Astrophysical Letters, 1986, Vol. 25, pp. 67–72
0004-6388/86/2502-0067/$10.00

General Relativistic Decollimation of a Jet

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(Received October 2, 1985; in final form April 1, 1986)

This paper discusses the effect of defocusing introduced by spacetime curvature in a narrow conical beam of particles emanating from the vicinity of a compact mass, applicable to the phenomenon of jets, provided particles are accelerated sufficiently close to it.
KEYWORDS general relativity jets

INTRODUCTION

The review by Rees *et al.* (1981) describes how the central core of the compact radio sources, which involves a large concentrated mass, most likely a black hole $\sim 10^8\,M_\odot$ accreting material from its neighbourhood, produces fast moving plasma in the form of directed beams (jets). In this region ($\sim 20m$; $m = GM/c^2$) bulk velocities are relativistic and conditions are very favourable for accelerating particles to ultra relativistic energies. The beam is neutral and electrons and ions (or positrons) have essentially the same density and move with very similar speeds. The apparent opening angles are $\gtrsim 5°$ which have been observed in VLBI jets. The flow is basically fluid like and collimation is assumed to be established on scales $r_0 \sim 20m$. The works of Abramowicz and Piran (1980) and Sikora and Wilson (1981) deal extensively with jet collimation within a relativistic framework.

Our purpose here is to investigate the contribution of gravitational effects to the collimation process choosing r_0 as the size of the region over which collimation is assumed to be established. This is shown by calculating the deflection in particle trajectories brought about by the attraction of the central compact mass which widens the beam (the decollimation). This produces reduction in the particle density and the effective luminosity in the beam by a factor of an order of magnitude (Kapoor, 1986).

PARTICLE TRAJECTORIES AND DECOLLIMATION

In what follows, we assume that the particles are accelerated to relativistic energies near the black hole in a preferential direction and that they follow geodesics i.e. the jet material is considered dust like for the purpose of demonstrating the decollimation. Let the black hole be Schwarzschild. For reasons of symmetry, $\theta = \pi/2$. The geodesic equations corresponding to space time around a mass M

$$ds^2 = \left(1 - \frac{2m}{r}\right) dt^2 - \left(1 - \frac{2m}{r}\right)^{-1} dr^2 - r^2\, d\varphi^2 \tag{1}$$

give the t- and φ-components of the 4-velocity $u^\alpha = dx^\alpha/ds$ $[x^\alpha = (t, r, \theta, \varphi)]$

$$\frac{dt}{ds} = \frac{\gamma}{(1-2m/r)}; \quad \frac{d\varphi}{ds} = \frac{h}{r^2}. \tag{2}$$

Here h and γ are the constants of motion, being respectively the orbital angular momentum and energy of the particle per unit rest mass as measured at infinity. Equations (1) and (2) give the r-component of u^α:

$$\frac{dr}{ds} = \pm\gamma\left[1 - \left(1 - \frac{2m}{r}\right)\left(\frac{1 + h^2/r^2}{\gamma^2}\right)\right]^{1/2}. \tag{3}$$

The particle starts out from a point r_0, a few times the Schwarzschild radius of the black hole. Since its trajectory is slightly non-radial (unless emitted along the axis), it suffers a net bending

$$\varphi_0 = \int_{r_0}^{\infty} \frac{d\varphi/ds}{dr/ds}\, dr. \tag{4}$$

An evaluation of this integral requires a knowledge of the impact parameter of the particle which we define as

$$q = \frac{h}{\gamma}. \tag{5}$$

To evaluate q, we note that the two physical components of the particle velocity as according to an observer at rest in the Schwarzschild field with respect to his proper reference frame can be split up as follows:

$$v_r = \left(-\frac{g_{11}}{g_{00}}\right)^{1/2} \frac{dr/ds}{dt/ds} = \frac{1}{\gamma}\frac{dr}{ds}, \tag{6}$$

$$v_\varphi = \left(-\frac{g_{33}}{g_{00}}\right)^{1/2} \frac{d\varphi/ds}{dt/ds} = q\frac{(1-2m/r)^{1/2}}{r}, \tag{7}$$

such that

$$v_r^2 + v_\varphi^2 = v^2. \tag{8}$$

(Note that $v_\varphi \to 0$ and $v_r \to 1$ as $r \to 2m$.) Let η denote the angle at which the particle is emitted with respect to the axis of the jet. Then we can write

$$v_r = v\cos\eta, \quad v_\varphi = v\sin\eta. \tag{9}$$

At the emission location $r = r_0$, the impact parameter is thus evaluated

$$q = \frac{v r_0 \sin\eta}{(1-2m/r_0)^{1/2}}. \tag{10}$$

Also from Equations (3), (6), (7) and (8), we have

$$\gamma = \left(\frac{1-2m/r_0}{1-v^2}\right)^{1/2}. \tag{11}$$

In order that particle reaches infinity (with $\gamma \geq 1$), we must have $v^2 \geq v_{esc}^2 = 2m/r_0$. For instance, if $r_0 = 6m$, $v > 3^{-1/2} \simeq 0.6$. Since $\gamma = (1 - v_\infty^2)^{-1/2}$, we have $v_\infty^2 = v^2 - 2m/r_0$. The capture angle is defined thus: the critical h for capture is $h = 4m$ (cf. Shapiro and Teukolsky, 1983). So, for $q = 4m/\gamma$,

$$\sin \eta_c = \frac{4m(1 - v^2)^{1/2}}{v r_0}. \tag{12}$$

for instance, if $r_0 = 6m$, $v = v_{esc} = 3^{-1/2}$, $\eta_c \cong 70°$. Hence, for the jet opening angles we are concerned with, namely $0 \leqslant \eta \leqslant 5°$, escape of a particle ($v < 1$) is assured provided $v \geq v_{esc}$.

Equation (4) can now be explicitly written

$$\varphi_0 = \int_{r_0}^{\infty} f(r, q)\, dr \tag{13}$$

where

$$f(r, q) = \frac{q}{r^2\left[1 - \left(\frac{1 - 2m/r}{1 - 2m/r_0}\right)\left\{1 - v^2 + \frac{v^2 r_0^2 \sin^2 \eta}{r^2}\right\}\right]^{1/2}}.$$

For $v \rightarrow 1$, Eq. (13) describes the bending of photons in the gravitational field of the black hole (Kapoor, 1981). It is possible to reduce the integral in Weirestrass form.

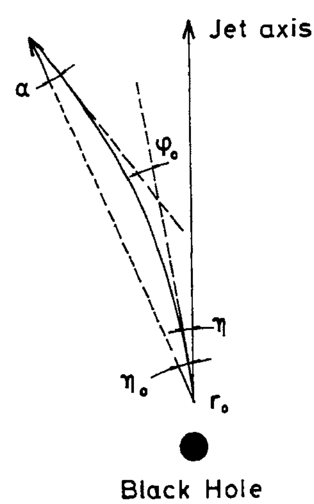

FIGURE 1 Schematic illustration of the trajectory of a particle starting from r_0 near the black hole and its bending.

Refer now to Figure 1. The bending can be written as

$$\varphi_0 = \eta_0 - \eta + \alpha \tag{14}$$

where

$$\tan \alpha = -r \frac{d\varphi}{dr}\bigg|_{r\to\infty} \to 0. \quad (15)$$

Hence the new value of η is

$$\eta_0 = \varphi_0 + \eta. \quad (16)$$

This in essence implies a decollimation of the beam. The degree of decollimation is measured by a quantity

$$\Delta = \frac{d\eta_0}{d\eta}\bigg|_{r=r_0} \quad (17)$$

$$= 1 + \cot \eta \left(\varphi_0 + \int g(r, q)\, dr\right) \quad (18)$$

where

$$g(r, q) = \frac{q^3}{r^4\left[1 - \left(\frac{1-2m/r}{1-2m/r_0}\right)\left\{1 - v^2 + \frac{v^2 r_0^2 \sin^2 \eta}{r^2}\right\}\right]^{3/2}}. \quad (19)$$

As a consequence of conservation of mass flux, the particle density ρ in the beam (or intensity if the beam consists of photons) drops at a given value of r_0 to its new value ρ_0 according to

$$\rho_0 = \frac{r_0^2\, d\Omega}{r_0^2 d\Omega_0}\rho = \frac{1}{\Delta}\frac{\sin \eta}{\sin \eta_0}\rho = \varepsilon\rho \quad (20)$$

where ε can be called attenuation factor. Note that $\varepsilon \to 1/\Delta^2$ as $\eta \to 0$. We wish to mention that this same effect has been found to produce a divergence in the width and a deamplification in the intensity of a pulse from a pulsar in the context of beacon model for pulsed emission (Datta and Kapoor, 1985; Kapoor and Datta, 1985).

DISCUSSION

We have evaluated the integrals in Eqs. (13) and (17) for various values of r_0, with v as the input parameter. The results are presented in Table I and Figure 2. Decollimation is larger for smaller values of r_0 and v considered. We find that the attenuation factor $\varepsilon \simeq 0.1$–0.2, implying a reduction in the particle density in the beam by an order of magnitude. This result holds for values of r_0 upto several times the radius of the marginally bound circular orbit round a Schwarzschild black hole. The effective luminosity inside the beam, $\simeq L/\eta^2$ (Abramowicz and Piran, 1980) accordingly is reduced by a factor $\varepsilon :\simeq L/\eta_0^2$. Since $\Delta(\eta) \simeq$ const for a given r_0 and the range of η of interest here, $\varepsilon(\eta)|_{r_0}$ is const too. Thus decollimation does not lead to a density gradient over the cross-section of the beam at a given value of r_0. The conclusion that can be drawn is that, in the local rest frame, the beam would start out with a density and an effective luminosity

TABLE I

Azimuthal angle of emission and the attenuation factor for various values of r_0, the radial location where acceleration takes place

r_0	v	η	η_0	ε
	0.800	2°	5.29	0.143
		5	13.22	0.144
$4m$	0.900	2	4.93	0.165
		5	12.32	0.166
	0.999	2	4.75	0.177
		5	11.88	0.178
	0.800	2	4.29	0.217
		5	10.73	0.218
$8m$	0.900	2	4.23	0.224
		5	10.57	0.225
	0.999	2	4.19	0.229
		5	10.46	0.229
	0.800	2	4.07	0.241
		5	10.18	0.242
$12m$	0.900	2	4.04	0.245
		5	10.10	0.246
	0.999	2	4.02	0.248
		5	10.05	0.249
	0.800	2	3.95	0.256
		5	9.89	0.257
$16m$	0.900	2	3.93	0.259
		5	9.83	0.259
	0.999	2	3.92	0.261
		5	9.80	0.261

~10 times larger and opening angle smaller by a factor ~2 than assumed, provided $r_0 \leqslant 20m$.

The conclusion, we stress, depends critically on the assumption of geodesic motion and that collimation is decided in a region as close to the black hole as $r_0 \leqslant 20m$. Given this, gravity would be overwhelming any effects in the vicinity of the black hole (say inward of ~$20m$) on the particle motion that may arise due to pressure and magnetic field. The collimation mechanisms invoking magnetic fields, radiative pressure etc. would, therefore, prevail over curvature effects in regions $\gg 20m$ where decollimation in the beam due to gravity serves as an input to the collimation process. It would be interesting to explore in detail such a transition which calls for a solution of the relativistic Euler equations and the baryon conservation equation. A preliminary study reveals t- and φ-components of the Euler equations to be identities, giving r dependent h and γ. Their ratio (the impact parameter of the particle), however, is a constant quantity. A knowledge of the velocity profile is then essential to estimate the degree of

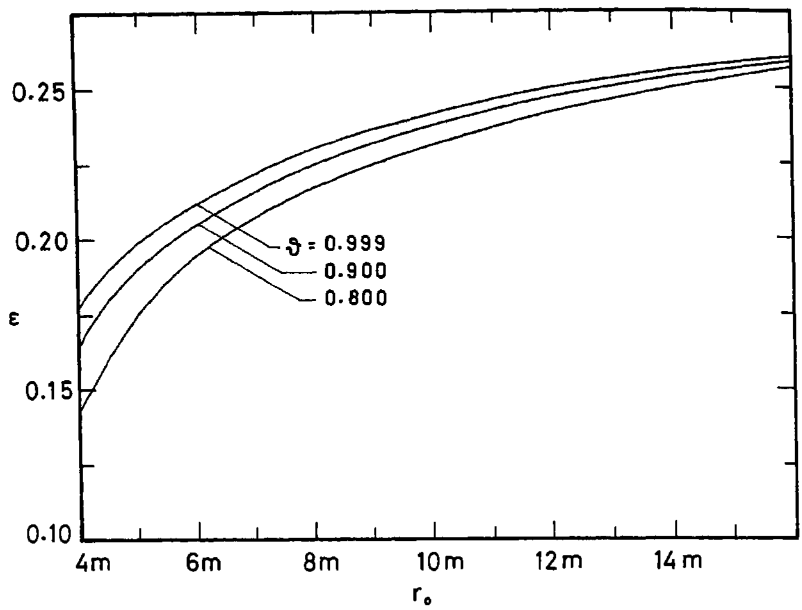

FIGURE 2 Attenuation factor versus r_0, the radial location where acceleration takes place.

gravity induced decollimation. Thus, although results of such a study may differ somewhat qualitatively as well as quantitatively from those presented here, this study underlines the importance of general relativistic decollimation to be a feature inherent to the beam models, leading to a large drop in the pressure and sound velocity in the beam and to the resistance to sideways compression in the vicinity of the black hole.

ACKNOWLEDGEMENTS

The author is grateful to Professor Paul Wiita and Dr. B. Datta for their comments. Thanks are due to Professor Abdus Salam for offering hospitality at the International Centre for Theoretical Physics, Trieste where these ideas were developed.

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