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RESEARCH ARTICLES

Resolution of the nonlocality puzzle in the EPR paradox

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Correlations of quantum-entangled multiparticle systems have been widely discussed in the context of the Einstein–Podolsky–Rosen (EPR) paradox. Bell's theorem prohibits a local realistic description of these correlations. The standard quantum mechanical derivation as well as the interpretation of the experiments suggest that a mysterious nonlocality is a basic feature of these correlations. We show that the correlations of space-like separated entangled particles can be reproduced starting from local probability amplitudes. The use of complex number amplitudes circumvents the widely discussed Bell's theorem. The result implies no-collapse-at-a-distance and resolves the EPR puzzle.

THE Einstein–Podolsky–Rosen (EPR) nonlocality puzzle is one of the most discussed fundamental problems in physics[1–4]. EPR considered a two-particle quantum system entangled and correlated in position and momentum and argued that under the assumption of locality and a reasonable definition of physical reality, the quantum mechanical description is incomplete. The conclusion of incompleteness was based on their reasoning that suggested the need for simultaneous existence of definite values for noncommuting observables for the individual particles before a measurement was performed. The EPR analysis was motivated by the desire to assign an objective reality to measurable properties in the microscopic world independent of the observer or apparatus. The Copenhagen interpretation of quantum mechanics rejected any such objective reality. EPR proposed that if the value of an observable could be predicted with certainty without the disturbance of a measurement, then there was an element of physical reality associated with that observable for the system. There is no way to probe experimentally such a physical reality for the single quantum, for example the reality of the spin component in a specific direction for a single

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photon or an electron. But if multi-particle correlated systems are considered, like two spin-$\frac{1}{2}$ particles propagating out from an initially spin-zero state, then it is possible to explore the consequences of assuming objective reality to observables.

Bell's theorem

David Bohm transcribed the EPR argument to the spin variables by considering the singlet entangled state of two spin-$\frac{1}{2}$ particles[5]. This state is described by the wave function

$$y_S = \frac{1}{\sqrt{2}}\{|1,-1\rangle - |-1,1\rangle\}, \tag{1}$$

where the state $|1, -1\rangle$ is a short form for $|1\rangle_1 |-1\rangle_2$, and represents an eigenvalue of $+1$ for the first particle and -1 for the second particle *if measured* in any particular direction. y_S is inherently nonlocal, describing both particles together, even when they are far apart in space-like separated regions. It is a superposition of two-particle product states and there is no specific state assigned to any of the particles individually. A measurement on *one* particle changes the whole wave function, collapsing the states of *both* particles to definite values.

If the spin component is measured in any direction on one of the particles, the same component can be predicted with certainty for the other companion particle using the conservation law that the total spin is zero. The EPR definition of reality then assigns objective reality before measurement to this spin component. Since the decision as to which component is to be measured is arbitrary and could be delayed till the particles are far apart, the assumption of locality – that one measurement should not influence the result of the other – implies that there should be physical reality for all the three components of the spin for the two particles individually. This is not allowed by quantum mechanics since the three spin components are mutually noncommuting. According to the EPR argument, this conclusion suggested the plausibility of a better theory, possibly with additional hidden variables that would assign specific values to the observables in each run of an experiment such that the statistical predictions of quantum mechanics are reproduced when averaged over the distribution of the hidden variables. Such a theory is a local (realistic) hidden variable theory.

John Bell analysed the EPR problem in the early sixties and established the Bell's inequalities obeyed by any local hidden variable theory of correlations of entangled particles[3,4,6]. He considered the measurement of the spin components on the two particles in two differ-

ent directions, in contrast to the EPR analysis that involved measurements in the same direction (see Figure 1). The result of such a measurement is two-valued in any direction. If A and B denote the outcomes $+1$ or -1 (written as $+$ or $-$) and $\mathbf{a}$ and $\mathbf{b}$ denote the settings of the analyser or the measurement apparatus for the first particle and the second particle, respectively, then the statement of locality is that

$$A(\mathbf{a}) = \pm 1, \qquad B(\mathbf{b}) = \pm 1. \tag{2}$$

The outcome for the first particle is decided by the local setting $\mathbf{a}$ for the analyser with which the first particle interacts, and for the second particle the outcome is decided by the local setting $\mathbf{b}$. The statement of existence of reality is that some hidden variable decides the outcomes even before the measurement, or equivalently the outcome has an objective existence even if the measurement is not actually carried out. This is encoded as

$$A(\mathbf{a},\mathbf{h}_1) = \pm 1, \qquad B(\mathbf{b}, \mathbf{h}_2) = \pm1, \tag{3}$$

where $\mathbf{h}_1$ and $\mathbf{h}_2$ are hidden variables associated with the outcomes. In an experiment involving the kind of measurement described, the experimenter can calculate a *correlation function of the outcomes* defined by

$$P(\mathbf{a},\mathbf{b}) = \frac{1}{N}\sum (A_i B_i). \tag{4}$$

This is a classical correlation function obtained by averaging the products of the form $(++ = +)$, $(-- = +)$, $(+- = -)$ and $(- + = -)$. Since the joint events $(++)$ and $(--)$ are coincidences and the events $(+ -)$ and $(- +)$ are anti-coincidences ('coincidence' denotes both particles showing the same value for the measurement and 'anti-coincidence' denotes those with opposite values), $P(\mathbf{a}, \mathbf{b})$ denotes the average of the quantity (*number of detections in coincidence – number of detections in*
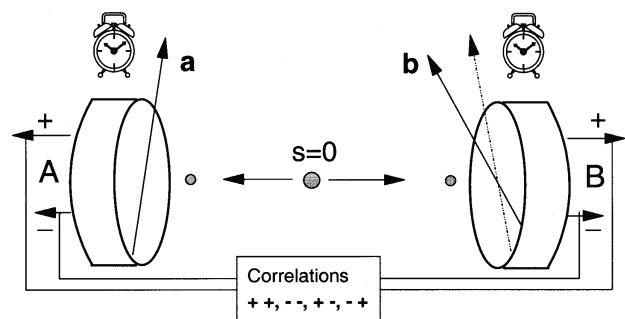

Figure 1. Schematic diagram of spin correlation measurements on entangled particles. The outputs of the two analysers are $+1$ or -1 and these are correlated in a coincidence unit. $\mathbf{a}$ and $\mathbf{b}$ are the two analyser directions. Dotted arrow on the right analyser shows the direction $\mathbf{a}$ for reference.

anticoincidence). The task of the theory is to calculate this function starting from suitable basic ingredients. Bell chose to calculate this correlation by multiplying $A(\mathbf{a}, \mathbf{h}_1)$ and $B(\mathbf{b}, \mathbf{h}_2)$ and integrating over the distribution of the hidden variable $\mathbf{h}$, since the assumption of realism demanded that the outcomes were 'there' even before the measurement was made. The Bell correlation is then $\int d\mathbf{h} r(\mathbf{h}) A(\mathbf{a}, \mathbf{h}_1) B(\mathbf{b}, \mathbf{h}_2)$, where $\int d\mathbf{h} r(\mathbf{h}) = 1$. The essence of Bell's theorem is that the function $P(\mathbf{a}, \mathbf{b})$ has distinctly different dependences on the relative angle between the polarizers for a local hidden variable description and for quantum mechanics.

The correlation predicted by a local realistic theory is bounded by the Bell's inequalities. The magnitude of a particular sum of correlations $P(\mathbf{a}, \mathbf{b})$ for different combinations of $\mathbf{a}$ and $\mathbf{b}$ is bounded by the value 2, and the same combination calculated in quantum mechanics using the entangled wave function and spin operators exceeds this bound, violating the inequality[3,6]. Also, experimentally measured correlations agree with the quantum mechanical predictions and violate the inequality[4,7]. The concept of local realism is not tenable in the Bell framework. These results have been interpreted as evidence for nonlocal influences across space-like separated events. The measurement of an observable on one of the particles causes the other particle to acquire a definite state consistent with the relevant conservation law. The standard quantum mechanical derivation of these correlations employs the nonlocal multiparticle wave function, and measurement on one particle is said to collapse the state of the companion particle to a definite eigenvalue. Though this nonlocal feature cannot be used for superluminal signal communication[8], there is a conflict with the spirit of relativity[9]. Also, such superluminal influences in the microscopic physical world are bizarre and at present beyond any understanding in terms of any physical mechanism.

The nonlocality puzzle can be resolved however, if the correct physical input is used for the calculation of the quantum correlations. The correct correlations can be reproduced if we start with the description of the relevant physical phenomenon, like the passage of the quantum particle through a polarizer, using local probability amplitudes[10]. In the local hidden variable theories of the form Bell considered, the correlations are calculated from eigenvalues and this procedure *does not preserve the phase information* and wave-like characteristics of the quantum system. The situation has some analogy to the description of interference in quantum mechanics. Any attempt to reproduce the interference pattern using locality and the information on 'which-path' will fail since the phase information is lost or modified in such an attempt. We calculate the correlation from the local amplitudes instead of from the eigenvalues. The wave nature of quantum systems is then explicitly used in the calculation of the correlations and the final probabilities are calculated by squaring a suitable inner product of the local amplitudes. (It is the use of complex functions with a phase that is interpreted as the 'wave nature' in this paper.) The realism in this theory is at a deeper level, concealed as a phase and not as actual outcomes before a measurement is made. This possibility was not considered in earlier local realistic theories, and turns out to be the crucial concept required to resolve the EPR puzzle.

A new calculus for correlations

Consider the breaking up of a maximally entangled state – photons entangled in orthogonal polarization states or spin-$\frac{1}{2}$ particles entangled in (up, down) state – as in the standard Bohm version of the EPR problem[4,5]. The two particles go off in opposite directions and are in space-like separated regions. Two observers make measurements on these particles individually with time stamps such that these results can be correlated later (Figure 1). We assume that strict locality is valid. *Measurement on one particle does not change either the magnitude or phase of the complex amplitude associated with the companion particle. In particular, measurement on one particle does not cause the companion particle to acquire a definite state.*

At each location the result of a measurement is two-valued, denoted by (+) and (–) for each particle (two mutually exclusive outcomes), for any angle of orientation of the analyser. We prescribe the local rules or amplitudes for the transmission through an analyser for these particles (we use the terms polarizer and analyser in a generic way. They could be Stern–Gerlach-like analysers for spin-$\frac{1}{2}$ particles or polarizers for photons). The local amplitudes for events + and – for the two particles individually are denoted by the complex functions C_{1+}, C_{1-}, C_{2+} and C_{2-}. Amplitudes C_{1+} and C_{1-} are mutually orthogonal and similarly C_{2+} and C_{2-} are mutually orthogonal. The statement of locality is at the level of these amplitudes, and can be written as

$$C_{1\pm} = C_{1\pm}(\mathbf{a}, \mathbf{f}_1), \quad C_{2\pm} = C_{2\pm}(\mathbf{b}, \mathbf{f}_2), \tag{5}$$

where $\mathbf{f}_1$ and $\mathbf{f}_2$ are the 'hidden variables'. These are internal variables associated with the individual particles and appear in the amplitudes as a phase. These can be thought of as the initial undetermined phase, associated with the spin of the individual quantum particles. A definite value for these variables does not imply a definite state for the particles before the measurement.

The locality assumption also implies that

$$A(\mathbf{a}, \mathbf{f}_1) = \pm 1, \quad B(\mathbf{b}, \mathbf{f}_2) = \pm 1. \tag{6}$$

But, this has a meaning different from the one in standard local realistic theories. Here, it means that the

outcomes, *when measured*, depend only on the local setting and the local internal variable.

In this framework the correlation function is not $P(\mathbf{a},\mathbf{b}) = (1/N)\Sigma(A_i B_i)$ or $\int d\mathbf{h}r(\mathbf{h})A(\mathbf{a}, \mathbf{h}_1)B(\mathbf{b}, \mathbf{h}_2)$. We calculate the correlations from the local amplitudes. These correlations are of the form $U(\mathbf{a}, \mathbf{b}) = \mathrm{Real}(NC_iC_j^*)$, where N is a normalization factor. It is the square of this correlation function that would give a joint probability. *This new calculus of local amplitudes ensures that all the probabilities are positive definite.* (There are local theories which reproduce the correct correlations using probability distribution of the hidden variables that are not positive definite. These theories are neither rooted in quantum concepts nor in classical concepts, and are really 'out of this world'.) The correlation function is analogous to the two-point amplitude correlations of two independent classical electromagnetic fields. The expression can be generalized to situations where there are more than two particles.

Now that we have outlined the general scheme and assumptions as well as the point of departure in calculating the correlation, let us consider the maximally entangled singlet system described by eq. (1), the most widely discussed example in the context of non-locality. We prescribe the local amplitudes as $C_{1+} = \frac{1}{\sqrt{2}}\exp\{is(\mathbf{q}_1 - \mathbf{f}_1)\}$ for the first particle at the first polarizer and $C_{2+} = \frac{1}{\sqrt{2}}\exp\{is(\mathbf{q}_2 - \mathbf{f}_2)\}$ for the second particle at the second polarizer. There are corresponding amplitudes, C_{1-} and C_{2-} for the events denoted by $-$, and they differ only in the phase for the maximally entangled state.

In these amplitudes, $\mathbf{q}_1$ and $\mathbf{q}_2$ are the directions of the two polarizers, and s is the spin of the particle (1 for photons and $\frac{1}{2}$ for the spin-$\frac{1}{2}$ particles). The explicit dependence of the amplitude on the spin of the particle is motivated by the fact that we are dealing with systems with phases and the phase associated with the spin rotations (a geometric phase) is a necessary input in this description[11]. The correlation at source is encoded in the relative value or the difference $\mathbf{f}_0$ of the internal variables. $\mathbf{f}_0$ is a constant for all the entangled pairs. The locality assumption is strictly enforced since the two amplitudes depend only on local variables and on an internal variable generated at the source and then individually carried by the particles without any subsequent interaction of any sort. The individual measurements at each end will now separately give the correct result for transmission for any angle of orientation. These probabilities are

$$C_1 C_1^* = C_2 C_2^* = \frac{1}{2}. \tag{7}$$

Events of both types $(++)$ and $(--)$ contribute to a 'coincidence'. The correlation function for an outcome of

either $(++)$ or $(--)$ of two maximally entangled particles is

$$U(\mathbf{q}_1, \mathbf{q}_2, \mathbf{f}_0) = 2\mathrm{Re}(C_1 C_2^*) = \cos\{s(\mathbf{q}_1 - \mathbf{q}_2) + s\mathbf{f}_0\}. \tag{8}$$

It is normalized such that its square will give the conditional joint probabilities of the type 'outcome $+$ for the second particle, given that the outcome for the first particle is $+$', etc. All references to the individual values of the internal variable $\mathbf{f}$ have dropped out.

We now derive the relation between this correlation function and the experimenter's correlation function $P(\mathbf{a}, \mathbf{b}) = (1/N)\Sigma (A_i B_i)$. Since $U^2_{++} = U^2_{--}$ for the maximally entangled state, $U^2(\mathbf{q}_1, \mathbf{q}_2, \mathbf{f}_0)$ is the probability for a coincidence detection $(++$ or $--)$, and $(1 - U^2(\mathbf{q}_1, \mathbf{q}_2, \mathbf{f}_0))$ is the probability for an anticoincidence (events of the type $+-$ and $-+$). Since the average of the quantity (number of coincidences – number of anticoincidences) is

$$U^2(\mathbf{q}_1, \mathbf{q}_2, \mathbf{f}_0) - (1 - U^2(\mathbf{q}_1, \mathbf{q}_2, \mathbf{f}_0))$$
$$= 2U^2(\mathbf{q}_1, \mathbf{q}_2, \mathbf{f}_0) - 1, \tag{9}$$

the correspondence between $P(\mathbf{a}, \mathbf{b})$ and $U(\mathbf{q}_1, \mathbf{q}_2, \mathbf{f}_0)$ is given by the expression,

$$P(\mathbf{a}, \mathbf{b}) = 2U^2(\mathbf{q}_1, \mathbf{q}_2, \mathbf{f}_0) - 1$$
$$= 2\cos^2\{s(\mathbf{q}_1 - \mathbf{q}_2) + s\mathbf{f}_0\} - 1. \tag{10}$$

Examples and applications

Singlet spin-$\frac{1}{2}$ particles and photons

Consider the singlet state breaking up into two spin-$\frac{1}{2}$ particles propagating in opposite directions to spatially separated regions. Since the total spin is zero in any basis we set $\mathbf{q}_0 = \mathbf{p}$. Then the correlation function and $P(\mathbf{a}, \mathbf{b})$ calculated from this function are

$$U(\mathbf{q}_1, \mathbf{q}_2, \mathbf{f}_0) = \cos\{s(\mathbf{q}_1 - \mathbf{q}_2) + s\mathbf{f}_0\}$$
$$= \cos\left\{\frac{1}{2}(\boldsymbol{q}_1 - \boldsymbol{q}_2) + \boldsymbol{p}/2\right\}$$
$$= -\sin\frac{1}{2}(\boldsymbol{q}_1 - \boldsymbol{q}_2), \tag{11}$$

$$P(\mathbf{a}, \mathbf{b}) = 2\sin^2\left(\frac{1}{2}(\boldsymbol{q}_1 - \boldsymbol{q}_2)\right) - 1$$
$$= -\cos(\mathbf{q}_1 - \mathbf{q}_2) = -\mathbf{a}\times\mathbf{b}. \tag{12}$$

This is identical to the quantum mechanical prediction obtained from the singlet entangled state and the Pauli

spin operators[4]. We have reproduced the correct correlation function using local amplitudes.

For the case of photons entangled in orthogonal polarization states we get, by setting $s = 1$ and $f_0 = p/2$ to represent orthogonal polarization,

$$U(q_1, q_2, f_0) = \cos\{(q_1 - q_2) + p/2\}$$
$$= -\sin(q_1 - q_2), \qquad (13)$$

$$P(\mathbf{a}, \mathbf{b}) = 2\sin^2(q_1 - q_2) - 1 = -\cos(2(q_1 - q_2)), \qquad (14)$$

which is the correct quantum mechanical correlation.

Two-particle interferometry

The cases of particles entangled in other sets of variables like momentum and position, and energy and time can be mapped onto the spin problem with two-valued outcomes and the local amplitudes reproduce the correct correlations.

Consider a pair of EPR-entangled particles which are propagating in opposite directions. Each particle encounters a double slit arrangement (Figure 2) on their way[12]. It is easy to see that there will not be any single particle interference pattern in this case on either screen. Near-perfect momentum correlation implies an extended source since a small source leads to uncontrollable uncertainties in momentum. Then the spatial coherence is not sufficient to produce the single particle interference pattern. But there is a two-particle interference pattern observable in the coincidence in two detectors, which could be in space-like separated regions. Usually the results in multiparticle interferometry are interpreted as evidence of bizarre nonlocality[12] since the pattern depends on the difference of the coordinates of both the detectors, just as the spin correlations depend on the difference in settings of the two analysers.

In the experiment, particles pass the slit planes and are detected with two counting detectors, one on each side. As in the case of spins, we assume the existence of an internal variable, denoted by x_0, and we assume that the two correlated particles have the same value for the internal variable. For the spin-$\frac{1}{2}$ case, the relative rotation required to go from a maximum of transmission through the analyser to a minimum is p. In the case of double-slit interference, the situation is similar. The relative rotation in phase from the center of the bright fringe to the centre of the dark fringe is p, for the angular variable defined by $q = akx$, where $k = 2p/l$, and x is the coordinate of the detector along the fringe pattern. a is a scale factor representing the distance of the slits from the source and the detectors. So, the double-slit interference problem can be mapped to the spin-$\frac{1}{2}$ singlet problem.

The corresponding local amplitudes are

$$C_1 = \frac{1}{\sqrt{2}}\exp(iak(x_1 - x_0)/2),$$

and

$$C_2 = \frac{1}{\sqrt{2}}\exp(iak(x_2 - x_0)/2), \qquad (15)$$

where x_1 and x_2 are the detector positions. The single particle detections on either side separately do not show any interference. The correlation function is

$$U(x_1, x_2) = \cos(ak(x_1 - x_2)/2). \qquad (16)$$

Probability for coincidence detection is

$$P(x_1, x_2) = \cos^2(ak(x_1, x_2)/2)$$
$$= \frac{1}{2}(1 + \cos ka(x_1 - x_2)). \qquad (17)$$

This is the two-photon interference pattern (it is more appropriate to call it a *two-photon correlation pattern*) with 100% visibility. The photon which is being detected at one detector has no nonlocal influence on the photon detected at the other detector.

Three-particle GHZ state correlations

So far we have been discussing examples in which the *statistical* predictions of quantum mechanics were compared with the prediction from a local theory. There have been examples where a single measurement of a perfect correlation, assuming perfect detectors, etc. is enough to demonstrate the conflict between quantum mechanics and a local realistic theory[13,14]. Experiments are yet to be done for these cases. We now show that the theory with local amplitudes and the spin internal variable deals with the correlations in such cases in a remarkably simple and transparent way.

Consider the three-particle GHZ state[13] defined as

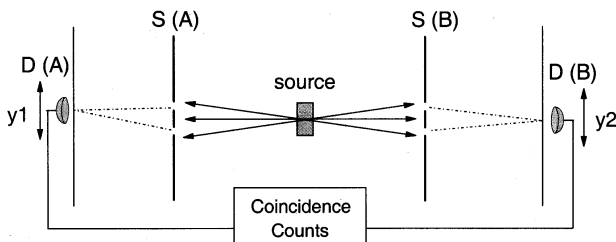$$|y_{GHZ}\rangle = \frac{1}{\sqrt{2}}(|1,1,1\rangle - |1,-1,-1\rangle), \qquad (18)$$

Figure 2. Two-particle dual double slit experiment. S(A) and S(B) are the double slits and D(A) and D(B) are the movable detectors. The outputs of the detectors are correlated as in Figure 1.

where the eigenvalues in the kets are with respect to the z-axis basis. The conflict between a local realistic theory and quantum mechanics is the following statement: The prediction from quantum mechanics for the measurement of the x-component of spin on all the three particles represented by the operator $s_x^1 \otimes s_x^2 \otimes s_x^3$ is given by

$$s_x^1 \otimes s_x^2 \otimes s_x^3 |y_{GHZ}\rangle = -|y_{GHZ}\rangle. \tag{19}$$

Local realistic theories predict[13] that the product of the outcomes in the x direction for the three particles should be +1. This contradicts eq. (19).

We now show that the correct correlations can be reproduced using local amplitudes. The general idea is that the three-particle correlation, analogous to our scheme for two-particle states, is the real part of a complex number Z obtained as a suitable product of three complex amplitudes. We choose the different phases such that the correlation represented by $N(Z-,-,-)$ is ± 1, to satisfy the condition that the joint probability for the outcome $(-, -, -)$ is unity according to eq. (19). N is a normalization factor. For this we choose $(Z-, -, -)$ to be pure real. The rest of the correlations follow without any additional input since flipping the sign once (for example $(Z+, -, -)$) amounts to rotating Z through the phase $p/2$, and the corresponding probability $P(+,-,-) = \text{Real}(Z+,-,-))^2 = 0$. This is because the amplitudes for + and – are orthogonal. Clearly the joint probabilities are unity for the outcomes containing an odd number of (–), and zero for all other outcomes. This result is independent of the definition (once a definition is chosen the phases can be chosen to get the desired outcomes) of the complex correlation Z, though for convenience the correlation can be defined as $N\text{Real}(C_1 C_2{}^* C_3{}^*)$.

Other applications

Hardy's puzzle

Another important case is the one discussed by Hardy[15] involving two non-maximally entangled particles and four observables, in which four separate correlations predicted by quantum mechanics cannot be reproduced in the local realistic hidden variable theory. The nonlocality demonstration uses three zero joint probabilities and one nonzero joint probability constructed from three of the measurement possibilities. The local amplitudes which give the correct joint probabilities for the Hardy problem have also been constructed.

K meson beams

The analysis based on local physics presented in the earlier sections can be applied to the $K^0 - \overline{K}^0$ beam for

which an EPR effect has been discussed[16]. The analysis using local amplitudes shows that the probability for both the particles in the correlated beam decaying into the same mode, say $p^0 p^0$, at equal proper time is zero without the nonlocal 'EPR effect'.

Entanglement through measurement

The general framework presented here is suited also for analysing entanglement correlations between particles which have not interacted in the past, but get apparently entangled due to joint measurements on their companion particles[17]. If one considers two ensembles of entangled particle pairs (1, 2) and (3, 4) such that (1, 2) are entangled and (3, 4) are entangled, but (2, 3) or (1, 4) are not entangled, it is possible to make entanglement correlations between (1, 4) for example, by making Bell-type joint measurements on the pair (2, 3). In a Bell-state measurement, particles 2 and 3 are made to interfere on a beam-splitter and then it is possible to pick out joint states of the type $|y_{2,3}\rangle_{\pm} = \frac{1}{\sqrt{2}}(|1, -1\rangle \pm |-1, 1\rangle)$. Though there is no prior fixed relation between the internal variables of (1, 4), the Bell measurement on (2, 3) chooses a sub-ensemble in which there is an observed correlation between particles 2 and 3 and hence between particles 1 and 4, due to fixed prior relationship in internal variables of particle pairs (1, 2) and (3, 4). There is no nonlocality. The pair (1, 4) picked out using the observation of $|y_{2,3}\rangle$ state, for example, as a filter will also show the same behaviour as the pair (2, 3) in the state $|y_{2,3}\rangle$. Only correlations at source and subsequent filtering into sub-ensembles through Bell-type measurements or other similar operations are needed. The various schemes are yet to be studied in detail.

Discussion

Obtaining the correct correlations assuming locality implies that the measurement on one particle does not collapse the other particle to a definite state. There is a distinction between a real measurement and 'predictability with certainty' of an outcome. Quantum correlation at source and a measurement on one of the particles is enough to make this prediction using the conservation laws. But till an actual measurement is made the companion particle does not acquire a definite state. There is direct proof for this from the fact that while an actual measurement of position on one of the particles disperses its momentum according to the uncertainty principle, this measurement and the resulting 100% predictability of the companion particle's position do not cause corresponding dispersion in the momentum of the companion particle. This is the lesson from Popper's experiment[18–20].

Apart from resolving the nonlocality problem, we have also found an answer to the original EPR paradox of simultaneous reality of noncommuting observables. The paradox arises only from the necessity to assume reality for the outcomes before a measurement is made, to reproduce the correct perfect correlation. Since we found a way of getting these correlations without such an assumption, and without nonlocality, there is no EPR paradox. *There are physical systems that are beyond the scope of the EPR definition of reality.*

The approach we have taken has locality as a basic feature and the objective reality (reality of a physical quantity independent of observer or apparatus) is at the level of the internal variables or initial phases. There is no objective reality to the outcomes before the measurement is made. One may explore the relation between the reality of the phase variable and the actual outcomes, and that would be a major step towards understanding quantum measurements. Such an understanding amounts to conceptual determinism in quantum mechanics, even though the initial phase is unmeasurable. It may turn out that observers in the classical world will not be able to grasp the mapping between such phases and actual outcomes. These issues are being contemplated on.

In summary, we have reproduced the correct correlations of entangled particles under the assumption of strict locality of amplitudes. The correlations agreeing with experiments and quantum mechanical predictions emerge without one measurement causing a collapse of the state of the companion particle. This resolves the EPR nonlocality puzzle. The results have significant implications to the interpretation of all experiments involving entangled particles, including quantum teleportation and entanglement swapping. This approach introduces a new point of view in the physical and philosophical understanding of the nature of reality in the microscopic world.

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