuation of this width, would give rise to a mass spectrum given (cf. Sivaram *et al.*, 1974b) by

$$\Delta m = n\left(\frac{h^2 H_0 \pi}{6Gc}\right)^{1/3}, \tag{12}$$

where $n$ has integer value. As noted in our earlier work it is remarkable that several values of $n$, do give the observed elementary particle mass spectrum. For e.g. $n = 4$ gives $m_\pi$ (pion), $n = 3$ gives $m_\mu$ (muon), $n = 14$ gives $K$ meson mass, etc.

Comparing Equation (12) with Equation (1), we see that Weinberg's relation is contained in Equation (12). As stated by Weinberg in his book, Equation (1), is a clue pointing to the fact that parameters pertinent to particle physics are not solely determined by considerations of microphysics, but in part by the influence of the whole universe. Our approach in arriving at the parameters of the Universe by considerations of microphysics fully embodies this spirit.

## References

Dirac, P. A. M.: 1937, *Nature* **139**, 323.
Motz, L.: 1972, *Nuovo Cimento* **1213**, 239.
Sivaram, C.: 1981, *Physics Today* **54**, 108.
Sivaram, C.: 1982a, *Astrophys. Space Sci.* (in press).
Sivaram, C.: 1982b, *Am. J. Phys.* **50**, 279.
Sivaram, C.: 1982c, *Am. J. Phys.* (in press).
Sivaram, C. *et al.*: 1974a, *Phys. News* **5**, 146.
Sivaram, C. *et al.*: 1974b, *Nuovo Cimento Letters* **9**, 740; **10**, 227; *Curr. Sci.* **43**, 165.
Sivaram, C. *et al.*: 1975, *Nuovo Cimento Letters* **13**, 357.
Weinberg, S.: 1972, *Gravitation and Cosmology*, John Wiley and Sons, New York.