1989Ap&SS.158..347D

THE NEED FOR AN ENERGY-DEPENDENT TORSION-COUPLING CONSTANT IN THE EARLY UNIVERSE

(Letter to the Editor)

VENZO DE SABBATA[1] and C. SIVARAM[2]
World Laboratory, Genève, Switzerland

(Received 6 March, 1989)

Abstract. We present arguments to show that torsion-coupling constant (which depends on energy as E^{-2}) can pass through the values of the coupling constants of the other interactions during the evolution of the Universe. An energy-dependent torsion-coupling constant helps in a natural way to understand the ratios of the coupling strengths of the different fundamental interactions.

It has been emphasized by many authors (Hehl and Datta, 1971; Sivaram and Sinha, 1975; Kaempffer, 1979; de Sabbata and Gasperini, 1979, 1980) that the general structure of metric-torsion theories allows parity violating interactions, i.e., the torsion of space-time might be responsible for parity violation in weak interactions. It has been recognized that the torsionic contact interaction between two Dirac particles has a formal analogy with weak interaction Lagrangian with the usual Einstein–Cartan static term for the field and can be written as

$$\mathscr{L} = iV_\mu(\Psi)J^\mu(\psi') - A_\mu(\psi)J^{5\mu}(\psi') = $$
$$= (-3/16)\chi[J_\mu(\psi)J^\mu(\psi') + J^5_\mu(\psi)J^{5\mu}(\psi')], \tag{1}$$

$$J^\mu = \bar{\psi}\gamma^\mu\psi, \qquad J^{5\mu} = \bar{\psi}\gamma^5\gamma^\mu\psi, \qquad V_\mu = Q^\alpha_{\mu\alpha},$$
$$A_\mu = (1/4)\varepsilon_{\mu\alpha\beta\gamma}Q^{\alpha\beta\gamma} \quad (Q_{\alpha\beta\gamma} \text{ being the torsion tensor}),$$
$$V_\mu = i(3/16)\chi J_\mu, \qquad A_\mu = (3/16)\chi J^5_{\mu'} . \tag{2}$$

This may be written in the standard $(V - A)$ form if at least one of the two fermions is massless (i.e., described by a two-component spinor $\gamma^\mu_{,\mu}\psi' = 0$ and $(1 - \gamma^5)\psi' = 2\psi'$), we have

$$\mathscr{L} = -(3/32)\chi\bar{\psi}'\gamma_\mu(1 - \gamma^5)\psi'(J^\mu + J^{5\mu}) = $$
$$= -(3/32)\chi\bar{\psi}'\gamma_\mu(1 - \gamma^5)\psi'\bar{\psi}\gamma^\mu(1 - \gamma^5)\psi . \tag{3}$$

Thus we have a torsionic interaction Lagrangian which is formally identical to the weak interaction four-fermion $(V - A)$ Lagrangian except for the value of the coupling

[1] Dept. of Physics, University of Ferrara, Italy, and Istituto di Fisica Nucleare, Sezione di Bologna, Italy.
[2] Indian Institute of Astrophysics, Bangalore, India.

Astrophysics and Space Science **158**: 347–352, 1989.
© 1989 *Kluwer Academic Publishers. Printed in Belgium.*

© Kluwer Academic Publishers • Provided by the NASA Astrophysics Data System

1989Ap&SS.158..347D

constant. This raises the possibility that torsion may provide a geometrical model for weak interactions just like curvature does for gravitational forces. However, the coupling constant in the above Lagrangian (3) is

$$3\chi/32 = 32\pi K_g \hbar^2/4c^2 \approx 10^{-81}\ \text{erg cm}^3\,, \tag{4}$$

whereas the Fermi weak coupling constant for the four-fermion or $(V - A)$ theory is

$$G_\text{F}/\sqrt{2} \approx 10^{-50}\ \text{erg cm}^3\,. \tag{5}$$

Therefore, in order to have a complete identification of torsionic and weak interactions we must postulate a spin-torsion coupling constant which is different from the mass-curvature coupling in Einstein's equations by a factor (Sivaram and Sinha, 1974, 1975; Kaempffer, 1976; de Sabbata and Gasperini, 1978) of

$$\chi'/\chi = \sqrt{8/9}\,(G_\text{F}c^2/\pi K_g \hbar^2) \approx 10^{31}\,. \tag{6}$$

One has the possibility of giving arbitrary value to the torsionic coupling constant. There is no compelling *a priori* reason why it should have the same value as the gravitational constant.

In fact one can give some arguments to justify identifying the torsion coupling G_T to the Fermi G_F. For instance, the similarity to the four-fermionic interaction (i.e., of the torsion interaction of two Dirac particles): i.e., with the action

$$\int \mathrm{d}^4x (G_T/\sqrt{2})\bar{\psi}_1 \psi_2 \bar{\psi}_3 \psi_4 \tag{7}$$

implying that G_T has the dimensionality of mass to a *negative* power. More precisely: the action is to be dimensionless; ψ being a spinor has dimensions $(\text{mass})^{3/2}$ so that $(\psi)^4$ has dimensions $(\text{mass})^6$, d^4x has dimensions of $(\text{mass})^{-4}$, so that G_T must have dimension of $(\text{mass})^{-2}$.

On the contrary, in the Hilbert Lagrangian for gravity

$$\int \mathrm{d}^4x (1/16\pi G)\sqrt{-g}\,R\,, \tag{8}$$

the coupling $(1/16\pi G)$ has the dimension of $(\text{mass})^2$; since $\sqrt{-g}\,\mathrm{d}^4x$ has dimensions $(\text{mass})^{-4}$, R the curvature scalar has dimensions $(\text{mass})^2$, then for the action to be dimensionless $(1/16\pi G)$ must have dimensions of $(\text{mass})^2$. This means that the dominant contributions to $1/16\pi G$ would arise from the higher mass states, the highest masses (i.e., mediating or vitual particles of highest mass) contributing the most. Thus for the Newtonian gravity coupling, the Planck mass states would chiefly determine the value $1/16\pi G_\text{N}$.

G_T on the other hand in Equation (7) having dimension of $(\text{mass})^{-2}$, would receive dominant contributions from the lower mass states, i.e., from the intermediate weak bosons (W, etc.). This would fix its value at the Fermi constant G_F, for intermediate boson masses ≈ 100 GeV. The whole hierarchy of heavier mass bosons up to Planck

© Kluwer Academic Publishers • Provided by the NASA Astrophysics Data System

1989Ap&SS.158..347D

mass would not significantly contribute to G_T as their effect in determining its value would drop as $(\text{mass})^{-2}$.

Therefore, if torsionic coupling constant receives its contributions from the intermediate weak bosons it could have the value $\approx G_F$. Another justification for identifying G_T with G_F, may arise in accounting for the rather intriguing coincidence (Sivaram, 1982)

$$e^2/2m_p c^2 = (G_F/\hbar c)^{1/2} \approx 6 \times 10^{-17}\ \text{cm}\,. \tag{9}$$

This suggests some link between the electromagnetic interaction (electric charge e) and the weak interaction through the proton mass m_p. We can arrive at this kind of relation by considering a model for the proton wherein it may be held together by the spin-torsion force between its constituents acting against the electromagnetic forces. At the proton electromagnetic radius $(e^2/2m_p c^2)$, the spin-torsion interaction energy (from the Einstein–Cartan theory) is (see de Sabbata and Gasperini, 1982; de Sabbata and Sivaram, 1989) given by

$$(4\pi G_F/3c^2)\,(\hbar \cdot \hbar/2)/(4\pi/3)\,(e^2/2m_p c^2)^3 = E_Q\,. \tag{10}$$

This is to be balanced by the electromagnetic self-energy $m_p c^2/2\alpha$ (α is the fine structure constant: i.e., from the uncertainty principle at the distance $e^2/2m_p c^2 = (1/2\alpha)\,(\hbar/m_p c)$, we have the energy $(m_p c^2/2\alpha)$. Equation E_Q in Equation (10) to $m_p c^2/2\alpha$, we then obtain the relation given by Equation (9). However, earlier (de Sabbata and Gasperini, 1981; de Sabbata and Sivaram, 1989; Sivaram and Sinha, 1979) we have considered torsion in connection with strong gravity, with a coupling constant $G_f = 10^{38}\,G_N$ – i.e., of the strength of the strong interactions. Many properties of the strong interactions could be explained by invoking strong gravity with torsion (Sivaram and Sinha, 1979). How do we then understand the torsion coupling constant being used in the description of both weak and strong interactions?

The answer lies again in the fact that, as suggested by Equation (7), the torsion coupling constant gets its contributions from the lowest mass states. We know that the spin-2f-mesons mediate strong gravity. Their mass is $m_f \approx 1$ GeV, i.e., about a hundred times smaller than the intermediate weak bosons. Thus as the coupling G_T scales as $(\text{mass})^{-2}$, we have the corresponding coupling constant for strong torsionic gravity as $\approx 10^4$ times $G_F \approx$ strong interaction coupling constant.

This suggests that the torsionic coupling constant should be energy dependent and would scale with the corresponding mass M of the mediating bosons as

$$G_T \sim 1/M^2\,. \tag{11}$$

Thus $G_T \equiv G_F$, the Fermi constant, if $M \approx M_w$, the intermediate boson mass. For $M \approx M_{\text{Planck}} \approx 10^{19}$ GeV, i.e., if the energy increases to such high values that particles of mass $\approx M_{\text{Planck}}$ mediate the interaction, then

$$G_T \equiv G_{\text{Newt.}} = G_F(M_w/M_{\text{Planck}})^2\,; \tag{12}$$

thus explaining the large difference in weak and gravitational couplings. For

© Kluwer Academic Publishers • Provided by the NASA Astrophysics Data System

1989Ap&SS.158..347D

$M_f \approx 1$ GeV, the f-meson mass, the coupling becomes large, i.e., $G_T \approx G_f$, which is the strong gravity constant. Thus

$$(G_f/G_{\mathrm{F}}) \approx (M_w/M_f)^2 \quad \text{or} \quad (G_f/G_{\mathrm{Newt}}) = (M_{\mathrm{Planck}}/M_f)^2 \approx 10^{38} . \tag{13}$$

Equations (12) and (13) enable us to understand the ratios of the coupling strengths of strong, weak, and gravitational interactions.

Now, Grand Unified Theories (GUT) predict nucleon decay to occur through effective four-fermion interactions $\sim G_{\mathrm{GUT}}(\psi)^4$, mediated by exchanges of super-heavy gauge bosons, mass $M_x \approx 10^{15}$ GeV. Therefore, the corresponding coupling constant G_{GUT} by analogy with the expression for Fermi weak interaction $(g_w/8m_w^2 \approx G_{\mathrm{F}}/\sqrt{2})$ can be written as:

$$G_{\mathrm{GUT}}/\sqrt{2} = g_w^2/8M_x^2 . \tag{14}$$

The nucleus decay rate by this four-fermion interaction is

$$\Gamma \sim 1/t_N \sim G_{\mathrm{GUT}}^2 m_N^5 \sim m_N^5/M_x^4 , \tag{15}$$

where m_N is the nucleon mass. For

$$G_{\mathrm{GUT}} \approx G_{\mathrm{F}}(M_w/M_x)^2 \approx 2 \times 10^{-75} \text{ erg cm}^3 ; \tag{16}$$

this gives

$$t_N \approx 10^{32} \text{ yr} .$$

At GUT energies ($\approx M_x$), quantum gravity effects would not much alter proton decay rates, as contributions of bosons of $M > M_x$ to G_{GUT}, scale as $1/M^2$. So that $M \approx M_{\mathrm{Planck}}$ mass bosons would change G_{GUT} coupling constant by one part in 10^8 only! This suggests that the same torsionic coupling constant at $M_x \approx 10^{15}$ *GeV*, i.e., $G_T \approx G_{\mathrm{GUT}}$ can be responsible for nucleon decay.

It has been suggested that all forces are due to a spin-curvature coupling (Harris, 1980). That is, in curved space with torsion there is a coupling of the spin-tensor $S_{\mu\nu}$ and the curvature tensor $R_{\mu\nu\alpha\beta}$ to give a force $(1/2)R^{\mu\nu}{}_{\alpha\beta}S_{\mu\nu}\dot{x}^{\beta}$. It has been shown (Harris, 1980) that all the forces that are derivable from gauge theories are of this form: i.e., they are due to a coupling of spin and curvature of the above form when the concepts of spin and curvature are suitable generalized. The question then as to whether the coupling constant of the spin-curvature interaction can describe the strengths of the basic interactions has been answered if one considers an energy-dependent torsion coupling constant. We have seen that Equations (12)–(16) above do suggest that the coupling strengths of all the fundamental interactions can be accomodated in the energy-dependent torsion coupling constant G_T. Now it was suggested earlier (de Sabbata and Gasperini, 1982; Sivaram and Sinha, 1979; de Sabbata and Rizzati, 1979) that in the early universe, during the hadron era (in connection with the Dirac two-metric theory) the gravity constant was appropriately the strong gravity one (i.e., G_f). So the above relations would suggest the evolution of the torsionic coupling constant as follows: during the Planck epoch it was identical with the Newtonian

© Kluwer Academic Publishers • Provided by the NASA Astrophysics Data System

1989Ap&SS.158..347D

constant G_{Newt}. During the GUT's epoch it had the value of the GUT's coupling (G_{GUT} which was the same for strong, weak, and electromagnetic), during the electroweak era, it was G_F (Fermi constant) and during the hadron era it was G_f. It then reduced by expansion of the Universe in accordance with Dirac relation $G \sim t^{-1}$ to its present value of G_{Newt} (see Figure 1). If one does not like to believe in variation of G, then after the hadron era, there was a phase transition from G_f to Newtonian value G_{Newt} (see Figure 2).

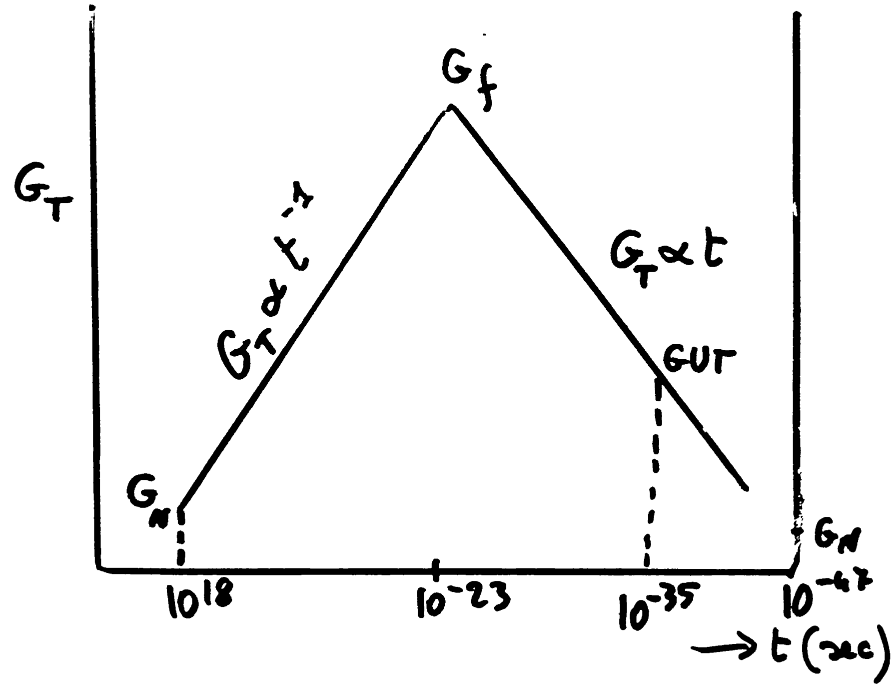

Fig. 1. Variation of torsionic coupling constant with time in the case that after hadronic era the gravitational constant follows Dirac hypothesis ($G \sim t^{-1}$).

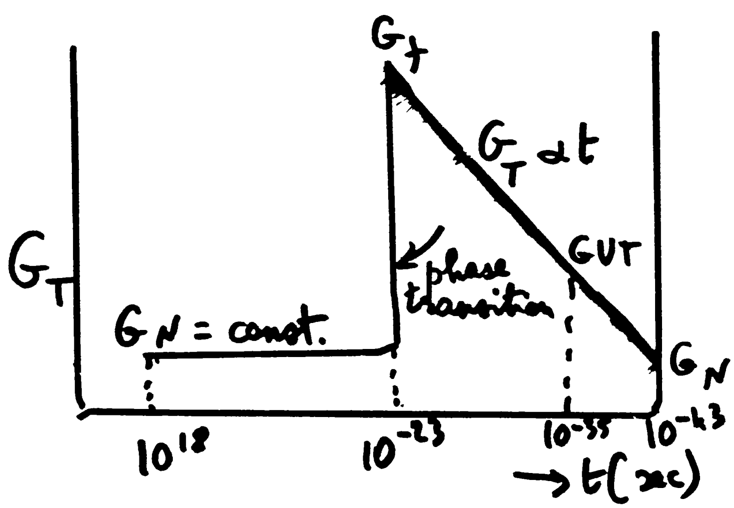

Fig. 2. Variation of torsionic coupling constant with time in the case that after hadronic era a phase transition from G_f to G_N has occurred.

© Kluwer Academic Publishers • Provided by the NASA Astrophysics Data System

1989Ap&SS.158..347D

The energy-dependent behaviour of the torsion coupling during the evolution of the Universe can be diagramatically represented as follows with respect to time t: before hadron era $G_T \sim T^{-2} \sim t$ in R–W universe temperature T and time are (cf. Sivaram and Sinha, 1975) related by $T \sim t^{-1}$. The maximum value of G_T would be reached for the lowest mass of the mediating meson, i.e., for m_π the pion mass. Once the Universe expands cooling further, energy is too low for producing any other particles and G_T drops off as t^{-1} to its present value of G_{Newt}.

References

de Sabbata, V. and Gasperini, M.: 1978, *Nuovo Cimento* **21**, 328.

de Sabbata, V. and Gasperini, M.: 1979, *Gen. Rel. Grav.* **10**, 731.

de Sabbata, V. and Gasperini, M.: 1980, in Peter G. Bergmann and Venzo de Sabbata (eds.), *Spin, Torsion, Rotation and Supergravity*, Plenum Press, New York, p. 139.

de Sabbata, V. and Gasperini, M.: 1982, in Venzo de Sabbata (ed.), *Origin and Evolution of Galaxies*, World Science, Singapore, p. 181.

de Sabbata, V. and Rizzati, P.: 1979, *Nuovo Cimento Letters* **20**, 525.

de Sabbata, V. and Sivaram, C.: 1989, *Nuovo Cimento* **101A**, 273.

Harris, E. G.: 1980, *Nuovo Cimento* **55B**, 205.

Hehl, F. W. and Datta, H. K.: 1971, *J. Math. Phys.* **12**, 1334.

Kaempffer, F.: 1976, *Gen. Rel. Grav.* **7**, 327.

Sivaram, C.: 1982, *Astrophys. Space Sci.* **85**, 507.

Sivaram, C.: 1985, *Bull. Astron. Soc. Ind.* **13**, 339.

Sivaram, C. and Sinha, K.: 1974, *Curr. Sci.* **43**, 165.

Sivaram, C. and Sinha, K.: 1975, *Nuovo Cimento Letters* **13**, 357.

Sivaram, C. and Sinha, K.: 1979, *Phys. Reports* **51**, 111.

© Kluwer Academic Publishers • Provided by the NASA Astrophysics Data System