ENERGY ENIGMAS IN THE EARLY UNIVERSE AND RELATED PROBLEMS IN NEWTONIAN COSMOLOGY

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Abstract. Some paradoxical situations regarding the balance between the different forms of energy present and the velocities of expansion in the early stages of the big bang are discussed. Solutions are suggested in the framework of Newtonian gravitation.

The standard big bang model seems to have found general acceptance as a viable theory for the evolution of the Universe. The all pervading presence of the isotropic microwave black-body radiation is taken as strong evidence for a dense hot early phase of the Universe. Further support comes from the observed helium abundance as the big bang predicts about 0.25 He as arising naturally from the thermonuclear reactions occurring between neutrons and protons at a few seconds after the start of the expansion and over about hundred seconds later. Little input is involved (about the only parameters are the neutron-proton mass difference, the neutron life time and the proton to baryon ratio) and no alternative model explains both of the above observations so naturally. However, it must be pointed out that recently some possible incompatibilities between the measured values of the abundances of D, He^4, Li^7, etc., and the theoretical predictions of standard big bang nucleosynthesis have been hinted at (Vidal-Madjar and Gry, 1983). Again recent data (Butcher, 1987) on the Th-232/Nd relative abundance implies inconsistency between stellar age and the Hubble time-scale. Despite the above notable successes there are several severe theoretical problems with the standard big bang especially at the earliest epochs. For instance we have the so-called 'Flatness' and 'Horizon' problems which have been explained to some extent by the inflationary models (Sivaram, 1986a, b, c, 1987). Now the usual Friedmann equation for the expanding scale factor, i.e., $(\dot{R}/R)^2 = 8\pi G\rho/3$ is well known to be obtainable from Newtonian gravity if the kinetic and potential energy terms are balanced, i.e., $\dot{R}^2 = 2GM/R = (8\pi G\rho/3)R^2$ (M being the total mass and ρ the average density of the Universe). $\dot{R}$ would be the velocity of expansion. An integration constant can also be added, i.e., kc^2. We would not consider the k term for the present, i.e., $k = 0$ is assumed. We would discuss the $k \neq 0$ case later. One usual way of stating the flatness problem is to say that the kinetic energy term $(\dot{R}/R)^2$ and the potential energy term $8\pi G\rho/3$ in the above equation must have been equal and opposite to about one part in 10^{18} at the time $t \simeq 1$ s, when helium was being formed, which implies a density very close to the critical closure density and a total energy (kinetic plus potential) of zero. This is the common interpretation. We can write for the expansion velocity

$$\dot{R} = (8\pi G\rho/3)^{1/2} R . \tag{1}$$

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We will consider some specific epochs, i.e., specific values of time t. For, e.g., take $t \simeq 10^{-24}$ s. The corresponding temperature (obtained from $\rho = aT^4$, a = const., in the radiation dominant era) is $T \approx 10^{10}/t^{1/2} \simeq 10^{22}$ K, giving a corresponding $\rho \simeq 10^{74}$ erg cm^{-3} (10^{53} g cm^{-3}). As baryon number would still be conserved, each of the $\sim 10^{78}$ baryons would have an energy $\sim 10^{10}$ times the rest energy, at this temperature. So at the above density this would give a size of $R \simeq 10^3$ cm for the Universe at this epoch. However, Equation (1) shows that for the above ρ, $\dot{R}$ would approach the light velocity c, even for R as small as $R_c \approx 10^{-13}$ cm. This is essentially the horizon size at this epoch. The equality between kinetic and potential energies would apply to only regions of this size; i.e., $R \approx 10^{-13}$ cm. As the gravitational potential increases as R^2 (for given ρ) this would imply that for the Universe of size $R \approx 10^3$ cm at this epoch, the potential energy in some 10^{32} times the total kinetic or thermal energy of the constituents, this being, as seen above, 10^{10} times the rest energy of the baryons. This can also been seen as follows: At the above temperature the total thermal energy of the 10^{78} baryons is $\approx 10^{85}$ ergs. The gravitational potential energy (Newtonian) is $\sim GM^2/R \simeq 10^{117}$ ergs ($R \approx 10^3$ cm being the size of the Universe at this time and total energy $M \approx 10^{64}$ g, all the above thermal energy contributing to gravity). Thus the gravitational energy is $\approx 10^{32}$ times the total thermal energy at a temperature of $\simeq 10^{22}$ deg. So at this epoch the Universe consists of $\sim 10^{48}$ regions, all expanding at exactly the same rate and having the same density! There is equality between kinetic and potential energies over each of these smaller regions but for the Universe as a whole the potential energy is $\sim 10^{32}$ times the total thermal energy at this epoch. As another illustration take $t \approx 10^{-4}$ s. This would correspond to a density of $\simeq 10^{17}$ g cm^{-3}. The thermal energies of the baryons would be comparable with their rest energies at this epoch which would imply a size of $R \simeq 10^{13}$ cm, for the Universe at this epoch. The expansion velocity would approach c for regions of size $R_c \simeq 10^5$ cm (the horizon size). The gravitational potential energy for the Universe as a whole at this epoch would be $\approx 10^{16}$ times the total thermal energy, the Universe consisting of $\sim 10^{24}$ regions over which kinetic and potential energy terms are equal, i.e., 'flat' regions.

Now it would be noted that in all these usual discussions of equality between kinetic and potential energies following from Equation (1) the non-relativistic formula (i.e., $\frac{1}{2}\dot{R}^2$) for kinetic energy was used. As seen from the above examples, for the early epochs, the expansion velocity approaches light velocity over infinitesimally small regions and moreover at the corresponding temperatures the baryons are ultra relativistic. So one would guess that in this Newtonian framework one should use for consistency the special relativistic formula for kinetic energy in Equation (1). Thus the corrected relativistic equation is

$$\frac{c^2}{\sqrt{1 - \dot{R}^2/c^2}} = \frac{4}{3}\pi G\rho R^2 . \tag{2}$$

This would imply a γ-factor (Lorentz factor) of expansion

$$\gamma = 1/\sqrt{1 - \dot{R}^2/c^2} = (4\pi G\rho R^2/3c^2) . \tag{3}$$

Now take the epoch, $t \approx 10^{-24}$ s when $\rho \simeq 10^{53}$ g cm^{-3} and $R \approx 10^3$ cm. This implies a γ of $\approx 10^{32}$, which means the K.E. is enhanced by a factor of 10^{32}. We saw earlier that the use of the non-relativistic formula implied a gravitational energy 10^{32} times larger than the kinetic energy for the Universe as a whole at this epoch. So with this γ-factor of 10^{32} in the relativistic formula, the relativistic kinetic and potential energy terms become the same for the Universe as a whole. Similarly for the epoch at $t \simeq 10^{-4}$ s the γ-factor turns out to be $\approx 10^{16}$ which is just the discrepant factor needed as we have seen above, the potential term being 10^{16} times larger when the non-relativistic formula was used. We are now in a position to understand the flatness problem, i.e., why k is apparently zero. If we now retain the integration constant kc^2 ($k = \pm 1$) in Equation (2), which we had earlier dropped, then we can write

$$\gamma c^2 = 4\pi G \rho R^2/3 \pm kc^2 . \tag{4}$$

We saw from the above examples that at the early epochs, $\gamma \gg 1$, ($\gamma \simeq 10^{32}$ and 10^{16}, respectively) and the γc^2 (relativistic K.E.) and gravitational term (i.e., the first term on the right-hand side in Equation (4)) are comparable. Thus the kc^2 term is very much smaller than the other two terms in Equation (4), as a consequence of ultra-relativistic expansion; i.e., the Universe was flat to begin with. If we had used the wrong non-relativistic formula (with $\gamma \ll 1$) this would not be apparent (as in Equation (1)). Again from Equation (2), we obtain the relation for $\dot{R}$ as

$$\frac{\dot{R}}{R} = \frac{C}{R}\left[1 - \left(\frac{3c^2}{4\pi G \rho R^2}\right)^2\right]^{1/2} . \tag{5}$$

For the very early epochs, the term $3c^2/4\pi G\rho R^2 \ll 1$, and $\dot{R} \simeq C$, giving the expansion with time of the Universe as $R_U \approx ct$. Now the horizon problem arises as is known because the Universe at the earliest epochs was assumed to expand as $R_U \propto t^{1/2}$ so that $R(\text{hor})/R_U \propto t^{1/2} \to 0$ as $t \to 0$, i.e., we have smaller and smaller non-communicating regions. With $R_U \simeq ct$, the horizon problem is no longer present. Thus it may turn out that (at least in the Newtonian cosmoogy framework) the solution to the horizon and flatness problems may lie in using the correct relativistic equation of motion for the kinetic energy of expansion in the early Universe! In usual discussions of the flatness problem, the equality between the non-relativistic kinetic energy ($\ll$ rest energy) and the gravitational potential energy is stated as in Equation (1), which does not consider the rest mass energy. Equations (2), (3) and (4) would settle this dilemma. In this context it is to be noted that in the usual discussion of Mach's principle in Newtonian cosmology the equality between total rest mass energy (rather than kinetic energy) and gravitational energy is considered to arrive at a cosmological interpretation of inertia (e.g., Callebaut *et al.*, 1984). Use of the relativistic kinetic energy as in Equations (2)–(5), would combine both these issues besides providing possible solution of the flatness problem, the 'flatness' (vanishing curvature k-term) as seen above being a consequence of ultra-relativistic K.E.) (i.e., $\gamma c^2 \gg kc^2$) at the early epochs.

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