Radiative capture of polarized neutrons by polarized protons

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A model-independent irreducible tensor approach to $\vec{p}(\vec{n},\gamma)d$ is presented and an explicit form for the spin-structure of the matrix $\boldsymbol{M}$ for the reaction is obtained in terms of the Pauli spin-matrices $\boldsymbol{\sigma}(n)$ and $\boldsymbol{\sigma}(p)$. Expressing the multipole amplitudes in terms of the triplet $\rightarrow$ triplet and singlet $\rightarrow$ triplet transitions, we point out how the initial singlet and triplet contributions to the differential cross section can be determined empirically.

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The study of radiative capture of neutrons by protons is of interest not only to nuclear physics as a testing ground for theories of NN interaction, but also to astrophysics, where the fusion reaction is part of the proton-proton chain responsible for the generation of solar energy and production of elements in the early universe [1]. The cross section for the process was directly measured [2] for the first time at neutron energy of 550 keV, although measurements [3] with thermal neutrons have been carried out earlier. The 10% discrepancy noted between such measurements and theory was accounted for with surprising accuracy by the inclusion [4] of meson exchange currents (MEC). Measurements [5] at higher energies were used to test effects of ρ and ω exchanges and relativistic corrections to the impulse approximation [6]. Although a thorough review of the inverse reaction $d(\gamma,n)p$ over a wide range of energies is found in [7], the energy region just above threshold does not seem to have received much attention. Photodisintegration experiments at low energies focused attention [8] on the relative $M1$ and $E1$ contributions. Experiments employing polarized photons have been reported between 5 to 10 MeV [9] and more recently [10] at 3.58 MeV. The angular distribution as well as polarization of the neutron in $d(\gamma,p)n$ were measured [11]. Though the measured angular distribution of the neutron was found [12] to be in good agreement with theory at 2.75 MeV, the angular distribution and neutron polarization in [11] differ from theory which includes the MEC contributions. Employing polarized neutrons at 6 MeV and 13.4 MeV, the analyzing powers [12] in $p(\vec{n},\gamma)d$ was measured which differed from theory. Recent cross section calculations [13, 14], which agree with each other within 5% deviation, are found to differ from the 1967 estimates of Fowler et al [1]. The cross section was obtained in [13] by fitting the existing data with a polynomial expansion, while the calculation in [14] includes MEC's, isobar currents and pair currents. The theory is in good agreement with cross section data for neutrons

TABLE I: Transitions from initial states of np system, from threshold onwards, to the final ^2H state with spin-parity 1^+ and isospin $I=0$, together with the corresponding ΔI, Δs and multipole characteristics of the radiation emitted in the fusion reaction.

Initial state	ΔI	Δs	Allowed multipolarities of emitted radiation
$^1S_0$ $I=1$	1	1	$M1$
$^3S_1$ $I=0$	0	0	$M1$ $E2$
$^1P_1$ $I=0$	0	1	$E1$ $M2$
$^3P_0$ $I=1$	1	0	$E1$
$^3P_1$ $I=1$	1	0	$E1$ $M2$
$^3P_2$ $I=1$	1	0	$E1$ $M2$ $E3$

... and so on.

with energy above 14 MeV, but deviates by about 15% from older $d(\gamma,p)n$ measurements between 2.5 MeV and 2.75 MeV [15], which correspond respectively to neutron energies 550 keV and 1080 keV in $p(n,\gamma)d$. Apart from the fact that uncertainties in $p(n,\gamma)d$ cross sections at energies below about 600 keV lead to dominant uncertainties in the determination of the relative abundances of elements in the early universe, it is interesting to study the fusion reaction for its own sake, especially since a beginning has also been made [2] to study $p(n,\gamma)d$ precisely in the region of energies of interest to astrophysics. This recent study [2] has shown experimentally that the $M1$ transition from S-wave capture, which dominates at lower energies and the $E1$ transition from P-wave capture at higher energies are comparable at 550 keV.

It is interesting to observe that the neutron capture in $p(n,\gamma)d$ from the $^3S_1$ state is from the isospin $I=0$ state, which is the same for the deuteron, while that from the $^1S_0$, $I=1$ state is characterized by a $\Delta I=1$ transition. Likewise, capture from $^1P_1$, $I=0$ state leads to a $\Delta I=0$ transition, whereas those from $^3P_{j=0,1,2}$, $I=1$ states are characterized by $\Delta I=1$ transitions (see TABLE I). The cross section at very low energies is dom-

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inated by the $\Delta I = 1$ amplitude for the $M1$ transition from the ${}^1S_0$ state, since the NN scattering length in the ${}^1S_0$ channel is large and the $M1$ amplitude is proportional to the isovector magnetic moment of the nucleon, which is more than five times larger than the isoscalar magnetic moment. It was, however, noted quite early [16] that the spin-dependent effects in the fusion reaction are sensitive to the small $\Delta I = 0$ amplitudes $M1$ and $E2$. Hence a "polarized-target-beam test" was proposed for the ${}^3S_1 \rightarrow {}^3S_1$ radiative transitions in thermal np capture. The isoscalar $M1$ and $E2$ amplitudes have also been studied theoretically recently [17], using effective field theory, since the circular polarization [18] of photons emitted in the capture of polarized neutrons by unpolarized protons and the angular distribution [19] in the capture of polarized neutrons by polarized protons are sensitive to the presence of these amplitudes. Parameter-free predictions, employing the Weinberg [20] scheme of power counting, have also been made [21] for the spin-dependent observables in $\vec{p}(\vec{n},\gamma)d$ at threshold energies. However, the recently [19] measured value of $\eta = (1.0 \pm 2.5) \times 10^{-4}$ for γ-anisotropy at 50.5% polarization of neutrons and protons in $\vec{p}(\vec{n},\gamma)d$ cannot test the effective field theory predictions at 10^{-7}. Although this measurement provides the first experimental value for a spin-dependent observable in $\vec{p}(\vec{n},\gamma)d$, spin-observables in elastic $\vec{N}\vec{N}$ scattering have been measured for more than two decades [22]. More recently, attention was focused on such observables in $\vec{N}\vec{N}$ scattering, with a veiw to determine the exact strength of the tensor interaction and the case of π° production in $\vec{p}\vec{p}$ collisions [24] for which the threshold itself is high. Even at higher energies, it is clear that the singlet and triplet radiative captures from any arbitrary initial partial wave, ℓ lead to $p(n,\gamma)d$ characterized respectively by $\Delta I = 0$ and $\Delta I = 1$ or $\Delta I = 1$ and $\Delta I = 0$ depending on whether the initial parity $(-1)^\ell$ is odd or even. Since the NN interaction, as elicited from elastic scattering data, conserves channel spin, s (apart from total angular momentum j) as well as isospin I, it would be of interest to study experimentally the relative strengths of the initial singlet and triplet contributions to the $p(n,\gamma)d$ reaction at any given energy, since the fusion reaction leads to transitions in the two-nucleon system, which change spin s as well as isospin I. It is also worth noting that np fusion near threshold bears some resemblance to the case of pd fusion, where the validity of the so-called "no-quartet theorem" [25] was questioned and led to several incisive theoretical studies [26] in later years. More recently, it was pointed out [27] that a non-zero tensor analyzing power in $\overrightarrow{p}(\overrightarrow{d},\gamma)\,{}^3He$ by itself provides a clear signature to the contributions from the quartet amplitudes. Moreover, it was shown [28] that it is possible to determine empirically the individual doublet and quartet differential cross sections at any given energy through appropriate measurements of the relevant spin-observables in $\vec{N}\vec{d}$ fusion. In contrast to the difficult $\vec{N}\vec{d}$ fusion experiments suggested in [28], it should be technically more feasible to carry out the $\vec{p}(\vec{n},\gamma)\,{}^2H$ experiments [19].

The purpose of this Rapid Communication is to present a model-independent theoretical approach to the np fusion process based on the irreducible tensor formalism and identify observables in $\vec{p}(\vec{n},d)\gamma$ experiments that facilitate determination of the singlet and triplet cross sections empirically for the fusion reaction at any energy.

Let $\boldsymbol{p}(p,0,0)$ and $\boldsymbol{k}(k,\theta,0)$ denote respectively the neutron and photon momenta in a right-handed c.m. frame [29] and let $\langle m_d; \boldsymbol{k}\mu|\boldsymbol{T}|\boldsymbol{p}; m_n, m_p\rangle$ denote the elements of the on-energy-shell $\boldsymbol{T}$-matrix for the fusion reaction $p(n,\gamma)d$ where m_n, m_p, m_d denote respectively the spin-projections of the neutron, proton and deuteron and $\mu = \pm 1$ denote the left- and right-circular states of photon polarization, as defined by Rose [30]. The unpolarized differential cross section for the reaction is then given by

$$\frac{\mathrm{d}\sigma_0}{\mathrm{d}\Omega} = \left(\frac{k}{2\pi E}\right)^2 \frac{E_n E_p E_d}{p}\,\frac{1}{4} \sum_{m_n,m_p=-\frac{1}{2}}^{\frac{1}{2}} \sum_{m_d=-1}^{1} \times \sum_{\mu=-1,1} |\langle m_d; \boldsymbol{k}\mu|\boldsymbol{T}|\boldsymbol{p}; m_n, m_p\rangle|^2, \tag{1}$$

where E_p, E_n and E_d denote the c.m. energies of the proton, neutron and ${}^2H$ respectively, when the reaction takes place at c.m. energy E. Expressing the initial np system in terms of isosinglet and isotriplet states and making use of the standard multipole expansion [30] for the photon in the final state and the usual partial wave expansion for relative motion in the initial state with channel-spin $s = 0, 1$, the $\boldsymbol{T}$-matrix may be expressed as

$$\boldsymbol{T} = \sum_{s=0,1} \sum_{\lambda=|1-s|}^{(1+s)} \sum_{\mu=-1,1} \left(S^\lambda(1,s)\cdot T^\lambda(\mu,s)\right), \tag{2}$$

where $S^\lambda_\nu(1,s)$ denote [31] irreducible tensor operators of rank λ in hadron spin-space. The irreducible tensor amplitudes $T^\lambda_\nu(\mu,s)$ in Eq. (2) are given explicitly by

$$\begin{aligned} T^\lambda_\nu(\mu,s) = {} & \sqrt{\frac{2}{3}}\,(2\pi)(-1)^{1-s} \sum_{\ell=0}^{\infty} \sum_{L=1}^{\infty} \sum_{j=|\ell-s|}^{(\ell+s)} (\mathrm{i})^{\ell-L} \\ & \times (-1)^{\lambda-L}\,[\ell]\,[L]\,[j]^2\, d^L_{\nu\mu}(\theta)\, C(\ell L\lambda; 0\nu\nu) \\ & \times W(\ell s L 1; j\lambda)\,\left[T^{j\,(\mathrm{mag})}_{L;\ell s}(E) - \mathrm{i}\mu\, T^{j\,(\mathrm{elec})}_{L;\ell s}(E)\right], \end{aligned} \tag{3}$$

where the energy dependence is carried entirely by the partial wave magnetic and electric 2^L-multipole amplitudes $T^{j\,(\mathrm{mag})}_{L;\ell s}(E)$ and $T^{j\,(\mathrm{elec})}_{L;\ell s}(E)$ respectively, while the angular dependence is contained entirely in $d^L_{\nu\mu}(\theta)$. We use the shorthand $[j] = \sqrt{(2j+1)}$ and the rest of the notations follow [30]. The multipole amplitudes are given in terms of the reduced matrix elements by

$$T^{j\,(\mathrm{mag})}_{L;\ell s}(E) = \frac{1}{2}\left\{(-1)^{L+1} + (-1)^{\ell}\right\}\,\langle\, ||\boldsymbol{T}\,||\,\rangle$$
$$T^{j\,(\mathrm{elec})}_{L;\ell s}(E) = \frac{1}{2}\left\{(-1)^{L} + (-1)^{\ell}\right\}\,\langle\, ||\boldsymbol{T}\,||\,\rangle\,, \tag{4}$$

where $\langle\, ||\boldsymbol{T}\,||\,\rangle$ denotes

$$\langle\, ||\boldsymbol{T}\,||\,\rangle = \sum_{I=0,1} (-1)^{I}\,[I]^{-\frac{1}{2}}\,\left[1-(-1)^{\ell+s+I}\right] \times C(\tfrac{1}{2}\tfrac{1}{2}I;-\tfrac{1}{2}\tfrac{1}{2}0)\Big\langle(1L)j;I_d=0\Big|\Big|\boldsymbol{T}\Big|\Big|(\ell s)j;I\Big\rangle\,. \tag{5}$$

Expressing the irreducible tensor operators $S^{\lambda}_{\nu}(1,s)$ of rank λ in terms of the Pauli spin-matrices $\boldsymbol{\sigma}(n)$ and $\boldsymbol{\sigma}(p)$ and unit 2×2 matrices $\sigma^0_0(n)$, $\sigma^0_0(p)$ for neutron, proton respectively following [31], we may rewrite Eq. (1) in the elegant form

$$\frac{\mathrm{d}\sigma_0}{\mathrm{d}\Omega} = \frac{1}{4}\,\mathrm{Tr}\left(\boldsymbol{M}\boldsymbol{M}^{\dagger}\right), \tag{6}$$

where $\mathrm{Tr}(\equiv \sum_{m_d}\sum_{\mu})$ denotes the Trace or Spur and the matrix $\boldsymbol{M}$ for $p(n,\gamma)d$ has the form

$$\boldsymbol{M} = \sum_{\lambda_1,\lambda_2=0}^{1}\;\sum_{\lambda=|\lambda_1-\lambda_2|}^{(\lambda_1+\lambda_2)} \left(\left(\sigma^{\lambda_1}(n)\otimes\sigma^{\lambda_2}(p)\right)^{\lambda}\cdot M^{\lambda}(\lambda_1,\lambda_2;\mu)\right), \tag{7}$$

in terms of irreducible tensor amplitudes $M^{\lambda}_{\nu}(\lambda_1,\lambda_2;\mu)$ of rank λ given by

$$M^{\lambda}_{\nu}(\lambda_1,\lambda_2;\mu) = \left(\frac{k}{2\pi E}\right)\left(\frac{E_n E_p E_d}{p}\right)^{\frac{1}{2}}\,\frac{3}{2}\,[\lambda_1][\lambda_2] \times\sum_{s=0}^{1}\,[s]\begin{Bmatrix}\frac{1}{2} & \frac{1}{2} & 1\\ \frac{1}{2} & \frac{1}{2} & s\\ \lambda_1 & \lambda_2 & \lambda\end{Bmatrix} T^{\lambda}_{\nu}(\mu,s), \tag{8}$$

where $\{\ \}$ denote Wigner-$9j$ symbols [32]. We may explicit Eq. (7) as

$$\begin{aligned}\boldsymbol{M} =& A + B\,(\boldsymbol{\sigma}(n)\cdot\boldsymbol{\sigma}(p)) + (\boldsymbol{\sigma}(n)+\boldsymbol{\sigma}(p))\cdot\boldsymbol{C}\\ &+ (\boldsymbol{\sigma}(n)-\boldsymbol{\sigma}(p))\cdot\boldsymbol{D} + (\boldsymbol{\sigma}(n)\times\boldsymbol{\sigma}(p))\cdot\boldsymbol{E}\\ &+ \left((\boldsymbol{\sigma}(n)\otimes\boldsymbol{\sigma}(p))^{2}\cdot F^{2}\right),\end{aligned} \tag{9}$$

where the coefficients are related to (8) through

$$\begin{aligned}&A = M^0_0(0,0;\mu)\ ;\ B = -\frac{1}{\sqrt{3}}M^0_0(1,1;\mu)\ ;\\ &C^1_{\nu} = \frac{1}{2}\left[M^1_{\nu}(1,0;\mu) + M^1_{\nu}(0,1;\mu)\right];\\ &D^1_{\nu} = \frac{1}{2}\left[M^1_{\nu}(1,0;\mu) - M^1_{\nu}(0,1;\mu)\right];\\ &E^1_{\nu} = \frac{\mathrm{i}}{\sqrt{2}}M^1_{\nu}(1,1;\mu)\ ;\ F^2_{\nu} = M^2_{\nu}(1,1;\mu)\end{aligned} \tag{10}$$

and $C^1_{\nu},\ D^1_{\nu},\ E^1_{\nu}$ denote respectively the spherical components of $\boldsymbol{C},\ \boldsymbol{D},\ \boldsymbol{E}$. Comparing Eq. (9) with $\boldsymbol{M}$ for elastic NN scattering [33] shows clearly that the fourth and fifth terms containing $(\boldsymbol{\sigma}(n)-\boldsymbol{\sigma}(p))$ and $(\boldsymbol{\sigma}(n)\times\boldsymbol{\sigma}(p))$ are the ones which induce transitions from the initial spin-singlet state to the final spin-triplet state of the deuteron. To estimate the singlet and triplet cross sections empirically we observe that the differential cross section for $\vec{p}(\vec{n},\gamma)d$ is given by

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \mathrm{Tr}\left(\boldsymbol{M}\rho\boldsymbol{M}^{\dagger}\right), \tag{11}$$

where the density matrix

$$\rho = \frac{1}{4}\left[1+\left(\boldsymbol{\sigma}(n)\cdot\boldsymbol{P}(n)\right)\right]\ \left[1+\left(\boldsymbol{\sigma}(p)\cdot\boldsymbol{P}(p)\right)\right] \tag{12}$$

describes the initial spin state if $\boldsymbol{P}(n)$ and $\boldsymbol{P}(p)$ denote respectively the neutron and proton polarizations. Rewriting Eq. (11) as

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{1}{4}\sum_{\alpha,\beta=0,x,y,z} P_{\alpha}(n)\,P_{\beta}(p)\,B_{\alpha\beta}, \tag{13}$$

where $P_0(n)=1, P_0(p)=1$ and

$$B_{\alpha\beta} = \mathrm{Tr}\left(\boldsymbol{M}\sigma_{\alpha}(n)\sigma_{\beta}(p)\boldsymbol{M}^{\dagger}\right), \tag{14}$$

we readily see that the unpolarized differential cross section (6) is given by $(1/4)B_{00}$. Noting [34] further that

$$\boldsymbol{\pi}(1,1) = \frac{1}{4}\left[1+\sigma_z(n)+\sigma_z(p)+\sigma_z(n)\sigma_z(p)\right] \tag{15}$$

$$\boldsymbol{\pi}(1,0) = \frac{1}{4}\left[1+\sigma_x(n)\sigma_x(p)+\sigma_y(n)\sigma_y(p)-\sigma_z(n)\sigma_z(p)\right] \tag{16}$$

$$\boldsymbol{\pi}(1,-1) = \frac{1}{4}\left[1-\sigma_z(n)-\sigma_z(p)+\sigma_z(n)\sigma_z(p)\right] \tag{17}$$

$$\boldsymbol{\pi}(0,0) = \frac{1}{4}\left[1-\sigma_x(n)\sigma_x(p)-\sigma_y(n)\sigma_y(p)-\sigma_z(n)\sigma_z(p)\right] \tag{18}$$

are the projection operators $|sm\rangle\langle sm|$, $s = 0,1$; $m = +s,\ldots,-s$, we readily identify

$$\frac{\mathrm{d}\sigma_{1,1}}{\mathrm{d}\Omega} = \frac{1}{16}\left[B_{00} + B_{z0} + B_{0z} + B_{zz}\right] \tag{19}$$

$$\frac{\mathrm{d}\sigma_{1,0}}{\mathrm{d}\Omega} = \frac{1}{16}\left[B_{00} + B_{xx} + B_{yy} - B_{zz}\right] \tag{20}$$

$$\frac{\mathrm{d}\sigma_{1,-1}}{\mathrm{d}\Omega} = \frac{1}{16}\left[B_{00} - B_{z0} - B_{0z} + B_{zz}\right] \tag{21}$$

$$\frac{\mathrm{d}\sigma_{0,0}}{\mathrm{d}\Omega} = \frac{1}{16}\left[B_{00} - B_{xx} - B_{yy} - B_{zz}\right] \tag{22}$$

as the triplet and singlet contributions $(\mathrm{d}\sigma_{s,m})/(\mathrm{d}\Omega)$ which add up to $(\mathrm{d}\sigma_0)/(\mathrm{d}\Omega)$ given by Eq. (1). Eq. (11) can also be expressed in the form

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \sum_{k_n,k_p,k} \left(\left(P^{k_n}(n)\otimes P^{k_p}(p)\right)^k \cdot B^k(k_n,k_p)\right) \tag{23}$$

$$= \frac{\mathrm{d}\sigma_0}{\mathrm{d}\Omega}\Bigg[1 + \boldsymbol{P}(n)\cdot\boldsymbol{A}(n) + \boldsymbol{P}(p)\cdot\boldsymbol{A}(p) + \sum_{k=0}^{2}\left(\left(\boldsymbol{P}(n)\otimes\boldsymbol{P}(p)\right)^k\cdot A^k\right)\Bigg], \tag{24}$$

where the irreducible bilinear amplitudes in Eq. (23)

$$B^k_q(k_n,k_p) = \frac{3}{2}\left(\frac{k}{2\pi E}\right)^2 \frac{E_nE_pE_d}{p}\,(-1)^{k_n+k_p-k}\,[k_n]\,[k_p] \times \sum_{s,s',\lambda,\lambda',\mu} (-1)^\lambda\,[s]\,[s']\,[\lambda]\,[\lambda']\,W(s'k1\lambda;s\lambda') \times \begin{Bmatrix} \frac{1}{2} & \frac{1}{2} & s \\ \frac{1}{2} & \frac{1}{2} & s' \\ k_n & k_p & k \end{Bmatrix} \left(T^\lambda(\mu,s)\otimes T^{\dagger^{\lambda'}}(\mu,s')\right)^k_q \tag{25}$$

are related to the spherical components of the analyzing powers in Eq. (24) through

$$\frac{\mathrm{d}\sigma_0}{\mathrm{d}\Omega}A^1_q(n) = B^1_q(1,0) \tag{26}$$

$$\frac{\mathrm{d}\sigma_0}{\mathrm{d}\Omega}A^1_q(p) = B^1_q(0,1) \tag{27}$$

$$\frac{\mathrm{d}\sigma_0}{\mathrm{d}\Omega}A^k_q = B^k_q(1,1), \tag{28}$$

with $B^0_0(0,0) = (\mathrm{d}\sigma_0)/(\mathrm{d}\Omega)$, the unpolarized differential cross section. From Eq. (3), we observe that the $T^\lambda_\nu(\mu,s)$ satisfy the symmetry property

$$T^\lambda_\nu(\mu,s) = (-1)^{\lambda-\nu}\,T^\lambda_{-\nu}(-\mu,s). \tag{29}$$

This in turn implies through Eq. (25) that the bilinear amplitudes satisfy the property

$$B^k_q(k_n,k_p) = (-1)^{k-q}\,B^k_{-q}(k_n,k_p) \tag{30}$$

so that

$$B^1_0(k_n,k_p) = 0 \tag{31}$$

and from Eqs. (26) and (27),

$$A_z(n) = A_z(p) = 0. \tag{32}$$

Using these results, we can now write the channel-spin differential cross sections $(\mathrm{d}\sigma_{s,m})/(\mathrm{d}\Omega)$ in terms of the analyzing powers as

$$\frac{\mathrm{d}\sigma_{1,1}}{\mathrm{d}\Omega} = \frac{\mathrm{d}\sigma_{1,-1}}{\mathrm{d}\Omega} = \frac{1}{4}\frac{\mathrm{d}\sigma_0}{\mathrm{d}\Omega}\left[1 - \tfrac{1}{\sqrt{3}}A^0_0 + \sqrt{\tfrac{2}{3}}A^2_0\right] \tag{33}$$

$$\frac{\mathrm{d}\sigma_{1,0}}{\mathrm{d}\Omega} = \frac{1}{4}\frac{\mathrm{d}\sigma_0}{\mathrm{d}\Omega}\left[1 - \tfrac{1}{\sqrt{3}}A^0_0 - 2\sqrt{\tfrac{2}{3}}A^2_0\right] \tag{34}$$

$$\frac{\mathrm{d}\sigma_{0,0}}{\mathrm{d}\Omega} = \frac{1}{4}\frac{\mathrm{d}\sigma_0}{\mathrm{d}\Omega}\left[1 + \sqrt{3}A^0_0\right]. \tag{35}$$

The analyzing powers, moreover, are measurable readily in an experimental setup such as in [19], where $\boldsymbol{P}(n) = \boldsymbol{P}(p) = P\widehat{\boldsymbol{P}}$ with $P = 50.5\%$ and $\widehat{\boldsymbol{P}}$ could be longitudinal as well as transverse. With $\widehat{\boldsymbol{P}}$ chosen parallel and antiparallel to the beam, it is clear that

$$\frac{\mathrm{d}\sigma_{+z}}{\mathrm{d}\Omega} + \frac{\mathrm{d}\sigma_{-z}}{\mathrm{d}\Omega} = 2\frac{\mathrm{d}\sigma_0}{\mathrm{d}\Omega}\left[1 + \frac{P^2}{\sqrt{3}}(\sqrt{2}\,A^2_0 - A^0_0)\right] \tag{36}$$

which yields $\left(\sqrt{2}\,A^2_0 - A^0_0\right)$.

Likewise, if $\widehat{\boldsymbol{P}}$ is chosen parallel and antiparallel to a direction (say $\hat{x}$ of $\hat{y}$) perpendicular to the beam, we have

$$\frac{\mathrm{d}\sigma_{+x}}{\mathrm{d}\Omega} + \frac{\mathrm{d}\sigma_{-x}}{\mathrm{d}\Omega} = 2\frac{\mathrm{d}\sigma_0}{\mathrm{d}\Omega}\left[1 - \frac{P^2}{\sqrt{3}}(A^0_0 + \tfrac{1}{\sqrt{2}}A^2_0) + A^2_2\right] \tag{37}$$

$$\frac{\mathrm{d}\sigma_{+y}}{\mathrm{d}\Omega} + \frac{\mathrm{d}\sigma_{-y}}{\mathrm{d}\Omega} = 2\frac{\mathrm{d}\sigma_0}{\mathrm{d}\Omega}\left[1 - \frac{P^2}{\sqrt{3}}(A^0_0 + \tfrac{1}{\sqrt{2}}A^2_0) - A^2_2\right], \tag{38}$$

where we have made use of the fact that $A^2_2 = A^2_{-2}$ from Eqs. (28) and (30). By adding Eqs. (37) and (38), one readily obtains $\left(A^0_0 + \frac{1}{\sqrt{2}}A^2_0\right)$. Since $\left(\sqrt{2}\,A^2_0 - A^0_0\right)$ is already known from Eq. (36), one can determine A^0_0 as well as A^2_0. Hence all the $(\mathrm{d}\sigma_{s,m})/(\mathrm{d}\Omega)$ for $m = -s,\ldots,s$ and $s = 0,1$ are determinable from experiment empirically at any given energy. We may perhaps add that these measurements need not have to be carried out at accuracies of 10^{-7} to determine $(\mathrm{d}\sigma_{s,m})/(\mathrm{d}\Omega)$. Moreover, the discussion presented here aims to extend the program of Breit and Rustgi [16] to actual individual determinations of the three triplet and one singlet cross sections for the important [1] fusion process at the differential level itself rather that estimate the relative importance of the triplet amplitude vis-a-vis the singlet amplitude. It would, therefore, be desirable to extend the recent experiment [19] on $\vec{p}(\vec{n},\gamma)d$ to measure the observables (36) to (38) in order to determine the differential cross sections $(d\sigma_{s,m})/(d\Omega)$ individually and hence study the fusion reaction more incisively at any given energy.

It is perhaps not out of place to mention here that the highly successful NN potential models like Nijmegen1 and Nijmegen2 potentials [35], the Argonne potential [36] and the Bonn potential [37] which reproduce the elastic NN scattering data with a remarkable χ^2/datum $\approx$ 1, have been subjected to an interesting study of the deuteron properties by Polls et al. [38], who say, *It is well known that the off-shell behavior of NN potentials cannot be pinned down by on-shell data...Moreover, within a given model, pinning down the on-shell point limits the range of variation for the off-shell behavior. The four modern high-precision models considered in this study pin down the on-shell T-matrix as much as by all means possible since they fit the NN scattering data with the perfect χ^2/ datum $\approx$ 1. Furthermore, it may be reasonable to believe that the four models cover about the range of diversity that there is to realistic physical models for the NN interaction. Based upon these premises, one may then conclude that the off-shell uncertainties revealed in this study are what we have to deal with, at the current status of theoretical nuclear physics. At this time, we do not know of any other objectively verifiable aspects that could further reduce the off-shell uncertainties. Tighter constraints for the off-shell behavior of NN may emerge in the future...* In this context, it is perhaps pertinent to point out that $np \to d\gamma$ involves necessarily off-shell NN interactions. Therefore it would also be of interest to examine the fusion reaction with a view to throw light on the off-shell uncertainties. Further work is in progress.

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