The Problem of a Primordial Black Hole Hydrogenlike Atom

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Abstract
It is shown that the recently proposed system composed of an electron and a primordial black hole forming a hydrogenlike atomic system cannot be stable, because the radiation pressure acting on the electron from Hawking radiation of the black hole far exceeds the binding force. On the contrary, if the black hole has a zero Hawking temperature, either due to its charge or spin, the required values of these parameters are so large that the electron would still not be stably bound to the black hole. However, if the black hole has a magnetic moment due to its spin, then electrons passing by the black hole could have spin-flip transitions producing high-energy gamma rays of $\sim 10^{14}$ *eV -* 10^{15} *eV.*

Key words: cosmology, primordial black hole, Hawking radiation, radiation pressure, magnetic moment, spin-flip transition, high-energy gamma rays, electron density, hyperfine atomic transition, background radiation

In a recent paper[1] Zeng proposed a hydrogenlike atomic system consisting of an electron and a primordial black hole (PBH) of mass $\sim 10^{14}$ - 10^{15}g. He noted that the electrostatic and gravitational forces between an electron and such a PBH are of the same order, the proton in the hydrogen atom being replaced by a PBH with a Schwarzschild radius about equal to the proton radius, that is, a PBH with a mass $\sim 10^{15}$ g has a radius $\approx 10^{-13}$ cm.

He then applies the Schrödinger equation to solve for the energy levels and transitions for this PBH hydrogenlike atom taking both electrostatic and gravitational forces into account, noting that the potential energy of this system differs from that of the usual hydrogenlike atom by only a numerical coefficient

$$B = 1 + a_s/2Zr_e,$$

where a_s is the Schwarzschild radius of the PBH, and r_e is the classical electron radius $r_e = e^2/m_e c^2$.

After giving formulas for the energy transitions of such a system (*mutatis mutandis* the hydrogen atom), Zeng goes on to discuss the cosmological implications of the system in Sec. 4 of the paper.[1]

For this he considers PBHs of mass 10^{14} g (whose evaporation time due to Hawking radiation is comparable to the age of the universe, $t \approx 1.5 \times 10^{10}$ y) to make estimates of such a system's contribution to the background cosmic radiation and concludes that the photon radiation of a PBH with a hydrogenlike atomic system mechanism (i.e., bound to an electron) makes a significant contribution to the cosmic background radiation (CBR) and may even explain the excess submillimeter radiation of the CBR.

However, in considering such a stable bound hydrogenlike atomic system of a PBH with an electron, he neglects the fact that the PBH is continually emitting intense Hawking radiation which would exert radiation pressure on any orbiting electrons. So it is not simply a question of replacing the proton in the hydrogen atom by a PBH of Schwarzschild radius equal to the proton radius. Let us estimate the radiation pressure exerted by the evaporating PBH (of mass $M_H \sim 10^{14}$ g) emitting Hawking radiation and compare it to the gravitational force between it and the electron (of mass m_e) orbiting at a distance r as in the hydrogen atom.

Now the evaporation time scale of a PBH (i.e., $t_H \approx G^2M_H^3/\hbar c^4$) due to Hawking radiation implies, as is well known, a rate of energy emission due to evaporation, that is, a luminosity L of

$$L \approx \hbar c^6/G^2M_H^2 \tag{1}$$

(note that L scales as $1/M_H^2$, the appearance of $\hbar$ indicating that it is a quantum effect). For a hole of mass $M_H \approx 10^{14}$ g, Eq. (1) implies a luminosity of

$$L \approx 2 \times 10^{22} \text{ erg/s}. \tag{2}$$

The gravitational binding force between the PBH and the electron is

$$F_G = GM_H m_e/r^2. \tag{3}$$

The force due to radiation pressure exerted by a source with luminosity L on an electron at a distance r is given by

$$F_R = L\sigma_T/4\pi r^2 c, \tag{4}$$

where σ_T is the electron–photon Thompson cross section given as

$$\sigma_T = (8\pi/3)(e^2/m_e c^2)^2. \tag{5}$$

Thus the balance between the radiation pressure force given by Eq. (4) and the gravitational force given by Eq. (3) would imply that the maximal Eddington luminosity of the PBH should be (for stability)

$$L_E = GM_H m_e 4\pi c/\sigma_T = 4\pi G M_H m_e c/\sigma_T. \tag{6}$$

Thus L in Eq. (4) must be much less than L_E for the stability of the system.

For a PBH of mass $M_H \approx 10^{14}$ g, Eq. (6) implies

$$L_E \approx 10^{15} \text{ erg/s}. \tag{7}$$

Comparing Eqs. (2) and (7), we see that $L \gg L_E$, which implies that the force due to radiation pressure from evaporating PBH will blow away the electron *whatever* its orbital radius (since r cancels out in the above expressions). So it is impossible to have a stable hydrogenlike system of a PBH with a mass 10^{14} g - 10^{15} g with an electron. This is the PBH mass range required in Ref. 1 to get significant contributions to the CBR and other important implications.

However, it is possible to have a PBH with zero Hawking temperature in two cases. First, it must have an electric charge Q given by[2] $Q^2 = GM_H^2$, that is,

$$Q = (G)^{1/2} M_H. \tag{8}$$

For $M_H \cong 10^{14}$ g, this would give $Q \cong 10^{21}$ e, where e is the electric charge. However, this would imply that a PBH with such a large Q would exert an enormous electrostatic force on any electron orbiting it. Moreover, it is well known even in atomic physics that any pointlike nucleus with a charge Z greater than $Z \cong 137$ ($\cong 170$ taking finite nuclear size into account) would have unstable electron orbits, that is, the Dirac equation would give negative electronic energy if $Z\alpha > 1$ ($\alpha = 1/137$), so that the orbits would collapse.[2] So here we effectively have $Z > 10^{21}$, so it is impossible to have an electron forming a bound state with such a PBH like a hydrogen atom.

The second possibility of the PBH having a zero Hawking temperature (ZHT) is when its spin J is given by[2]

$$J_H = GM_H^2/c. \tag{9}$$

For $M_H \cong 10^{14}$ g, this would give

$$J_H \cong 2 \times 10^{37} \hbar, \tag{10}$$

but a PBH with such a large J_H would give rise to enormous spin-orbit and spin-spin coupling on any electron orbiting it, so it is impossible in this case to have a stable PBH electron, hydrogen atomlike, bound system.

However, because of its large intrinsic spin as given by Eq. (9), such a hole would have a magnetic moment given by[2,3]

$$\mu_H = (G/c)^{1/2} J_H, \tag{11}$$

with J_H given by Eq. (10); this would imply

$$\mu_H \cong 2 \times 10^{-4} \text{ erg/G}. \tag{12}$$

Since electrons have an intrinsic magnetic moment μ_e (given by the Bohr magneton $\mu_B = e\hbar/2m_e c$), one can have a situation similar to hyperfine atomic transitions, when electrons passing by the nucleus can undergo spin-flip transitions due to interactions between electronic and nuclear magnetic moments.

The Hamiltonian for this would be given (for pointlike interacting components with magnetic moment, which is a good approximation for this system) as

$$H = (\mu_H \cdot \mu_e/\pi a^3 n^3)\bar{\sigma}_p \cdot \bar{\sigma}_e \tag{13}$$

(in the usual case, $a \cong$ Bohr radius $\sim 10^{-8}$ cm, $\bar{\sigma}_p \cdot \bar{\sigma}_e = 3$ or 1, depending on whether it is a triplet or singlet state).

For a μ_H given by Eq. (12), Eq. (13) would imply that the energy of the photon emitted in such a spin-flip transition undergone by the electron when interacting with the hole moment μ_H is

$$E_\gamma \approx 5 \times 10^{14} \text{ eV}, \tag{14}$$

that is, corresponding to a very high-energy gamma ray. So such interactions of PBHs in interstellar space may be observable even if they have ZHT. Gamma rays in energy ranges of 10^{14} eV - 10^{15} eV are known to emanate in cosmic rays and from sources like Cygnus X-3, and their origin is not known with certainty.

However, since PBHs can have a whole range of masses M_H, and since μ_H increases with M_H, one can have high-energy gamma rays with a whole range of energy. For 10^{14} g we would have $\cong 5 \times 10^{14}$ eV. For 10^{13} g we would have 5×10^{12} eV, since in Eq. (13) μ_H is involved and depends on M_H^2 [Eq. (9)]. For higher M_H, intensity drops considerably as $\sim 1/M_H^2$ [Eq.(1)]. In fact, high-energy gamma rays are seen from 10^{10} eV - 10^{17} eV. Still higher energy gamma rays are cut off by interaction with the cosmic background radiation. The intensity of the radiation would depend on the number density of the electrons present in the source, and from the observed intensities it should be possible to estimate the electron density.

As shown in Ref. 2, the PBH, with spin given by Eq. (10), would have a magnetic field B_H (due to torsion) associated with it of

$$B_H = (8\pi/3c)(2\alpha G)^{1/2}\sigma_H, \tag{15}$$

where σ_H, the spin density, is given by dividing J_H [given by

Eq. (10)] by the Schwarzschild volume of the black hole. This gives a field of $B_H \approx 10^{35}$ G at the surface of the black hole. At a distance of Bohr radius $R_B \cong 10^{-8}$ cm, this field (being dipolar as torsion gives dipolar field) is

$$B_0 \approx B_H(R_H/R_B)^3, \tag{16}$$

where R_H, the Schwarzschild radius, is $\cong 10^{-13}$ cm; then

$$B_0 \cong 10^{20} \text{ G}. \tag{17}$$

This gives the spin-flip time for an electron with $\mu_e \approx 10^{-20}$ erg/s interacting with this field generated by a torsion of

$$t_{s\text{-}f} \cong \hbar/(\mu_e B_0) \cong 10^{-27} \text{ s}. \tag{18}$$

This $t_{s\text{-}f}$ agrees with the uncertainty principle estimated from the energy of spin-flip transition as given by Eq. (14), which gives $t_{s\text{-}f} \approx 10^{-28}$ s.

Now the time scale for collapse onto the black hole of an electron moving with speed c at a distance of $\approx 10^{-8}$ cm (R_B) is

$$t_G \cong R_B/c \approx 10^{-18} \text{ s}. \tag{19}$$

Thus $t_{s\text{-}f} \ll t_G$, which means *spin-flip will occur well before* electron collapse via gravitational attraction on the black hole.

Moreover, the magnetic force on the electron (magnetic field generated by spin torsion) is (e is the electron charge)

$$\begin{aligned} F_B &\approx eB_0c \\ &\approx 5 \times 10^{-10} \times 10^{20} \times 3 \times 10^{10} \cong 10^{21} \text{ dyn}. \end{aligned} \tag{20}$$

The gravitational force between the black hole and the electron (at a distance of $R_B \approx 10^{-8}$ cm) is

$$F_G = GM_H m_e/R_B^2 \approx 5 \times 10^{-5} \text{ dyn}. \tag{21}$$

Thus $F_B \gg F_G$, that is, the *magnetic force is much greater than the gravitational force* at a distance when spin-flip can occur. Even at the Schwarzschild radius of the black hole $F_B \gg F_G$ so that the electron cannot collapse on the black hole before spin-flip.

Now the Hawking–Page bound on the number of evaporating PBHs from the gamma ray background is[4]

$$n_{PBH} < 10^{11} \text{ pc}^{-3}. \tag{22}$$

Incidentally, this is much smaller than what is assumed for the cosmological implications in Ref. 1. The *interstellar* electron density from pulsar dispersion measures is $\cong 0.1$ cm^{-3}. The geometric cross section σ for the process is about $\pi \times (10^{-13})^2$ cm^2 (since both the PBH and the electron have sizes $\sim 10^{-13}$ cm).

Then assuming a velocity $\sim c$, the flux of the gamma rays due to the spin-flip can be estimated in the usual way as (rate $\cong n_H n_e \sigma v$)

$$F_\gamma \approx 10^{-17} \text{ cm}^{-2}\text{s}^{-1}. \tag{23}$$

For example, for a specific source like Cygnus X-3, the electron density would be much higher, $\sim 10^2$ cm^{-3}, in which case the flux from the source would be 10^3 times higher, which is consistent with the observed gamma ray pulses.

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Résumé

Il est démontré que le système proposé récemment, selon le modèle d'un hydrogénoïde, ayant comme composante un électron et un trou noir primordial, ne peut être stable parce que la pression de radiation provenant du trou noir est beaucoup plus grande que celle de la force de liaison. D'autre part, dans le cas où le trou noir a une température d'Hawking égal à zéro, les valeurs requises pour la charge ou le spin, sont trop grandes pour que l'électron puisse être lié d'une façon stable au trou noir. Toutefois, si le trou noir a un moment magnétique dû à son spin, alors les électrons, passant par le trou noir, pourraient être sujets à un renversement du spin, causant ainsi l'émission de rayons gamma à des énergies de $\sim 10^{14}$ *eV -* 10^{15} *eV.*

References

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