VARIATIONS OF THE COUPLING CONSTANTS OF FUNDAMENTAL INTERACTIONS IN DIRAC'S COSMOLOGY

(Letter to the Editor)

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Abstract. It is pointed out that several interesting coincidences and relationships connecting the parameters of cosmology and elementary particle physics suggest that the coupling constants of the weak, strong and electromagnetic interactions do not change with time in Dirac's cosmology. This would be consistent with the stringent limits imposed on their variation by recent data on isotopic abundances.

Two remarkable consequences of Dirac's Large Numbers Hypothesis (LNH) are that the total number of nucleons in the Universe N is predicted to vary as t^2 and the gravitational constant G is predicted to decrease as t^{-1}, t being the age of the Universe (Dirac, 1937, 1973). Thus the ratio of the electromagnetic to gravitational interaction between a proton (mass m_p) and an electron (mass m_e), i.e. $e^2/Gm_pm_e \approx 10^{40}$, is approximately of the same order as the age of the Universe ($1/H$, H being the Hubble constant) expressed in units of the atomic time, $e^2/m_ec^3 \approx 10^{-23}$ s, $1/H$ being about 10^{18} s. Again the total number of particles in the Universe (N) is of the order of the square of this ratio – i.e. $\sim 10^{80}$. Thus if e^2 is constant, $G \propto t^{-1}$ and $N \propto t^2$, H being related to epoch as $1/t$. This is Dirac's cosmology where G and N have the above epoch dependences. The Large Numbers Hypothesis has been recently extended to include the weak and strong interaction coupling constants also by invoking dimensionless relations connecting these constants to cosmological parameters (Sivaram, 1983a). Two dimensionless relations involving the universal Fermi constant of beta-decay, $G_\mathrm{F} = 1.5 \times 10^{-49}$ erg cm^3 are

$$\left(\frac{Gm_e^2}{\hbar c}\right) = H\left(\frac{G_\mathrm{F}}{\hbar c^3}\right)^{1/2} = 2 \times 10^{-45}\,, \tag{1}$$

(the inverse giving the large number) and

$$\sqrt{N}\left(\frac{Gm_e^2}{\hbar c}\right) = \left(\frac{G_\mathrm{F}}{\hbar c}\right)\left(\frac{m_pc}{\hbar}\right)^2; \tag{2}$$

suggesting a link between weak interactions and cosmology in the Machian sense as explored in the work of Hayakawa (1965), who interprets the electric charge as arising from $1/\sqrt{N}$ fluctuations of the electron number of the Universe. Here we can similarly picture $1/\sqrt{N}$ fluctuations of the total gravitational charge (NGm_e^2, gravitational charge

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defined as in Motz (1972)) of the electrons as giving rise to the weak interaction force between electrons, the right-hand side of (2) being the dimensionless weak charge. Explicit relations between weak and gravitational interaction constant arise in unified theories of weak and gravitational interactions (Sivaram, 1975; Kaempffer, 1976) and in estimating helium abundance in the nucleosynthetic stage of the big bang (Sivaram, 1983b). Now if we consider Equations (1) and (2) in the context of Dirac's cosmology, where G and N are epoch dependent (G varies as t^{-1} and N varies as t^2), it is seen that these variations of G and N, imply that G_F is independent of time and does not vary with epoch. (It is to be remembered that H is t^{-1}. Also to be noted that, $(G_F/\hbar c)^{1/2} = 7 \times 10^{-17}$ cm is the characteristic beta decay length and $(G_F/\hbar c^3)^{1/2}$ is the corresponding time scale.) In an earlier paper (Sivaram, 1982a) it was pointed out that the gross parameters of the Universe such as the overall size and mass can be arrived at from microphysical considerations involving the fundamental interactions of elementary particle physics. Interesting relations inter-relating the parameters of cosmology and elementary particles were obtained. In particular, the Hubble radius, $R_H = \hbar^2/Gm_pm_em_\pi$, was obtained as the maximal range associated with energy fluctuations corresponding to exchange of gravitons of virtual mass (m_g) given by

$$m_g = \frac{Gm_pm_em_\pi}{\hbar c}, \tag{3}$$

where m_π being the pion mass and m_π in turn can be related to cosmological parameters H and G through the relations (cf. Sivaram, 1982a, b; 1983c; Weinberg, 1972) given by

$$m_\pi = \left(\frac{\hbar^2 H}{Gc}\right)^{1/3} \tag{4}$$

and

$$\left(\frac{3G\hbar}{32\pi c^2 H}\right)^{1/3} = \frac{\hbar}{m_\pi c} = \frac{g^2}{2m_pc^2} = \frac{e^2}{2m_ec^2}, \tag{5}$$

$(g^2/\hbar c) \approx 14$ is the pion-nucleon strong interaction constant, $e^2/\hbar c = 1/137$).

Equation (5) suggests that in Dirac's cosmology, where $G \propto t^{-1}$; (H is $1/t$); g^2 and e^2 are both constant and from Equation (4) m_π (which fixes the range of the nuclear forces) is also constant. Thus the strong and electromagnetic coupling constants do not vary in time. Again another curious coincidence implied by the unification of weak and electromagnetic interactions (elaborated in the earlier papers), – i.e., $e^2/2m_pc^2 = (G_F/\hbar C)^{1/2}$; also implies that since G_F is epoch independent (as seen from Equations (1) and (2)), e^2 also does not vary with time. Other interesting relations between cosmological and particle physics parameters can also be obtained. Consider Equation (3) for the graviton mass, which would be corresponding to the smallest possible proper mass. A mass m in general relativity cannot be localized in space to a distance smaller than Gm/c^2. Thus with m_g in Equation (3), we can obtain the smallest possible distance or length scale, i.e. Gm_g/c^2. On the contrary, in quantum mechanics a particle or system of mass M cannot be localized over a distance smaller than $\hbar/Mc$ – i.e. the position of the particle cannot be specified to a distance less than this. Thus in contrast to classical mechanics where a point particle can be identified with particles of smaller and smaller mass (i.e., localisable distance is proportional to m in Gm/c^2), in quantum mechanics the localization is inversely proportional to m (in $\hbar/mc$). In the quantum picture, therefore, the smallest possible length scale would correspond to the largest possible mass which if we assume as the mass of the Universe M_{U} would correspond to $\hbar/M_{\text{U}}c$. Now if we insist for consistency that these two smallest length scales (defined in different ways) be the same we would have

$$Gm_g/c^2 = \hbar/M_{\text{U}}c\ ,$$

or

$$Gm_g m_{\text{U}} = \hbar c\ , \tag{6}$$

or

$$m_g m_{\text{U}} = m_{\text{Pl}}^2\ ; \tag{7}$$

where $m_{\text{Pl}}^2 = (\hbar c/G) = (\text{Planck mass})^2$. Also substituting for m_g, from Equation (3), we get the elegant relation

$$Gm_p m_e m_\pi m_{\text{U}} = m_{Pl}^2 \hbar c\ ,$$

or

$$m_p m_e m_\pi m_{\text{U}} = (\hbar c/G)^2 = m_{\text{Pl}}^4\ , \tag{8}$$

as a relation connecting the masses of the electron, proton, and pion with the mass of the Universe through $\hbar$, c, and G. Writing m_{U} as Nm_p, from (8) we have

$$N = \frac{m_{\text{Pl}}^4}{m_p^2 m_\pi m_e}\ . \tag{9}$$

From Equations (8) and (9), it follows that if $G \propto t^{-1}$, then N or m_{U} would go as t^2 which is of course consistent with Dirac's cosmology. Again from the equality: $e^2/2m_e c^2 = \hbar/m_\pi c$, (see Equation (5)), we can rewrite Equations (8) and (9) as

$$m_p m_e^2 m_{\text{U}} = \frac{\alpha}{2}\, m_{\text{Pl}}^4$$

or

$$N = \frac{\alpha}{2}\left[\frac{m_{\text{Pl}}^2}{m_p m_e}\right]^2 . \tag{10}$$

As in Dirac's cosmology, $G \propto t^{-1}$ and $N \propto t^2$, it follows from Equation (10), that the fine structure constant α is independent of time t and the electric charge does not change with epoch.

We shall now remark on the constancy of the Planck's constant h and the velocity of light c. The action associated with the total gravitational charge is NGm_p^2/c. Local $1/\sqrt{N}$ fluctuations would give a local change of $\sqrt{N}\,Gm_p^2/c$ which would be identified as h. Indeed with $N = 10^{78}$, we have

$$\sqrt{N}\,Gm_p^2/c = 6 \times 10^{-27}\ \text{erg s} = h\,.$$

Thus with N going as t^2 and G as t^{-1}, h is invariant with epoch. The gravitational potential $\phi = GNm(1/R)$, acting on a particle remains constant for a radius R increasing linearly with time; i.e. for an expanding universe with $R \propto t$, $G \propto t^{-1}$, we have with $N \propto t^2$ ($\therefore \phi$ is constant) which is Dirac's cosmology. If we postulate that ϕ apart from local fluctuations should equal c^2, then c is also invariant with epoch. The potential being equal to c^2 everywhere, would imply that the negative gravitational potential energy added to the rest mass energy gives zero, so that new particles could be created without violating energy conservation.

Our conclusion that the coupling constants of the strong, weak, and electromagnetic interactions do not vary with epoch finds justification in some recent data on isotopic abundances. The tightest limits claimed for the constancy of the weak, strong and electromagnetic couplings are based upon the abundance ratio of samarium isotopes Sm^{149} and Sm^{148} from the Oklo Uranium Mine (Shlyakhter, 1976; Maurette, 1976). In the Oklo sample the ratio of these isotopes is ~ 0.02 as compared to the natural ratio ~ 0.9, the depletion being due to the bombardment received from thermal neutrons over periods of many millions of years during the running of the natural 'reactor'. The capture cross-section for thermal neutrons on Sm^{149} is dominated by a strong capture resonance and the Oklo samples apparently imply that this resonance could not have shifted by more than 0.02 eV over a period of 2×10^9 yr. As the position of this resonance sensitively determines relative binding energies of the different samarium isotopes with respect to weak (W), strong (S), and electromagnetic (E) couplings, this would imply time variations constrained by $\dot{E}/E \leq 10^{-17}\ \text{yr}^{-1}$, $\dot{W}/W \leq 10^{-12}\ \text{yr}^{-1}$, $\dot{S}/S \leq 5 \times 10^{-19}\ \text{yr}^{-1}$.

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