Some implications of quantum gravity and string theory for everyday physics

C. Sivaram

Quantum gravity is still an enigmatic and exotic subject and overhelming opinion would indicate that it could possibly have no impact whatsoever on the workings of everyday physics governed by classical, statistical and quantum mechanics. Moreover there are several different approaches to the problem of quantizing gravity and a fully consistent, acceptable theory is yet to emerge. Even in the absence of such a complete theory, we point out that several model independent interesting implications of quantum gravity exist for a wide variety of phenomena in everyday physics. These include universal upper bounds on field strengths (including electric and magnetic fields), temperature, acceleration, particle energies and properties of bulk matter like density, elastic strength, surface tension, etc. as well as absolute upper limits on computational and information processing rates power generation, etc. Also lower limits on temperature (in a non-inertial frame), and on particle magnetic moments are discussed along with some recent ideas on large quantum gravity effects and vacuum entropy. Again possible implications of string theory for propagation of high energy photons from gamma ray bursts and other sources enable constraints to be put on string parameters. There are also effects from presence of extra spatial dimensions for deviations from Newton's gravity law, for corrections in atomic spectroscopy, etc. which are discussed. Possible effects in interferometry, gravitational wave detectors, k-meson decay, as well as implications for clock and computer performance are also discussed.

ALTHOUGH there has been a considerable spurt of recent interest in research in several formal aspects of quantum gravity including considerable mathematical progress, the subject still remains enigmatic and remote from other areas of physics. Despite several suggestions and complex models, no clear cut consistent consensus on uniting quantum theory and gravity has emerged. It would appear as if quantum gravity has no implications or impact on the rest of everyday mundane physics which depends on measurement or observation of well defined physical quantities or properties that characterize a system or a substance. We shall see that this is not strictly true. It is possible to carry out calculations of the effects of quantum gravity on certain systems and come out with numbers! This has been known for some time especially in the case of a weak field in a linearized theory. For instance, one can estimate[1] that in the sun from Coulomb collisions in the core plasma 10^9 W of thermal gravitational radiation can be generated. For the degenerate matter in white dwarfs this is 10^{15} W and from a newly born neutron star this could be as high as 10^{23} W in high frequency gravitational radiation[2]. It is amusing that one could actually estimate the number of gravitons (N_g) emitted in an asymmetric explosion of energy E as (ref. 3).

$$N_g \simeq \frac{G}{c^5}\frac{E^2}{\hbar}, \tag{1}$$

($\hbar$ is Planck's constant).

For a hundred-megaton nuclear explosion, this implies for instance[4], a dimensionless strain (at a suitably defined distance) of $h \sim d/l \sim 10^{-31}$.

As another example one can estimate the lifetime (i.e. probability) of a 3d $\to$ ls transition in hydrogen with emission of a graviton[1,2] to be

$$\sim \frac{GM_e^2}{\hbar c} w_{hyd} a^4 \sim 10^{35} \text{ s},$$

where m_e is the electron mass, w_{hyd} the frequency roughly corresponding to the 3d $\to$ ls transition (~ 12 eV), a the fine structure constant). Implications of this transition for detection of high frequency gravitational radiation (~ 10^{15} Hz) have been discussed in ref. 2. There are many other similar examples of low energy manifestations of quantum gravity. Gravitational quantum effects are very small at ordinary energies when $\hbar c/E >>$

C. Sivaram is in the Indian Institute of Astrophysics, Bangalore 560 034, India

GE/c^4, but as easily seen a natural cut off energy arises at $E_c \sim (1/G)^{1/2} \sim 10^{19}$ GeV, when the particle is trapped in its own gravitational field! This for instance implies an upper limit to cosmic ray particle energies $\sim 10^{28}$ eV, a few orders larger than the highest energies seen so far $\sim 10^{21}$ eV in experiments such as the Fly's Eye. This also tells us that virtual particles in quantum electrodynamics and other field theories cannot have arbitrarily high energies, so that self-energy integrals get truncated giving finite values for quantities like the electron mass of the form $a \sim \ln(\hbar c/GM_e^2)$ explaining the smallness of G relative to a. The wavelength corresponding to E_c implies a maximal curvature of $K_{max} \sim c^3/\hbar G$ which imposes the ultimate quantum limits on geometrical measurements giving a smallest spatial resolution $\sim K_{max}^{1/2}$. No physical experiment, however ingenious, can overcome this limit.

The maximal curvature implies in the language of geometry and gauge theory a maximal field strength of[5].

$$E_{max} \sim \frac{c^{7/2}}{(\hbar G)^{1/2}}, \tag{2}$$

(by the equivalence principle, this is also the maximal permissible physical acceleration).

Eq. (2) implies that for a gravitating body of mass M, the minimal radius to which it can collapse in a comoving frame is given by:

$$R_{min} = \left(\frac{GM}{E_{max}}\right)^{1/2} \simeq \left(\frac{G^3\hbar}{c^7}\right)^{1/4} M^{1/2}. \tag{3}$$

E_{max} also implies the existence of a maximal density of $r_{max} \sim (c^5/G^2\hbar)$. This among other things gives an upper bound on strength of magnetic fields as $B_{max} \sim c^{7/2}/G\hbar^{1/2} \sim 10^{57}$ G as also on electric fields. These limits are independent of any detailed form quantum gravity theory might take. There is also an upper limiting temperature of $T_{max} \sim (\hbar c^5/G)^{1/2}(1/K_B) \sim 10^{32}$ deg and a corresponding maximal pressure. In turn this imposes limits on bulk elastic properties such as a limiting surface tension of $s_1 \sim 10^{80}$ dyne/cm^2 and a maximal elastic modulus of $\sim 10^{112}$ dyne/cm.

The smallest spatial resolution mentioned earlier, also implies a smallest time interval of $(\hbar G/c^5)^{1/2} \sim 10^{-43}$ s which in turn imposes a limiting angular frequency $w_{max} \sim 10^{43}$ Hz. The cutoff energy and the limiting temporal resolution then give the highest power that can be emitted or generated by any physical system as:

$$P_{max} \simeq C^5/G \sim 3\times 10^{59} \text{ ergs/s}. \tag{4}$$

An astronomical object emitting at this luminosity would have an absolute bolometric magnitude of –58 and even at a distance of ten gigaparsecs would have a flux comparable to the moon's brightness. This limiting power in turn implies a universal bound on information processing rate or rate of computation as given by information theory as:

$$f \simeq \sqrt{\frac{P_{max}}{\hbar}} \simeq 10^{44} \text{bits/s}. \tag{5}$$

Black hole thermodynamics implies a maximum information content of (energy E) $I \leq 2pER/\hbar c \ln 2$ (ref. 6), but the size of the system also enters (R). However eq. (5) is an ultimate upper bound giving $f \simeq (c^5/G\hbar)$ bits/s (independent of energy or size of system), a quantum gravity limit. As is well known, through Hawking's work[7] a manifestation of quantum gravity is the evaporation of black holes, with a time scale $t \sim (G^2M^3/\hbar c^4)$ (limit $\hbar \to 0$ gives infinite time for a classical black hole). Black holes in the mass range 10^{14}–10^{15} g are interesting as they have a lifetime comparable to the Hubble age and could be observable as a burst of gamma rays. Many experimental searches have been initiated to observe such events. So far limits have been put on the space density of such objects. A 10^{14} g black hole ($t \sim 10^{17}$ s), has a power generation of $\sim 10^{21}$ ergs/s, coincidentally comparable to the total commercial power generated on Earth! The temperature is proportional to the surface gravity and thus scales inversely with the black hole mass. The equivalence principle would thus suggest that temperature should also be associated with an accelerated frame. Indeed a detector uniformly accelerated through Minkowski Vacuum is heated due to interactions with the vacuum fluctuations and would detect radiation with a thermal spectrum with a temperature given by $T \simeq (\hbar a/2pcK_B)$ (a being the acceleration). For $a \simeq 1$ g, $T \simeq 10^{-19}$ deg.! Thus it is impossible to cool to absolute zero in an accelerated frame, i.e. there is a fundamental limitation imposed by quantum gravity as to how low a temperature one can achieve! The effect could be observable in situations involving very high accelerations such as in particle storage rings. Indeed Leinaas[8] suggested that electrons in a storage ring polarized spontaneously but not completely, the maximum polarization had been found to be $P = 0.92$. The departure from full polarization can be interpreted as due to heating of the electrons caused by their acceleration. Another possible manifestation is a cut off in the transverse momenta of colliding hadrons in high-energy experiments[9]. The Global Positioning System (GPS) has already brought GR to the masses and fractional frequency shifts of 1 part in 10^{14} are measureable[10]. GR effects are 4.5 parts in 10^{10}, equivalent to missing a heart beat in a lifetime! As temperature is also dependent on the local gravitational field (like time) and we can now achieve a few nanokelvin, it is conceivable that in the future, limitations in achieving lowest temperatures in gravitational fields can actually be monitored! There is also an argument for a minimal acceleration[11], based on the operational requirement that the gravitational self energy of a particle be measurable over a Hubble time (Ho^{-1}) which is the same as the a_0 in theories like MOND which have been proposed as an alternative to the existence of dark matter in galaxies and clusters with impressive results[12].

Again the riddle of blackhole entropy, may be tied up with another well-measured phenomenon, i.e. the Casimir force in which two conducting parallel plates are attracted by zero point vacuum fluctuations. The black hole entropy becomes a Casimir entropy, an entropy associated with a thermal contribution of zero-point modes[13]. For two parallel plates separated by a distance a, a Casimir entropy is essentially equal to the number of squares of edge 'a' required to fill the area of the plate[14]. Remarkably for a black hole the entropy is the number of planck squares required to fill the area of the horizon! This is again connected to the quantization of area in quantum gravity. A quantized Schwarschild black hole has energy levels of the form $E_n = \sqrt{N}Ep$. For rotating and charged black holes, the mass formulae mutatis mutandis resemble well-known mass formulae for mesons and baryons and things like Regge trajectories in a chew-Frautschi plot[9]. This also holds for strings. In fact based on this, new mass formulae for particles have been suggested. A maximal energy E_p also implies a lowest magnetic moment of $\hbar / c\sqrt{G} \sim 10^{-22}$ m_B, m_B, is the Bohr magneton[9]. Recently it has been pointed out that large quantum gravity effects can occur even in low curvature regimes, i.e. the quantum fluctuations in the geometry can be large unless the number and frequency of[15] photons satisfy $N(\hbar Gw)^2 << 1$, i.e.

$$\left(\frac{\Delta g_{Rw}}{< g_{RR}}\right) \simeq e^{N(G\hbar w)^2}.$$

We briefly consider situations where this condition can be realized, i.e. $N(\hbar Gw)^2 > 1$, to induce large geometry fluctuations. If there is a source of high energy gamma rays, with $w \simeq 10^{21}$ Hz, then one requires

$$N \sim \frac{1}{(\hbar Gw)^2} \sim 10^{44},$$

or a source with total energy emission of $\sim 10^{38}$ ergs. Many astrophysical sources can produce this kind of energy, and if the source is a neutron star, the curvature is small. If we have a laboratory source of 10^4 TeV particles, then one needs $N \sim (\hbar Gw)^{-2} \sim 10^{22}$ and a total energy of $\sim 10^{25}$ ergs, not too inconceivable in the future to produce large quantum gravity effects even in very low curvature regimes. The large number of examples given suggest implications for quantum gravity in several areas of everyday physics[16].

Talking of gamma ray bursts, it must be mentioned that recently it has been suggested that in a large class of quantum gravity approaches[17] (in which the existence of a minimum length $l_{\min} \sim E_{QG}$ is assumed) a deformed photon dispersion arises which can possibly yield observational constraints and effects based on gamma ray bursts (GRBs). Specially this is assumed to be of the form[17,18].

$$PC = E\sqrt{1 + E / E_{QG}},$$

where E_{QG} is the quantum gravity scale, which could be as low as 10^{-3} E_{planck}, i.e. $E_{QG} > 10^{16}$ GeV. One of the consequences could be that high energy photons, would not travel at the speed of light, but at a speed of:

$$V = C(1 - E/E_{QG}). \tag{6}$$

Thus we can write for d (for two photons of energies E_1 and E_2).

$$\Delta t \simeq \frac{D}{C(1 - E_1 / E_{QG})} - \frac{D}{C(1 - E_2 / E_{QG})}$$

$$\simeq \frac{D}{C}\left(\frac{E_1}{E_{QG}}\right), \text{ for } E_1 >> E_2. \tag{7}$$

In the case of GRB 990123, where optical detection followed hardly 20 s after the gamma ray burst, this was shown to imply[18] a lower limit as:

$$E_{QG} > \frac{DE_1}{c\Delta t} = 5 \times 10^{16} \text{ GeV}.$$

Similar constraints exist from TeV flares and other GRBs.

In the case of superstring theory, the same dispersion relation arises but with E_{QG} now related to the string scale l_s. As well known, duality invariance $l \to (a/l)$, implies a minimal length scale l_s for string theory. So eq. (7) could be translated into a constraint on the super string scale as:

$$l_s < 3 \times 10^{-31} \text{ cm} < 200\ L_{pl}, \tag{8}$$

(corresponding to the string scale $E_s > 5 \times 10^{16}$ GeV). Essentially for the first order effects, i.e. effects going as in first power of E/E_s one gets the corresponding limits imposed by superstring theory simply by substituting E_s in place of E_{pl} (in all the above quantum gravity imposed limits), i.e. highest CR energies would now be

~ 10^{26} eV, upper limiting temperature (early universe, etc.) ~ 10^{30} deg. etc.

It must be emphasized that just because one has a string scale (related to the string tension and gauge coupling as discussed below), this does not imply a fundamental length, which in the earlier literature was a very arbitrary quantity. Here as argued below, the duality invariance (a feature of string theories) implies a minimal length scale and a *modified* quantum uncertainty principle (to be discussed below), and a mixed commutator (see eq. (13)). It is to be noted that just a fundamental length arbitrarily proposed cannot give a modified Born-Infeld electrodynamics or give corrections or deviations to Newton's laws at submillimetre ranges. Nor can the above estimates of high frequency gravitons or fundamental limits on temperature follow from a fundamental length.

The string (or quantum gravity) modified dispersion relation could also have drastic effects on the propagation of TeV and PeV photons in intergalactic space. For instance, if E_{QG} or $E_s \sim 10^{16}$ GeV, photons with even a few TeV energy could travel freely (with reduced photon–photon scattering a higher order process) through any soft background of microwave or infrared photons[19].

Degradation by pair creation appears kinematically forbidden when energy of such a high energy photon exceeds $E_m \sim 2\sqrt{E_b E_s}$, E_b is the theory of the soft background photon (say 3°k background). For disintegration into pairs of mass m and momentum p, (e.g. through $\gamma + \gamma \to e^+ + e^- \to l^+ + l^-$, etc.), the modified dispersion relation implies:

$$m^2c^4 + p^2c^2 = E^2 + E^3/E_s.$$

With this modification, it can be shown that pair creation is allowed only if:

$$E_m \le 2(2E_b/E_s)^{1/2}, \tag{9}$$

(an equivalent way of looking at it is that there is a modification of standard Lorentz kinematics, i.e. explicit breaking of Lorentz invariance (for point particles) at the string scale l_s). This is similar to the situation in k distorted Poincare group[20]. The above relations when applied to the process of TeV photons (E_m) with soft IR background photons with $E_b \sim 0.5$ eV, implies no pair creation (for the process $\gamma_h + \gamma \to e^+ + e^-$) unless

$$E_m \le 400 \text{ TeV } (E_s/E_{pl})^{1/2}. \tag{10}$$

For the above constraint on E_s (from eq. (8)), $E_m < 30$ TeV. This shows that the string or quantum gravity modified dispersion relation implies that gamma photons larger than ~ 30 TeV energy would not be destroyed by pair creation onto soft background photons of eV energies. Thus powerful sources of TeV radiation from black hole binaries or AGNs could be detectable up to large red shifts. This would also have implications for the GZ cut off at high energies for the CR spectrum by interaction with the CMB background, i.e. we could have photons with energies ~ 10^{20} eV or larger.

Again it is to be remarked that the above dispersion relations have an energy dependence quite distinct from that in conventional electromagnetic plasmas[21], which decreases with increasing energy. For γ rays of a MeV range if one assumes an effect of ~ l_s/l, one gets for the effect a time shift in the waves ~ 10^{-4} s. For instance, features of about 1 ms have been reported in bursts of 0.1 s width so that is possible to look for some of these effects in noisy data. In addition to the above dispersion relations induced for example by stringy effects, it has been suggested that the polymer-like nature of space time predicted by canonical quantum gravity models could also lead to biregringent effects[22]. See also ref. 23. Corrections to Maxwell equations are obtained as:

$$d_t\bar{E} = -\nabla\times\bar{B} + \frac{k}{\sqrt{E_{QG}}}\nabla^2\bar{B},$$

$$d_t\bar{B} = \nabla\times\bar{E} - \frac{k}{\sqrt{E_{QG}}}\nabla^2\bar{E}. \tag{11}$$

If one seeks solutions with a given helicity:

$$E_\pm = Re((\bar{e}_1 \pm ie_2)e^{i}(\Omega_t t - \bar{k}.\bar{x}),$$

one finds

$$\Omega_\pm \sim (k)(1 \pm 2l_s\,|k|). \tag{12}$$

The group velocity has two branches, and the effect is of a shift of one l_s per wavelength. For TeV γ rays, if is of fractional order ~ 10^{-12}. Gravitational waves would also be affected by the above type of distortions. For space-based GW detectors with laser interferometers, because of the long wavelengths the effects are of ~ 10^{-27} or less, too small unfortunately to be detected. Superstring theory also implied a modification of the uncertainty principle. The uncertainties in the string position and momentum are[24]:

$$\Delta x \simeq (\hbar/T)^{1/2} \text{ and } \Delta p \simeq (\hbar T)^{1/2},$$

(where T is the string tension).

A similar situation exists for the harmonic oscillator where the position and momentum uncertainty behave rather symmetrically both scaling like the square root of the planck constant. The duality invariance $l \rightarrow l^{-1}$ implied existence of a minimum string scale and gives rise to extended uncertainty principles given as:

$$\Delta l = \frac{\hbar}{\Delta p} + \frac{\Delta P}{T}.$$

The existence of a minimal length may be related to a non-commutative geometry.

At a formal level a rather general procedure of modifying the uncertainty principles is provided by change of the mixed commutator in the Heisenberg algebra[25]

$$[x_{\mathrm{i}}, p_{\mathrm{j}}] = ih(d_{\mathrm{j}} + \varphi_{\mathrm{i}} p_{\mathrm{j}} + g d_{\mathrm{j}} p^2)$$

$$= ih(d_{\mathrm{j}} + \varphi_{\mathrm{i}} p_{\mathrm{j}} + g d_{\mathrm{j}} p^2), \qquad (13)$$

where $g = l_s^2/\hbar^2$ is a deformation parameter introducing the minimal length scale l_s. In the first perturbative order in g the consequences of this minimally modified algebra on the H-atom spectrum has been recently studied[26]. The result for the corrected energy eigen values can be put in the form[27].

$$En_{\mathrm{ij}} = B\left(-\frac{1}{n^2} + 4\left(\frac{l_{\mathrm{s}}}{a}\right)^2\right)\frac{4n - 3(l + 1/2)}{n^4(l + 1/2)}. \qquad (14)$$

The correction is always +ve and is maximal for the ground state, leading to a relative decrease $E_{\mathrm{L}} = -20(l_s/a)^2$ of the hydrogen ionization energy (a is Bohr radius) within each multiplet, effect is maximal for 1 = 0 levels. $\Delta E_{n_1 0}/E_{n_1 0} = -4(l_s/a)^2(8n - 3)/n^2$. The accuracy in the frequency data for the 1s – 2s transition is ~ 1 kHz. The precision in the energy difference between the two levels is thus about 10^{-12} eV which thereby implies $l_s < 0.01$ fermi. This would put atomic physics constraints on large extra dimensions, ($l_s >> L_{\mathrm{pl}}$) now in vogue[28]. For $l_s = L_{\mathrm{pl}}$, relative effect in Rydberg energy ~ 10^{-47} (<) and for the E_{QG} constraint from gamma ray burst this is of ~ 10^{-42}. The large extra dimensions postulated in many recent papers, imply deviations from Newton's law at submillimetre ranges. For $n > 1$, extra space dimensions, the long-range corrections can be approximated by a Yukuwa type interaction with a potential as[29]

$$V_{r>R} \simeq \frac{1}{R^{\mathrm{n}} r}(1 + 2n \times e^{-r/R}), \qquad (15)$$

where r is the distance and R a common compactification radius of n extra dimensions. We can compare these modifications, with the usual parametrization of the extra long-range forces in the literature (e.g. ref. 30): $V(\partial\ a(1/\partial)(1 + ae^{-\partial l})$ and get a definite prediction for the strength a of Yukawa type gravitational corrections, in the case of one extra compact dimension it is $a = 2$. Using the $a - l$ plot of ref. 31 which gives the experimentally determined region, one can conclude[28,29] that the allowed radius has an upper bound of the order of $l = R$ ~ 1 mm, see also ref. 32. It has recently been pointed out that new dimensions may have a size R much larger than the string scale perhaps as large as a millimetre[28,29]. This has the effect of diluting the strength of the 4D gravity observed at >> R. The 4D Planck scale is determined by the fundamental Planck scale M by Gauss's law $M_{\mathrm{pl}}^2 \sim M^{2+n}R^n$, n is the number of new dimensions. Large dimensions are not in conflict with experiment if the standard model fields are confined to a 3-brane. In the extra dimensions they lead to corrections to the Newtonian potential ~(TeV$^{-(n+2)}$ $\gamma^{-(n+1)}$ at all distances. Within the realm of the ordinary 3 + 1 dimensional space-time an important consequence of the existence of large extra dimensions would be the presence of a tower of Kaluza-Klein modes associated to gravitons. The weakness of the coupling between gravitons and other particles can be compensated by the large number of KK modes when the experimental energy resolution is much larger than the mass splitting between the modes, which for a small number of very large extra dimensions can be a weak requirement (e.g. for 6 mm wide extra dimensions[33], the mass splitting is a few MeV). This can lead to observably large effects at planned particle-physics colliders, particularly CERN's LHC. This is irrelevant at large distances but dominates the Newtonian potential at distances smaller than R where $M_{\mathrm{pl}}^2 \sim (\mathrm{TeV})^{n+2}R^n$. For $n \geq 2$, R is < 1 mm and so long distance Newtonian gravity is not affected. For an early suggestion of large extra dimensions in connection with strong gravity see ref. 32 and for a early connection to dual resonance see ref. 34. A large extra dimension at the TeV scale would also give a correction to the magnetic moment ~ $G_F^{1/2}$, G_F is the Fermi constant[35]. Again there can be corrections to the wave function and the energy spectrum of quantum particles due to the presence of the string tension. This introduces a generic modification of the energy spectrum[36] as

$$E = \left(m^2 + k^2 + \frac{l^2}{a^2 \mathrm{a}^2} - \frac{1}{(4a^2)}\right)^{1/2},$$

where a is related to the string tension, k, m are the momentum and mass, l is the angular quant. no., a is radial distance from the string. The Schrödinger equation can also be solved in the string background space time, with the string tension entering into the centrifugal barrier term as a correction and energy difference between any excited level and the ground state is $\sim l^2/2ma^2a^2$. Again the appearance of a Born–Infeld action is a key ingredient in string theory. For the dynamics of a D_p-brane embedded in space time of small curvature compared to l_s^2 we can write a term of the form

$$\int \mathrm{d}^{p+1} n e^{-f} \sqrt{-\det(G_{ab} - l_s^2 F_{ab}) + 2}$$

by comparing with the B–I action of form

$$\sim \left(1 - \frac{1}{2} \frac{F_{mn} F_{m0}}{E_{\max}^2}\right),$$

(ironically Born and Infeld proposed their modified electrodynamics with a maximal or limiting field strength to avoid divergences in classical electron self energies). Here the limiting field strength is connected to l_s^2 through $E_{\max} \sim g^2/l_s^2$, where g is the gauge coupling. This would give a finite size to collapsing charged configurations. There are clearly implications for the early universe[37].

In superstring cosmology the universe starts from a state of very small curvature, then goes through a long phase of dilaton-driven inflation reaching nearly Planck energy density and eventually reaches standard radiation dominated evolution. For a review see ref. 38. For a cosmology based on critical superstrings see ref. 39. The period of nearly Planckian energy density plays a crucial role in making the quantum gravity effects observable. The string cosmology involves a period in which quantum gravity effects are actually quite large and planned gravity-wave detectors such as LIGO might be able to detect faint residual traces of these effects[38]. The effects of the dilaton and antisymmetric fields in the superstring inflation phase could have left their imprints in the COBE spectrum and indeed the tilt of COBE can constrain aspects of superstring geometry as discussed in ref. 40.

Some additional notes

(Conceptual issues, consequences, etc.)

1. As was noted above, existence of a minimum string scale gives rise to an extended uncertainty principle. Thus combining general relativity with quantum mechanics is bound to lead to a reformulation of a very basic tenet of quantum physics, i.e. the uncertainty principle.
2. A well-tested aspect of classical GR is the Einstein equivalence principle. This implies among other things that orbits of classical *test* particles are independent of their mass (i.e. mass of test particle (m) cancels out in the force equation, $mv^2/r = GMm/r^2$, M being the source mass). Now if we invoke Bohr-Sommerfeld type 'quantized' stationary orbits (i.e. quantization of angular momentum, $mvr = n\hbar$), it is clear that the radii of the 'allowed' Bohr orbits are given by: (n the principal quantum no.)

$$r = \frac{n^2 \hbar^2}{G m^2 M}.$$

So that we can infer the mass of a test particle from its orbital radius in contrast with the classical case. Evidently we perhaps need an extended equivalence principle implying that this principle does not hold for particles in stationary 'gravitational Bohr orbits' (GBO), where the particles do not radiate (i.e. do not collapse unlike the classical situation) and energy transitions are only between these states. One can of course solve the relevant Schrödinger equation (with the gravitational potential) and get all the energy levels. So analogous to the usual correspondence principle, the classical situation (where the equivalence principle is valid) holds for only large quantum numbers $n \to a$. Area quantization, analogous to energy level quantization has been proposed so that quantized area changes lead to discrete energy changes.

In the superstring approach we have a minimal length (l_s) as felt above, whereas in loop quantum gravity we have a minimal area (analogous to minimal action $\hbar$ in the Bohr–Sommerfeld rule).

3. In any case since in quantum mechanics a particle of mass m cannot be localized to a distance $< \hbar/mc$, i.e. we have extended particles which are not strictly test particles, so that the equivalence principle cannot exactly hold.

If two quantum particles come closer and closer till they are separated by a distance r, then the uncertainty principle implies that their mutual gravitational force scales are:

$$F \simeq \frac{G}{r^2} \cdot \left(\frac{\hbar}{rc}\right) \cdot \left(\frac{\hbar}{rc}\right) \sim \frac{1}{r^4}, \text{ i.e.}$$

we have a $1/r^4$ dependence rather than the classical $1/r^2$. So at short distances, the gravitational force would be *very different* from the classical case. Same kind of arguments hold for interaction between strings which are *extended* objects of minimal size comparable to l_s, the string scale. The equivalence principle does not hold for extended objects. The tidal acceleration on an extended quantum particle will go as $1/m$, the mass of the particle. As the 'wavelength' (de Broglie) of particles scales with temperature as $l \simeq \left(\frac{\hbar^2}{2mkT}\right)^{1/2}$, it follows that very low temperatures, would give a large wavelength which in turn would imply a large r tidal acceleration which could probably be tested by cold atoms falling freely (Peters *et al.*, *Nature*, 1999,

400, 849). The gravitational force between such wave packets would scale as ~ m^3T (a temperature dependent force). Recently it has been shown that the classical gravitational self-energy obeys equivalence principle to a part in a thousand (Baessler *et al.*, *PRL*, 1999, **83**). As noted above, *quantum* gravitational self energy would introduce new features which could be tested in future experiments. For superstrings the tidal force would scale as l_s^2.

4. In experiments probing aspects of interplay between gravitation and quantum mechanics (like that of neutron interferometry) the description of matter dynamics (with wave function $y\,(r)$), in presence of the earth's gravitational potential $f(r)$ is given by the appropriate Schrödinger equation where the kinetic term $(\hbar^2/2M_I)\nabla^2$ and the potential $M_G\,f(r)$, involve M_I and M_G, the inertial and gravitational masses respectively. This again raises a puzzle with respect to the equivalence principle of general relativity, as even for $M_I = M_G$, the test mass does not cancel out in the quantum evolution equation. The above problem has not been fully addressed even within the most popular quantum gravity approaches, i.e. critical superstrings and canonical or loop quantum gravity. It is still not clear what role and which formulation of the equivalence principle would or should hold in quantum gravity.
5. Generalization of the Bohr Rosenfeld analysis to the measurability of gravitational fields of course confront the fact that the ratio between gravitational charge (mass) and inertial mass is fixed by the equivalence principle. This strongly suggests that the mechanics on which quantum gravity is based should accommodate a somewhat different relationship between 'system' and 'measuring apparatus' and should not rely on the idealized 'measuring apparatus' which plays a central role in ordinary quantum mechanics (e.g. the 'Copenhagen interpretation'). As mentioned above, interplay between gravitational and quantum properties of devices affects the measurability of distances. If one considers the gravitational properties of devices a conflict with ordinary quantum mechanics immediately arises because the classical device (infinite mass) limit is inappropriate for measurements concerning gravitational effects. As the devices get more and more massive they disturb the gravitational (geometrical) observables. This conflict between the infinite-mass classical formalism of ordinary quantum mechanics to describe outcome of experiments and the nature of gravitational interactions has not been addressed in both superstring and loop gravity approach. For completion one should mention the suggestion of Penrose that the reduction of the wave function in quantum measurements is initiated by quantum gravity effects. Again as noted above in quantum gravity any measurement that monitors a distance L for the time t^{obs} is affected by an uncertainty $\mathrm{d}L \leq \sqrt{l_s c.t_{obs}}$, owing to the extended uncertainty principle.
6. In a time of observation as long as the inverse of the typical gravity-wave interferometers frequency of operation, an extremely large number of minute quantum fluctuations could affect the distance between the test masses. The standard deviation increases with the time of observation while the displacement noise amplitude spectral density increases with inverse of frequency. The dynamical behaviour of superstrings in the very localized space-time fluctuations of a fuzzy space-time is yet to be formulated. It is plausible that the 'advanced phase' of LIGO achieves a displacement noise spectrum of less than 10^{-20} m/$\sqrt{\mathrm{Hz}}$ near 10^2 Hz and in principle this could proble valves as small as 10^{-32} cm (~ l_c). The present bound is believed to be at the level of ~ 10^{-27} cm (ref. 41). In this connection it is of interest to note that in certain 'M-theory motivated' scenarios with an extra length scale associated to the compactification from eleven to ten dimensions a hierarchy of quantum gravity scales may be involved which the above noise spectrum may probe in principle. Existing noise-level data obtained at the Caltech 40 m interferometer which has achieved[42] displacement noise levels with amplitude spectral density lower than 10^{-18} m/$\sqrt{\mathrm{Hz}}$ for frequencies between 200 and 2000 Hz. Again it has been conjectured that an effective large-distance description of some aspects of quantum gravity might involve quantum symmetries and non-commutative geometry. As noted above, the type of *in vacuo* dispersion which can be tested using observations of distant gamma ray sources is naturally encoded within a consistent deformation of Poincare symmetries. Evidence has been found in Liouville strings supporting the validity of deformed dispersion relations with the deformation going linearly with the Planck/string length.
7. Recent neutral-kaon experiments such as the ones performed by CPLEAR[43] have set significant bounds on quantum gravity associated CPT-violation effects and forthcoming experiments may improve bounds further. While in the interferometry-based and the GRB-based experiments the approach is to put together many quantum gravity effects, in the case of neutral-kaons the crucial element is provided by the very delicate balance of scales that characterizes the system. Particularly it happens that the dimensionless ratio setting the order-or-magnitude of quantum-gravity effects in the linear suppression scenario which is $c^2M_{l,s}/E_{pl}$ ~ 2×10^{-19} is not much smaller than some of the dimensionless ratios characterizing the neutral kaon system, i.e. ratio $|m_L - m_s|/m_{L,S}$ ~ 10^{-15} and the ratio $(g_L - g_S)/M_{L,S} \sim 10^{-14}$.

8. For extended objects like strings, Wigner's quantum inequalities may also be appropriate Wigner's first inequality giving the maximum time over which a clock of dimension L and mass m can remain accurate (i.e. $t_{\max} < (L^2M/\hbar)$) gets modified in the superstring case to:

$$L = \left(\frac{\hbar t_{\mathrm{mas}}}{M} + l_s^2\right)^{1/2},$$

using for the position spread of a 'string' clock, the relation $(\Delta L = L + \hbar t M^{-1} L^{-1} + l_s^2 L^{-1})$ and a similar relation for the bound on the clock mass (second inequality)[44].

9. There are also implications of string theories for computer capabilities as noted above. We saw above that classical gravity gives an upper limit as the amount of information (data) that can be stored in a system of linear dimension L and total energy E. While quantum gravity gives the minimal information processing rate possible in a gravitating system. For a solar mass gravitating computer, the maximum data that can be stored is $\sim 10^{77}$ bits, while the lowest processing rate ~ 1 megabit. Indeed the above Wigner clock inequalities do provide strongest constraints on the ultimate capability of femto-computer technologies involving accurately time-ordered sequences. The maximal rate of computation (f) with a mean input or supply of power P can be expressed as (this is independent of the *hardware technology*).

$$f \leq (P/\hbar)^{1/2}.$$

This gave rise to the ultimate upper quantum gravity bound of $\sim 10^{44}$ bit/s. Superstring theory gives the modified relation between the power input P required for a processing rate f as:

$$P \leq \hbar f^2 \left(1 + \frac{l_s^2 f^2}{c^2}\right) (f << c/l_s).$$

However, if the particle spectrum invoked in the quantum computer is described by string degrees of freedom, the number of states grows exponentially with mass as $n \sim m^{-3} \exp(m/T)$ and this would imply a maximal processing rate a few orders smaller than the upper limit above[45]. Product of P and f is an upper limit of $\sim (i^5/G\hbar).(l_s/l_p)^2$.

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