Comment on "Thermodynamics of Black Holes: An Analogy with Glasses"

In a recent paper [1], Nieuwenhuizen has pointed out drawbacks in the present equilibrium formulation of black hole thermodynamics which assumes the same temperature for black hole and heat bath. He has applied nonequilibrium thermodynamics developed for glassy systems to black holes by bringing in the cosmic background radiation (temperature $T_{\rm cb}$) as a heat bath. He shows that the well-known laws of black hole dynamics are in accordance with the laws of thermodynamics far from equilibrium obeyed, e.g., by glasses. This approach results in the entropy production rates on short and long time scales, the latter occurring at the Hawking temperature T_H (internal temperature). Again the third law of thermodynamics (i.e., vanishing of the entropy at $T_{\rm cb} = 0$ and final black hole evaporation) deviates from the third law of black hole dynamics which deals with extremal black holes with $T_H = 0$ but finite entropy. Thus it is important to consider not only the quantum evaporation of the black hole but also its absorption of the cosmic background radiation; so that for the present $T_{\rm cb} = 2.73$ K, only black hole masses smaller than $2.2 \times 10^{-8} M_\odot$ evaporate. Within the above framework, this Comment points out further that for an evaporating black hole of mass M to accrete (absorb) either ambient radiation or matter, its luminosity due to Hawking radiation (which is $L_H \sim \hbar c^6/G^2M^2$) must be less than the corresponding Eddington luminosity [$L_E \sim 10^{38}(M/M_\odot)$ ergs/s, $M_\odot$ being the solar mass] at which radiation pressure from the hole drives away the ambient medium. (L_E is obtained by equating the radiation pressure and the gravitational force.) The mass M^* at which $L_E = L_H$ is obtained from $\hbar c^6/G^2M^{*2} \sim 10^{38}M^*/M_\odot$ and this gives $M^* = 1.5 \times 10^{15}$ g (note that L_E/L_H scales as M^3). Thus for a black hole to *effectively* accrete or absorb energy from the ambient heat bath (radiation), it must have a mass $M \gg M^*$. In the early universe $T_{\rm cb}$ was large and in his Eq. (19), Nieuwenhuizen estimates the temperature at which the entropy of ordinary matter and the entropy of the same matter as a black hole was equal, which turns out to be $10^{12}k$. However, it is a known coincidence, for example [2] that the quantum mechanical entropy of a black hole ($S_{\rm bh} \propto M^2$) equals the classical entropy of the mass M, i.e., $S_{\rm cl}$ equals the number of nucleons in M, for a black hole mass $M \simeq 10^{14}$ g, which indeed corresponds to a temperature around 10^{12} K. It also holds that the Schwarschild radius of such a black hole equals the Compton wavelength of the proton r_p. Thus $S_{\rm bh}/S_{\rm cl} \sim (R_s/r_p) \sim 1$, for $T_{\rm bh} \sim 10^{12}$ K. It is probably a coincidence that such black holes (of mass $M \sim 10^{14}$ g and $T \sim 10^{12}$ K) with equal quantum and classical entropies evaporate over the Hubble age. For such cosmological constraints on black hole temperatures, see [3]. Moreover, in the early universe, which is radiation dominated, the radiation temperature T_R changes rapidly with time (t) and is known to follow a time-temperature relation of the form $T_R = [(10^{10}\ {\rm K})/t^{1/2}\ ({\rm s})]$, so that a radiation temperature of 10^{12} K, occurs at $t \simeq 10^{-4}$ s. The above relation follows from the standard cosmological picture (with the Robertson-Walker metric) where the scale factor evolves with the density as $\dot{R}/R = (8\pi G\rho/3)^{1/2}$, and substituting for $\rho = \sigma T_R^4/c^2$ (σ, the Stefan-Boltzmann constant) for the radiation-dominated energy density, the above temperature-time relation follows. It is supposed to hold throughout the radiation-dominated era to very early epochs.

In the early universe, black holes are formed when metric fluctuations exceed unity. As this is a short time phenomenon, it has nothing to do with black hole thermodynamics. Thus in the early universe, this could happen if the external radiation pressure forced material inside the Schwarzschild radius provided it began with a density sufficiently in excess of the ambient average density ρ [4]. Since the density in the radiation dominated era as a function of time is given by $\rho = 3/32\pi Gt^2$, the mass of radiation in causal contact after time t is $M_H = (4\pi t/c)c^3t^3\rho$, and substituting for ρ, this implies that at an epoch less than t, only black holes of mass $M_H = c^3t/8G$ form. For a radiation temperature of 10^{12} K, occurring at $t \simeq 10^{-6}$ s, only black holes with $M_H \simeq 10^{32}$ g can form, which have Hawking temperature $<10^{-6}$ deg. Combining the relations for M_H and T_R above it turns out that for black holes formed in the early universe, the Hawking temperature and the background radiation temperature are comparably the same only for $T_R = T_H \simeq 10^{20}(\pi K_B/\hbar) \simeq 10^{32}$ K, or for Planck mass black holes with $M \simeq 10^{-5}$ g at $t \simeq 10^{-43}$ s. Now at a time t, only black holes of mass $M_H = c^3t/8G$ form, which have a Hawking temperature T_H of $\hbar/\pi K_B t$ obtained by substituting for M_H in the Hawking formula [$T_H = (\hbar c^3/8\pi G M_H K_B)$]. Now equating T_R and T_H above, we have $\hbar/\pi K_B t \simeq 10^{10}/t^{1/2}$, giving $t^{1/2} = \hbar/\pi K_B \times 10^{10}$, which implies a $T_R = T_H \simeq 10^{20}\pi(K_B/\hbar) \simeq 10^{32}$ K. (This also implies a corresponding lower time limit.) Thus the conclusions in Ref. [1], for equality of T_H and T_R for $T_R = 10^{12}$ K [Eq. (19)] are not justified. Indeed, 10^{14} g black holes with $T_H \simeq 10^{12}$ K form at $t \simeq 10^{-24}$ s when ambient $T_R \simeq 10^{22}$ K.

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