TORSION AND INFLATION

(Letter to the Editor)

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Abstract. We show that considering the torsion in early universe, we are led to an inflationary expansion with only a massless scalar field, so avoiding all physical questions that we are facing when working with massive scalar field.

In some recent work (de Sabbata and Sivaram, 1990a) we had considered the simplest Einstein–Cartan generalization of standard Big Bang cosmology by considering the Universe filled with spinning fluid and solving the modified Einstein equations $G^{\alpha\beta}(\{\ \}) = \chi\theta^{\alpha\beta}$ (where $\{\ \}$ are Christoffel symbols, θ is $T + \tau$ with T the usual energy-momentum tensor and τ representing the contribution of an effective spin–spin interaction). In the comoving frame $u^{\mu} = (0, 0, 0, 1)$ the modified equations of the Robertson–Walker metric is of the general form

$$\dot{R}^2/R^2 = (8\pi G/3)\,[\rho - (2/3)\pi G\sigma^2/c^4] + \Lambda c^2/3 - kc^2/R^2\,, \tag{1}$$

where ρ, σ depend only on time. We note that the quantity within the square brackets corresponds to an effective density of the form

$$\rho_{\text{eff}} = [\rho - (2/3)\pi G\sigma^2/c^4]\,. \tag{2}$$

It was noted in the above work that the torsion term (i.e., the second term within the square brackets) was at the Planck epoch equal and opposite in sign to the cosmological term $\Lambda_{\text{Pl}}c^2 \approx 10^{87}$ at that epoch, i.e., we had $-(8\pi G/3)\,(2/3)\pi G\sigma^2/c^4 \approx -10^{87}$. Again the $8\pi G\rho/3$ term is also of comparable magnitude (with ρ Planck density). This raises the question of whether ρ_{eff} can become negative just around or before the Planck epoch. If ρ_{eff} and consequently (as in general, from kinetic considerations $p_{\text{eff}} \approx k\rho_{\text{eff}}c^2$, where k is a numerical coefficient ≈ 1) the pressure p_{eff} becomes negative, we have the required condition for inflation, as negative pressure drives inflation. So inflation would follow as a natural consequence of a spin-dominated (i.e., torsion-dominated) phase in the very early universe, spin being a basic property besides the mass. We can see that this could indeed be the case from Equations (1) and (2).

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Indeed in another recent work (cf. de Sabbata and Sivaram, 1989) we had pointed out the relation $G \sim E^{-2}$, i.e., an energy-dependent G in the very early universe. With G going as E^{-2}, $n_{\rm Pl}$ would go as $\sim E^2$ and using the relation J going as E^2 for the spin of the particles increasing in mass just prior to the Planck epoch (i.e., a Regge relation) we find that the second term in the square brackets of Equation (1) increases faster with energy than the first term before the Planck epoch. So we could easily have had an effective *negative* density at the earliest phase. This would also be tantamount to the particle masses being effectively negative thereby implying a *repulsive* gravitational field with a negative mass source. As follows from the weak equivalence principle, a negative mass *source* will repel *all* test particles (positive and negative). The condition for an inflationary expansion are thus satisfied. Again G going as E^{-2}, would also resolve a paradoxical situation with the uncertainty principle (as arises in string theories (cf. Veneziano, 1989) as the spectrum of masses, with multiples of the Planck mass, no longer have $Gm/c^2 = \hbar/mc$, so their Compton length decreases whereas their horizon increases). G varying with m^{-2}, would ensure equality of Gm/c^2 and $\hbar/mc$ at all energies.

With torsion present it also turns out that we could realise an inflationary expansion with only a *massless* scalar field with no quartic self-coupling. As is known, in the usual inflationary scenario, the presence of a massive scalar field is essential. A massive scalar field possesses several physical questions besides having arbitrary couplings and particle masses. A massless scalar field with an improved (with torsion) energy-momentum tensor is physically more satisfactory, with just one effective coupling. We shall consider the case of a massless scalar field coupled to the Einstein–Cartan Lagrangian. So the action density is of the form

$$\frac{1}{2\chi_{\rm eff}} \sqrt{g}\,[R(\Gamma) + 2\Lambda] + (1/2)\sqrt{-g}\,(\partial_\mu \phi\, \partial^\mu \phi - \eta\phi^2 R(\Gamma)) \tag{3}$$

($\eta \ll 1$). (We could also add higher-order terms like $R^2\phi^2$, RQ, etc., but we do not consider them here.)

The quantity $R(\Gamma)$ is constructed from the non-symmetric affine connection

$$\Gamma^\rho_{\mu\nu} = \left\{ {\rho \atop \mu\nu} \right\} - K_{\mu\nu}{}^\rho\,, \tag{4}$$

where $K_{\mu\nu}{}^\rho$ is the contorsion tensor, related to the torsion $Q_{\mu\nu}{}^\rho \equiv \Gamma_{[\mu\nu]}{}^\rho$ by

$$K_{\mu\nu}{}^\rho = -Q_{\mu\nu}{}^\rho - Q^\rho_{.\mu\nu} + Q^\rho_{\nu.\mu}\,, \tag{5}$$

The modified torsion tensor is

$$T_{\mu\nu.}{}^\rho \equiv Q_{\mu\nu.}{}^\rho + 2\delta^\rho_{[\mu} Q_{\nu]\rho.}{}^\rho\,. \tag{6}$$

The variation with respect to $g_{\mu\nu}$ and $K_{\mu\nu}{}^\rho$ gives

$$G_{\mu\nu}[\{\ \}] + (1 - \chi_{\rm eff}\eta\phi^2)^{-1}\Lambda g_{\mu\nu} = \chi_{\rm eff}(T_{\mu\nu} + \tau_{\mu\nu}) + (1 - \chi_{\rm eff}\eta\phi^2)\Lambda g_{\mu\nu} =$$
$$= \chi_{\rm eff} T_{\mu\nu} + 4\tau_{\mu[\beta|}^{\cdot\alpha\cdot}\tau_{\nu|\alpha]}^{\cdot\beta\cdot} + 2\tau_\mu^{\cdot\alpha\beta}\tau_{\nu\alpha\beta} - \tau^{\alpha\beta}_{\cdot\cdot\mu}\tau_{\alpha\beta\nu} -$$

$$- (1/2)g_{\mu\nu}(4\tau_{\alpha}^{\cdot\beta}{}_{[\rho}\tau^{\alpha\rho}_{\cdot\cdot\beta]} + \tau^{\alpha\beta\rho}\tau_{\alpha\beta\rho}) -$$

$$- (1 - \chi_{\text{eff}}\eta\phi^2)^{-1}\Big\{(1 - 2\eta)\,\partial_\mu\phi\,\partial_\nu\phi + (2\eta - (1/2))\phi\,\partial^\alpha\phi\,\partial_\alpha\phi g_{\mu\nu} -$$

$$- 2\eta\phi(T_{\mu\nu}{}^{\rho} + T_{\nu\mu}{}^{\rho})\,\partial_\rho\phi + 2\eta g_{\mu\nu}\phi\,\Box\phi - 2\eta\phi\left\{{\rho \atop \mu\nu}\right\}\partial_\rho\phi -$$

$$- 2\eta\phi K^{\rho}_{\cdot(\mu\nu)}\,\partial_\rho\phi + 2\eta\phi T_{(\mu}\phi_{\nu)}\Big\}. \tag{7}$$

We see that due to torsion-scalar field coupling, tensor $T_{\mu\nu}$ is related to the gradient of a function of the scalar field

$$T_{[\mu\nu]}{}^{\rho} = 2\delta^{\rho}_{[\mu}\delta_{\nu]}\ln(1 - \chi_{\text{eff}}\eta\phi^2) \tag{8}$$

and

$$\Box\phi + \eta\phi R(\Gamma) = 0\,. \tag{9}$$

For a homogeneous, isotropic space-time such as that described by the R–W metric

$$\mathrm{d}s^2 = \mathrm{d}t^2 - R^2(t)\,(\mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2)\,,$$

the torsion (and the scalar fields) are solely functions of time and in fact the only surviving terms are

$$T^{1}_{00} = T^{2}_{02} = T^{3}_{03} \simeq \eta\phi\dot{\phi}\chi_{\text{eff}}\,, \tag{10}$$

where χ_{eff} is the effective Einstein gravitational constant whose evolution in time with energy was considered in detail (de Sabbata and Sivaram, 1989). Here in the spirit of induced gravity where the gravitational constant is described as the VEV of the scalar field (see Sivaram, 1983, 1986) (in fact in the induced gravity formalism, the Newtonian constant is generated by a scalar field with a non-zero vacuum expectation value (VEV) $\langle\phi\rangle$ as $G_N \approx 1/\langle\phi\rangle^2$ related to the mass M of the field as $G \sim 1/M^2$ with $M \sim M_{\text{Pl}}$, $G \sim G_N$):

$$\chi_{\text{eff}} \cong \frac{1}{\eta\phi^2}\;;$$

or in terms of energy

$$\chi_{\text{eff}} \sim M^{-2}\,. \tag{11}$$

For the case of the above metric, the field equations (7) (with χ_{eff} varying with epoch as above) become

$$\left(\frac{\dot{R}}{R}\right)^2 = \chi_{\text{eff}}\left\{\phi^2 - 2\,\frac{\dot{R}}{R}\,\eta\phi\dot{\phi} + 8\chi_{\text{eff}}\eta^2\phi^2\dot{\phi}^2 + \frac{\Lambda}{3\chi_{\text{eff}}}\right\}, \tag{12}$$

$$2\frac{\ddot{R}}{R}+\frac{\dot{R}^2}{R^2}=\chi_{\mathrm{eff}}[-24\eta^2\phi^2\dot{\phi}^2-\chi_{\mathrm{eff}}(2\eta+(1/2))\dot{\phi}^2-$$

$$-2\eta\phi\ddot{\phi}-4\eta\phi\dot{\phi}(\dot{R}/R)\Lambda/3\chi_{\mathrm{eff}}]\,. \tag{13}$$

As $\eta \ll 1$, we have, after simplification

$$\dot{R}/R=\dot{\phi}\chi_{\mathrm{eff}}^2((1/6)^{1/2}-\eta\phi\chi_{\mathrm{eff}}^{1/2})\,; \tag{14}$$

and substituting for χ_{eff} we find that

$$\dot{R}/R=\frac{\dot{\phi}}{\eta^{1/2}\phi}((1/6)^{1/2}-\eta^{1/2})\simeq(1/6\eta)^{1/2}\frac{\dot{\phi}}{\phi}\,, \tag{15}$$

which admits of a solution

$$R(t)=R_0\{\phi(t)\}^{(1/6\eta)^{(1/2)}}\,. \tag{16}$$

If $\eta^{1/2}\simeq 10^{-1}$ (for arguments regarding this value of coupling of torsion see de Sabbata and Sivaram (1990b)) $R(t)\sim R_0\phi(t)^{(10)}$; ($\phi(t)\sim E_0/E(t)$ with E_0 the initial particle energy).

Again if $\dot{\phi}/\phi$ remains constant in some interval $(0, t)$,

$$R(t)=R_0\exp(kt/(6\eta)^{1/2}\quad\text{where}\quad k=\dot{\phi}/\phi\gg 1\,.$$

In terms of energy,

$$R(t)=R_0\exp(\beta/M)t\quad(\beta\text{ is constant }\sim 10)\,. \tag{17}$$

We thus have a mechanism to initiate inflationary expansion phase, with only a massless scalar field. Again anisotropic metrics of the type

$$\mathrm{d}s^2=-\mathrm{d}t^2+A^2\,\mathrm{d}x^2+B^2(\mathrm{d}y^2+\mathrm{d}z^2)\,,$$

where $B=B(t)$ and $A=A(x,y,z,t)$ with spin sources, can be shown to possess exponential expansion phases $B=B_0\exp(kt)$, $P=-3k^2$, in the early universe, spin again playing the role of a negative pressure (Berman, 1990). In the usual inflationary scenarios there are problems with the mechanism in anisotropic conditions.

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