STRING TENSION AND FUNDAMENTAL CONSTANTS IN THE EARLY UNIVERSE

(Letter to the Editor)

C. SIVARAM

Indian Institute of Astrophysics, Bangalore, India

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Abstract. An energy-dependent string tension could be connected to the fundamental physical and coupling constants. The role of Weyl gravity for sub-Planckian as well as macroscopic domains is explored and the existence of a hierarchy of scales is considered.

In several recent papers (Sivaram, 1986a, b, c, 1987a, b) a unification of the parameters underlying elementary particles and cosmology was outlined. Again in a recent paper (de Sabbata and Sivaram, 1989) it was shown that the torsion-coupling constant G_T depending on energy as E^{-2} can pass through the values of the coupling constants of the fundamental interactions during the evolution of the Universe, enabling the understanding of the ratios of the different coupling strengths. Thus we had the relations

$$G_{\text{Newt}} = G_{\text{F}}(M_W/M_{\text{Planck}})^2\,, \tag{1}$$

where G_{Newt} and G_{F} are the Newtonian gravitational and Fermi weak interaction constants, respectively, M_W and M_{Planck} being the intermediate weak boson and Planck masses. The large difference in weak and gravitational couplings are explained by the ratio $(M_W/M_{\text{Planck}})^2$, G being inversely proportional to $1/M^2$. Again as pointed out in de Sabbata and Sivaram (1989), for $M_f \approx 1$ GeV, the f-meson mass, the coupling becomes large, i.e., $G \approx G_f$ which is the strong gravity constant characterising the strength of strong interactions.

Thus

$$(G_f/G_{\text{F}}) \approx (M_W/M_f)^2 \quad \text{or} \quad \left(\frac{G_f}{G_{\text{Newt}}}\right) = \left(\frac{M_{\text{Planck}}}{M_f}\right)^2 \simeq 10^{38}\,. \tag{2}$$

Equations (1) and (2) enable us to understand the ratios of the coupling strengths of strong, weak, and gravitational interactions. Again at the grand unification scale, where the interactions are mediated by exchanges of super-heavy gauge bosons of mass M_x, the corresponding coupling constant is

$$G_{\text{GUT}} \simeq G_{\text{F}}(M_W/M_x)^2 \simeq 2 \times 10^{-75}\ \text{erg cm}^3\,;$$

whereas at the Planck scale $G = G_{\text{Newt}}$.

Now in string theories, the coupling is characterized by the so-called string tension

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T, which is the energy or mass per unit length. At the Planck scale, T is given by

$$T_{\text{Planck}} = c^2/G = 1.6 \times 10^{28}\ \text{g cm}^{-1}\,. \tag{3}$$

Thus T has units of G^{-1} ($c = 1$), is $T \sim G^{-1}$. For any other energy scale, M, T is given as

$$T = T_{\text{Planck}}(M/M_{\text{Planck}})^2\,. \tag{4}$$

(Equations (1) and (2) showed that $G \sim 1/M^2$, so that $T \sim G^{-1} \sim M^2$, T scaling as the mass squared). So the maximum value of T would be at M_{Planck}, i.e., $T_{\text{Planck}} = c^2/G = T_{\text{max}}$. The string length at this scale is the Planck length $L_{\text{Planck}} = (\hbar G/c^3)^{1/2}$, so that the string mass is $T_{\text{Planck}}L_{\text{Planck}}$ which is M_{Planck}.

If the string is now stretched beyond L_{Planck}, i.e., to $L \gg L_{\text{Planck}}$, then to conserve energy, T would be expected to fall as L^{-1}, otherwise M would increase as L increases. However, the quantum uncertainty principle would imply an M inversely proportional to L (as mass M cannot be localized to below h/Mc). If T falls as L^{-1}, M would be constant but if M is to scale as L^{-1} as required by the uncertainty principle, T should scale as $1/L^2$ or be proportional to M^2. As $T \sim G^{-1}$, this would imply that G scales as $1/M^2$ or as L^2, consistent with Equations (2) and (1)!

So GM^2 is an invariant and equals to $\hbar c$, i.e., $GM^2 = \hbar c$. So we have $G_{\text{Newt}}M^2_{\text{Planck}} = \hbar c$, $G_{\text{F}}M^2_W = \hbar c$, $G_f M^2_f = \hbar c$, $G_{\text{GUT}}M^2_{\text{GUT}} = \hbar c$, etc., or in units of $h = c = 1$, $GM^2 \approx 1$. So a combination of energy conservation and the uncertainty principle implies that if the string is stretched to $R \gg L_{\text{Planck}}$ then for all length scales larger than L_{Planck} (or all energy scales smaller than M_{Planck} or E_{Planck}) we have the scaling relations:

$$G \sim 1/M^2 \sim R^2\,, \tag{5a}$$

$$GM \sim R\,, \tag{5b}$$

$$M \sim R^{-1} \quad \text{(uncertainty principle)}\,, \tag{5c}$$

$$T \sim M^2 \sim G^{-1} \sim 1/R^2\,, \tag{5d}$$

$$GM^2 = \text{const.} = \hbar c = \text{universal 'charge' squared}\,. \tag{5e}$$

Equation (5e) implies that the 'underlying charge' squared irrespective of changes of length or energy scales is a universal constant which is just $\hbar c$. So although, at lengths as given by (5a)–(5d), the product $G_s M^2$ scale, where G_s is the value of G at any scale M_{scale} is still $\hbar c$.

Thus

$$G_{\text{Newt}}M^2_{\text{Planck}} = G_{\text{F}}M^2_W = G_f M^2_f = \text{G}_{\text{GUT}}M^2_{\text{GUT}} = \hbar c\,. \tag{6}$$

As the smallest mass scale (i.e., longest range) involved in strong interactions is the pion mass, M_π, $G_\pi M^2_\pi = \hbar c$, and G has its largest value, i.e., the pion–nucleon coupling strength is ≈ 10.

So, between L_{Planck} and $\hbar/M_\pi c \approx 1.4 \times 10^{-13}$ cm, Equations (5a)–(5e) hold and moreover $Gm \sim R$. Thus, in all the above relations (as summarized in Equation (6)) we have the coupling constants of various interactions represented by an 'equivalent gravitational constant'. In the case of strong interactions this was referred to as the strong gravity constant (de Sabbata and Sivaram, 1989). Here we have the whole hierarchy of interactions below M_{Planck} (above L_{Planck}) represented by the corresponding 'gravitational constant' G scaling with energy as E^{-2} or with scale length as R^2, however, GM^2 is a universal constant which is just $\hbar c$!

What about scale lengths below L_{Planck}? It is often thought that L_{Planck} is the smallest length one can construct from the fundamental constants. This is not true. There re a whole hierarchy of length scales smaller than the Planck length L_{Planck} which one could construct. For instance we have

$$\sqrt{G}\, e/c^2 = \sqrt{\alpha}\, L_{\text{Planck}} \simeq 10^{-34}\ \text{cm}\ (\alpha \approx \tfrac{1}{137}), \quad G = G_{\text{Newt}} \tag{7a}$$

(the corresponding mass is $e/\sqrt{G} = \sqrt{\alpha}\, M_{\text{Planck}}$).

Again if one has a 'weak charge' g_w (related to G_{F} as $g_w^2 = G_{\text{F}} m_p^2$) we have

$$\sqrt{G}\, g_w/c^2 = 3 \times 10^{-3}\, L_{\text{Planck}} \simeq 5 \times 10^{-36}\ \text{cm}, \tag{7b}$$

(the corresponding mass is $g_w/\sqrt{G} \simeq 3 \times 10^{-3}\, M_{\text{Planck}}$).

Again we can have

$$g_{we} \simeq \sqrt{G_{\text{F}}}\, m_e \simeq 10^{-3}\, g_w, \tag{7c}$$

so that one has another length

$$\sqrt{G}\, g_{we}/c^2 \simeq 5 \times 10^{-39}\ \text{cm}. \tag{7d}$$

One can go all the way down to the length corresponding to the gravitational charge $G_{\text{Newt}} M_p^2$ which would give

$$\sqrt{G}\sqrt{G}\, m_p/c^2 = G m_p/c^2 \simeq 2 \times 10^{-52}\ \text{cm}. \tag{7e}$$

(the corresponding mass is the proton mass m_p!). This is just the gravitational radius of the proton which is $10^{-19}\, L_{\text{Planck}}$!

Again for the electron one has

$$G m_e/c^2 \simeq 2 \times 10^{-55}\ \text{cm} \simeq 10^{-22}\, L_{\text{Planck}}. \tag{7f}$$

Equations (7a)–(7f) would imply that for all length scales R less than L_{Planck}, M scales as R, i.e., $M \sim R$. For instance, the ratio of $G m_p/c^2$ to L_{Planck} is the same as the ratio of m_p to M_{Planck}, etc. This is consistent in the string picture with the string tension T remaining constant at lengths below L_{Planck} so that the mass at all length scales $R \leq L_{\text{Planck}}$ is just proportional to the length. Thus, analogous to Equations (5), we have for all lengths $R < L_{\text{Planck}}$ (also for $R \ll L_{\text{Planck}}$) the scaling relations

$$G = \text{const.} = G_{\text{Newt}}, \tag{8a}$$

$$T = c^2/G = \text{const.} = T_{\text{Planck}}, \tag{8b}$$

$$M \sim R \,, \tag{8c}$$

$$GM \sim R \,, \tag{8d}$$

$$GM^2 \sim R^2 \,. \tag{8e}$$

A comparison of the scaling relation equations (5) for all $R > L_{\text{Planck}}$ and Equations (8) for $R < L_{\text{Planck}}$ would indicate that in both cases $GM \sim R$ (in fact $GM = Rc^2$ for the whole range).

$GM \sim R$ holds both above and below the Planck scale. Equations (8a) and (8e) imply asymptotic freedom for all lengths below L_{Planck}. Equation (8e) implies that the charge squared (GM^2 which was universally $\hbar c$ for all $R \gg L_{\text{Planck}}$ as implied by Equation (5e)) now scales as R^2 reaching a maximum of $\hbar c$ for $R = L_{\text{Planck}}$ and falling of as $1/R^2$ for all $L < L_{\text{Planck}}$. Thus, at the gravitational radius of the proton, $R_g \simeq 10^{-19} L_{\text{Planck}}$, $GM^2 = GM_p^2 = 10^{-38}\,\hbar c$, i.e., $GM_p^2/\hbar c \simeq (R_g/L_{\text{Planck}})^2$. It is GM_e^2 for $R_{ge} \simeq 10^{-22} L_{\text{Planck}}$ (Equation (7f), i.e., $GM_e^2/\hbar c \simeq (R_{ge}/L_{\text{Planck}})^2 \simeq 10^{-44}$, etc. Above L_{Planck} for all lengths between L_{Planck} and $\hbar/M_\pi c \simeq 10^{-13}$ cm, as noted above, $GM^{\,2} = \hbar c = \text{const.}$, $G \sim R^2 \sim 1/M^2$, $GM \sim R$, etc. Also we saw that $M \sim R^{-1}$ above L_{Planck} was consistent with the uncertainty principle (in turn implying $T \sim M^2$).

What happens to the uncertainty principle at lengths below L_{Planck}? In fact this is a problem even for string models as we have a contradiction for all length scales below L_{Planck}. That is, for all length scales below L_{Planck} the gravitational radius is much greater than the Compton length or localisation length implied by the uncertainty principle.

The equality between GM/c^2 and $\hbar/Mc$ (as required by consistency requirements on both GR and quantum mechanics) can be preserved at all lengths below L_{Planck} only if $\hbar$, i.e., the quantum of action, itself varies as M^2. Thus at M_{Planck} it has its usual value $\hbar$ so that $G_{\text{Newt}} M_{\text{Planck}}^2 = \hbar c$ and $GM^2 = \hbar c$ at all distance scales above L_{Planck} as noted in Equation (5e). But for all distances R below L_{Planck}, $\hbar$ reduces, i.e., goes as R^2. Thus $\hbar \sim M^2 \sim R^2$, so that $GM_p^2 \approx 10^{-38}\hbar c$ at the proton gravitational radius which is $\approx 10^{-19} L_{\text{Planck}}$ so that the effective $\hbar$ scaling as R^2 is $10^{-38}\hbar$! So we have another scaling relation: for lengths above L_{Planck}, $\hbar = \text{const.}$; lengths below L_{Planck}, $\hbar$ scales as R^2 (also $GM^2 \sim R^2$ and $GM^2/\hbar c = \text{const.}$).

Asymptotic freedom is implied for length scales below L_{Planck} as $GM^2 \to 0$ (the effective charge vanishing) as $R \to 0$, $\hbar \to 0$ as $R \to 0$. This would remove a contradiction wherein a zero-distance scale would correspond to infinite energy fluctuations.

Above L_{Planck}, contradiction with the uncertainty principle was avoided by having G scaling as M^2, with $\hbar c$ as constant. Energy conservation was also guaranteed. The scaling $M \sim R$, below L_{Planck} would again satisfy energy conservation, besides implying asymptotic invariance $M \to 0$ as $R \to 0$. The string tension T is constant below L_{Planck} and is (c^2/G). The scaling law $M \sim R$, below L_{Planck} would, moreover, be consistent with the expectation that gravity at length scales below L_{Planck} would be described by a Weyl-type theory (Sivaram, 1985, 1986a, b, c) rather than by the Einstein–Hilbert one. The Weyl theory has a quadratic curvature action $I_W \sim aC^{abcd}C_{abcd} + bR^2$ with a dimensionless coupling constant. For the scale-invariant Weyl gravity, the mass or energy grows linearly with distance, i.e., $M \sim R$. So, if the Weyl gravity dominates at $R < L_{\text{Planck}}$, we would expect $M \sim R$ and $GM \sim R$ and $GM^2 \sim R^2$. As pointed out in earlier works (Sivaram, 1985, 1986a, 1987a), breaking of scale invariance at L_{Planck}, induces a Hilbert term with a dimensional gravity constant G varying with scale as M^{-2} so that GM^2 = const. and $GM \sim R$ which is consitent with Equations (5) describing the behaviour at $L > L_{\text{Planck}}$. So Weil gravity with $M \sim R$ holds for $L < L_{\text{Planck}}$ and Einstein–Hilbert gravity for $L > L_{\text{Planck}}$. But for both cases $GM \sim R$ and GM/c^2 equals $\hbar/Mc$, so no contradiction between quantum and classical criteria for localizability.

Now several macroscopic structures on astronomical scales such as galaxy clusters and superclusters show evidence for considerable dark matter their rotation curves and velocity dispersions implying M rising linearly with R, i.e., $M \sim R$. One wonders whether Weyl gravity is also implied over such large scales which would naturally imply $M \sim R$. One can note some amusing macroscopic analogues to the length scales (7a)–(7e). If one considers that the Hubble radius $\sim 10^{28}$ cm is 10^{61} times the Planck length L_{Planck}, then if all the length scales in Equations (7) are multiplied by the same scaling factor (i.e., 10^{61}), then (7a) with the quark charge, would imply structures on the scale of ~ 100 Mpc (the Great Wall of Galaxies?). Equation (7b) would imply structures with lengths of ~ 10 Mpc (Large Super Clusters). Equation (7d) implies ~ 10 kpc (galaxies) and so on giving a whole range of macroscopic 'scaled-up' analogues of sub-Planck length scales! They would be self-similar in the sense of $M \sim R$, $\rho \sim 1/R^2$. This also implies a $\hbar \sim M^2$, so that for galactic structures with $M_{\text{gal}} \simeq 10^{50} M_{\text{Planck}}$ we have a 'galactic' Planck's constant of $\hbar_{\text{gal}} \simeq 10^{100} \hbar$ quantum ($\sim 10^{73}$ erg s) and has been independently postulated (for, e.g., by Cocke and Tifft, 1989) to explain several velocity and red-shift features! Note also that $M \sim R$ also implies an angular-momentum mass relation of the type $J \sim M^2$ consistent with that observed for a whole hierarchy of structures. The string picture also seems to imply this (Sivaram, 1987b).

And the largest structure predicted would be ~ 300 Mpc. Finally, from the geometrical picture of the string wherein the tension T is related to the deficit angle (i.e., geometry is locally that of planes joined by a fraction $(\phi/2\pi)$ of a cylinder, the string being described by a Wedge in space-time), $\psi = -2\phi$ by $T = \phi/8\pi G$, we can have an understanding as to why the equivalent G becomes large at scales $> L_{\text{Planck}}$, or equivalently why T becomes smaller at $R > L_{\text{Planck}}$. The deviation from Minkowski space is locally given by the deficit angle ϕ. For distances $R > L_{\text{Planck}}$, the deviation becomes smaller and smaller (i.e., the curvature is small) so that ϕ becomes small. So the string tension T proportional to ϕ becomes small and G being inversely proportional to T, becomes correspondingly larger, as $G \sim \phi^{-1}$. $\phi \sim R^{-2}$ (being related to curvature) and so $G \sim R^2$, accounting for the scaling relation equations (5). ϕ is very small at large $R \gg L_{\text{Planck}}$, so deviation from 2π is small, i.e., deficit angle small and, therefore, $T \to 0$ as effective G becomes large. At $R \leq L_{\text{Planck}}$, ϕ is constant and $T = C^2/G$ and we have asymptotic freedom with the effective charge or coupling g^2 (or GM^2) $\sim R^2$.

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