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Does Sunspot Activity Originate in Slow Global Oscillations of the Sun?

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Abstract. We present preliminary results of a spherical-harmonic-Fourier analysis of sunspot activity during the twenty-two years 1933–1954. The results indicate that the sunspot activity might be originating in global solar oscillations with periods of years and decades. However, except for the axisymmetric mode of degree 6, the set of other axisymmetric modes showing ~11 yr periodicities are different from one sunspot cycle to another. A more detailed analysis, preferably with larger data series, will be needed to arrive at a more definite conclusion.

Key words: Sun, global oscillations—sunspots—solar activity cycle

I. Introduction

It has often been suggested that the solar magnetic and activity cycles might be originating in some global oscillations of the Sun rather than in turbulent hydromagnetic processes in the convection zone (*e.g.* Walen 1947; Layzer, Rosner & Doyle 1979; Labonte & Howard 1982). The twenty-two and the eleven year periodicities of the magnetic and activity cycles could be related either to the periods of sufficiently slow global oscillations (*e.g.* Plumpton & Ferraro 1955) or to the timescales of modulation of fast global oscillations (*e.g.* Gokhale 1977, 1984). Theoretically, slow MHD modes of global oscillations could have periods in years and decades depending upon the effective depth and intensity of the magnetic field in the solar interior. In this paper we present preliminary results of a spherical-harmonic-Fourier (SHF) analysis of sunspot activity during the twenty-two years 1933–54, which was undertaken to determine whether some specific modes of global oscillations with ~11 yr or ~22 yr periods contribute predominently to the sunspot activity, assuming such oscillations to exist. Meanwhile, from SHF analysis of Mount-Wilson and Kitt-Peak magnetogram data (1959–1984), Stenflo & Vogel (1986) have shown that global oscillations with specific periods (~22 yr and smaller) do contribute predominently to the evolution of the largescale photospheric field. The present analysis of the sunspot data extends over first ten degrees ($l = 0$ to 9) and all azimuthal orders ($m = 0$ to l) of the spherical harmonics and over frequencies $\nu = 0$ to 9 in units of 1/11 or 1/22 yr^{-1} depending on the length of the data used.

The Fourier amplitudes or powers, (or both), of several harmonics show peaks in their variation with ν. Some of the peaks, though not all, seem to be statistically significant according to a crude preliminary test. Peaks in the amplitudes and powers of the axisymmetric harmonics show that if sunspot activity originates in global solar oscillations, then different sets of harmonic modes show ~11 yr periodicity during different sunspot cycles.

2. The data used and the method of analysis

We have used the data from Ledgers I and II of the Greenwich photoheliographic results. Since the transfer of data from the published volumes to magnetic tapes and its checking is extremely laborious, the data from only two sunspot cycles (1933–1954) could be analyzed so far.

The date and location of the first appearance of each sunspot group is noted as the nearest approximation to the time and location when and where the associated strong magnetic structure is produced inside the Sun. Spot groups born on the invisible side of the Sun and brought into view by solar rotation are omitted from the analysis in view of the large uncertainty in the date of their first appearance on the Sun's surface. This gives us a data set of 2670 spot groups during the twenty-two year period. The epochs t_i (in days, and fractions, from the zero-hour of 1933 January 1), and the heliographic colatitudes and longitudes θ_i and ϕ_i, of the 2670 spotgroups as indexed by i are noted. The maximum observed area $A_{\max}(i)$, corrected for foreshortening, during the entire life of the spotgroup (including the recurrent appearance, if any) is also noted as a measure of the magnetic flux in the structure producing the spot group i. The uncertainties in θ_i, ϕ_i and t_i are ~1°, 1° and 1 day respectively.

The following two functions are subjected to the spherical-harmonic-Fourier analysis:

$$A(\theta, \phi, t) = A_{\max}(i)\delta(\theta - \theta_i, \phi - \phi_i, t - t_i),$$

and

$$p(\theta, \phi, t) = \text{probability distribution of 'sunspot occurrence'},$$
$$\text{over the solar surface, with weights } A_{\max}(i) \text{ at } \theta = \theta_i, \phi = \phi_i, t = t_i,$$

where $\delta(\theta - \theta_i, \phi - \phi_i, t - t_i)$ is the three-dimensional delta function in the space (θ, ϕ, t). The harmonic amplitudes $H_\alpha(l, m, \nu)$ in the SHF analysis of a function $f(\theta, \phi, t)$, over a time-span T, are given by:

$$H_\alpha(l, m, \nu) = C_l^m \int_0^T \mathrm{d}t \int_S f(\theta, \phi, t)\, P_l^m(\cos\theta) {}^{\cos}_{\sin}(m\phi) {}^{\cos}_{\sin}(2\pi\nu t)\, \mathrm{d}S, \tag{1}$$

where l and m are respectively the degree and the azimuthal order of the spherical harmonic, ν is the frequency in units of $\Delta\nu = 1/T$, α is a *symbolic suffix* with values 'cc', 'cs', 'sc' or 'ss', depending upon the combination of cosines or sines of $m\phi$ and $2\pi\nu t$, $\mathrm{d}S$ is an element of area for integration over the surface S of a unit sphere and C_l^m are normalizing factors given by

$$C_l^m = \frac{1}{\sqrt{2\pi}} \left(\frac{2l+1}{2\pi}\right) \frac{(l-m)!}{(l+m)!} \qquad \text{for } m \neq 0 \tag{2}$$

and

$$C_l^o = \frac{1}{\sqrt{2\pi}} \left(\frac{2l+1}{4\pi}\right).$$

For the functions $A(\theta, \phi, t)$ and $p(\theta, \phi, t)$ defined above, expression (1) reduces to

$$H_\alpha(l,m,\nu) = C_l^m \sum_i A_{\max}(i) P_l^m(\cos\theta_i) {}^{\cos}_{\sin}(m\phi_i) {}^{\cos}_{\sin}(2\pi\nu t_i) \sin\theta_i \qquad (3)$$

and

$$H_\alpha(l,m,\nu) = C_l^m \sum_i A_{\max}(i) P_l^m(\cos\theta_i) {}^{\cos}_{\sin}(m\phi_i) {}^{\cos}_{\sin}(2\pi\nu t_i) \qquad (4)$$

respectively. The powers P_c and P_s in the harmonics involving $\cos(m\phi)$ and $\sin(m\phi)$ are calculated using

$$P_c(l,m,\nu) = \sqrt{[H^2_{cc}(l,m,\nu) + H^2_{cs}(l,m,\nu)]}$$

and

$$P_s(l,m,\nu) = \sqrt{[H^2_{sc}(l,m,\nu) + H^2_{ss}(l,m,\nu)]}. \qquad (5)$$

However, since the results for $A(\theta, \phi, t)$ and $p(\theta, \phi, t)$ are qualitatively similar, we present the results for $A(\theta, \phi, t)$ only. In these, the relative uncertainty in each value of H_α, P_c, P_s, is estimated to be < 10 per cent.

3. Results and conclusions

Fig. 1 illustrates the plots of H_{cc}, H_{cs}, H_{sc}, H_{ss}, P_c and P_s as functions of frequency ν given by SHF analysis of $A(\theta, \phi, t)$ for the first twelve harmonics ($l = 0$ to 3 and $m = 0$ to l; $l = 4$, $m = 1$ and 2) obtained from the data of 1281 sunspot groups during the sunspot cycle 1933–43. The abscissa is the frequency ν in units of $\Delta\nu = 0.00025\,\mathrm{d}^{-1}$ ($\sim 1/11\,\mathrm{yr}^{-1}$). The ordinate in each plot is scaled to the range of values of one of the functions represented in the plot. It can be seen that for some modes there are one or more significant peaks at certain frequencies.

Similar peaks are also present in Fig. 2 which shows a sample of the SHF analysis of an 'imaginary' data set produced by taking for θ_i, ϕ_i and t_i, three independent series, each of 1281 random values within the same limits as those for the real data. Hence merely from the presence of the peaks in the results for the real data, one cannot conclude that sunspot activity originates in long-period global oscillations of the Sun. Fig. 3 shows results for the cycle 1944–54.

In Fig. 4 we illustrate the results obtained from the combined data for all the 2670 spot groups during the entire period of the twenty-two years 1933–1954. Here the unit of frequency is $\Delta\nu = 0.000125\,\mathrm{d}^{-1}$ (*i.e.* $1/22\,\mathrm{yr}^{-1}$).

For many modes (especially the axisymmetric modes $m = 0$) in the results from the real data as well as those from the imaginary data it is possible to estimate the noise level, though crudely, by visual inspection. Among these there are several modes for which the results obtained from the real data show significant peaks at frequencies where the imaginary data yields either no peaks or only insignificant peaks. This fact gives an indication that the sunspot activity might be physically related to, (and perhaps originating in), global oscillations represented by the peaks. However a more detailed analysis, preferably with longer data series, will be needed to arrive at a more definite conclusion.

Here we discuss only the peaks at ~ 11 yr and ~ 22 yr periodicities in the Fourier amplitudes $|H_{cc}|$, $|H_{cs}|$, and in the power P_c of the axisymmetric modes (listed in Table 1). These peaks are considered more significant since none of the results from the imaginary data shows any peaks at these periodicities. The amplitudes as well as the

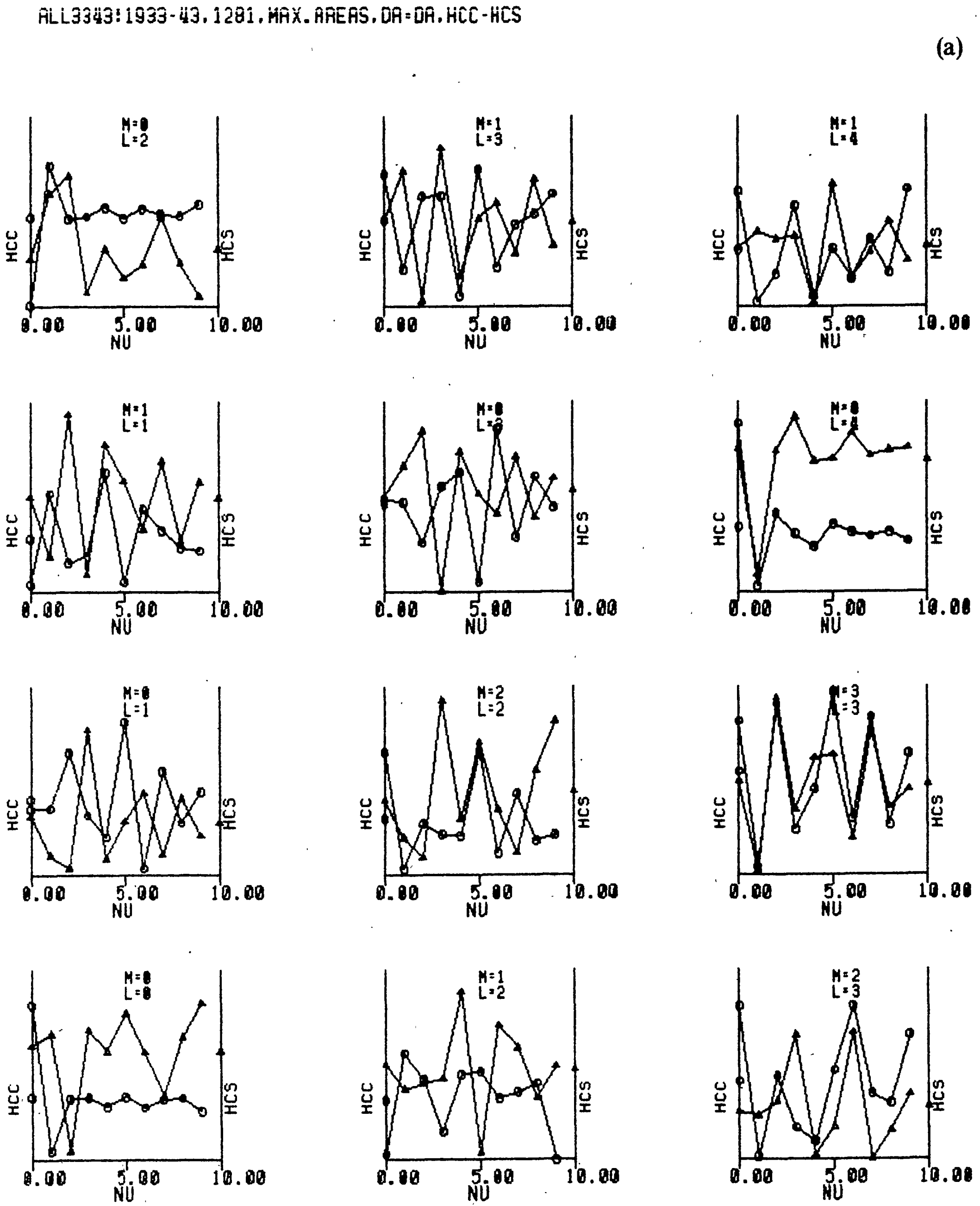
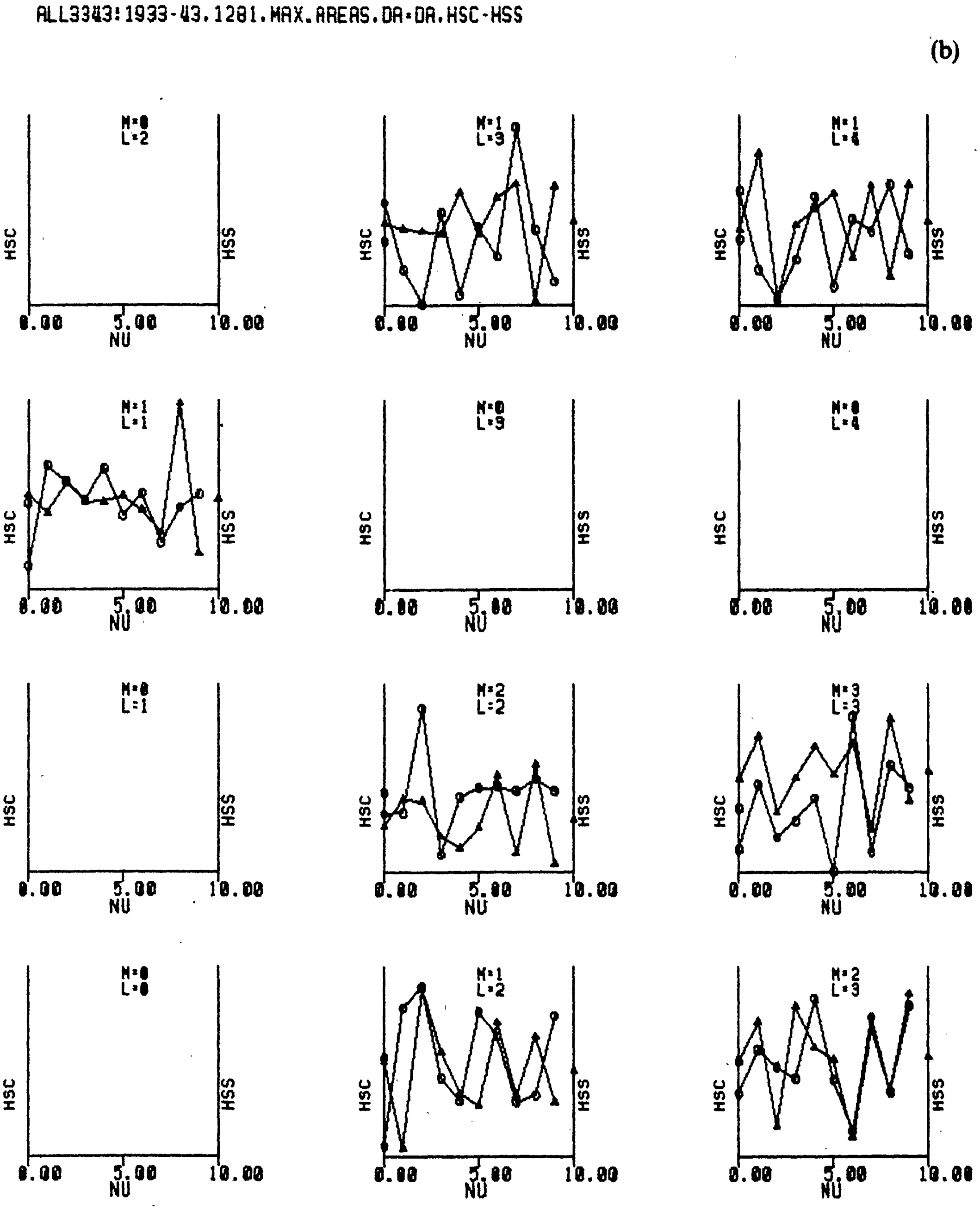


Figure 1. Plots of (a): H_{cc} (denoted by circle) and H_{cs} (denoted by triangle), (b): H_{sc} (circle) and H_{ss} (triangle), (c): P_c (circle) and P_s (triangle), as functions of frequency ν obtained from data for the 1281 spotgroups during 1933–1943. The scale on the ordinate is arbitrary in each plot and differs from one plot to another. The average values of the functions plotted are shown by the symbols circle and triangle on the ordinate. In (a) and (b) these also indicate approximate positions of the zeroes on the respective scales. For axisymmetric ($m = 0$) modes, H_{sc}, H_{ss} and P_s vanish identically. Hence the corresponding plots are blank.



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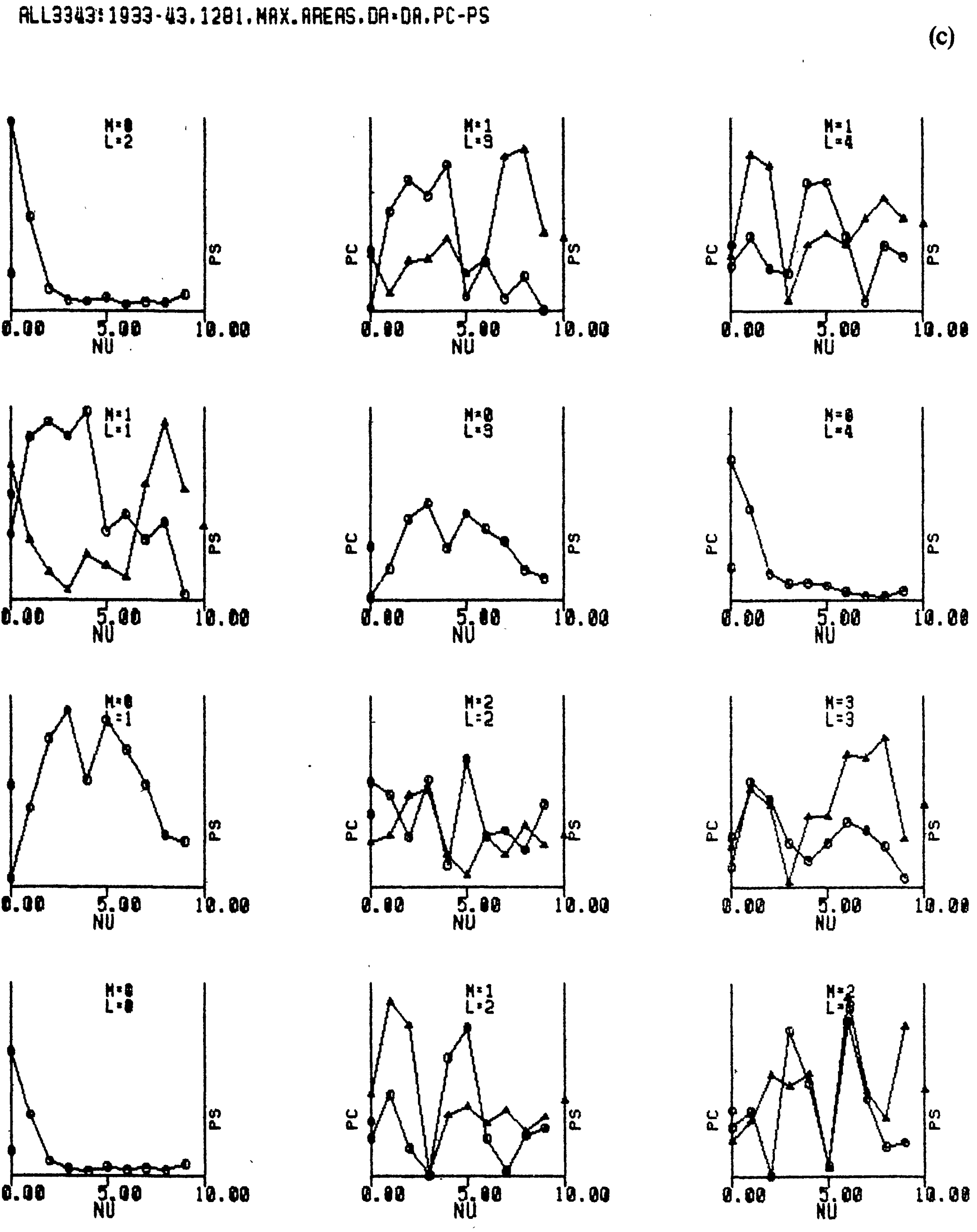

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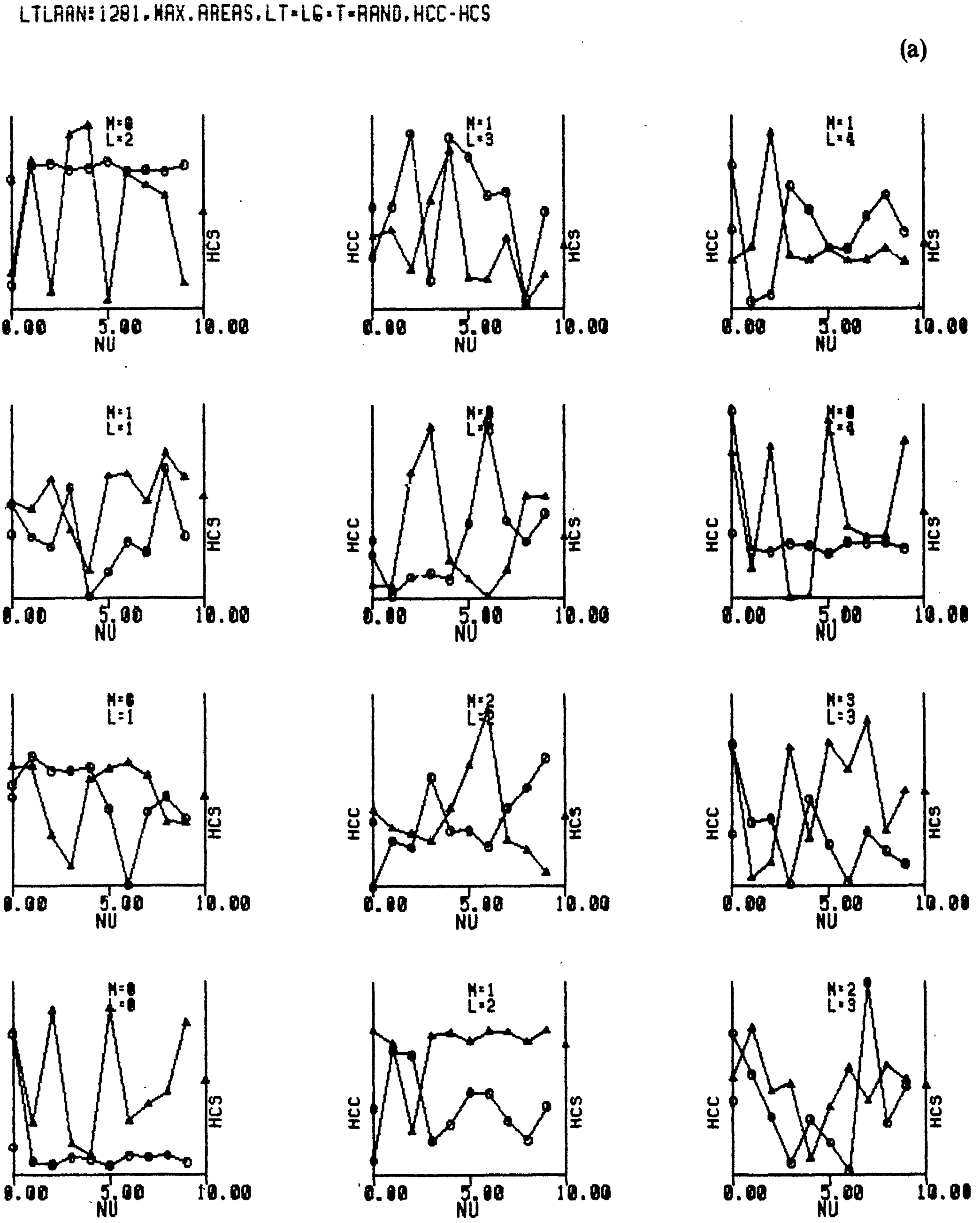
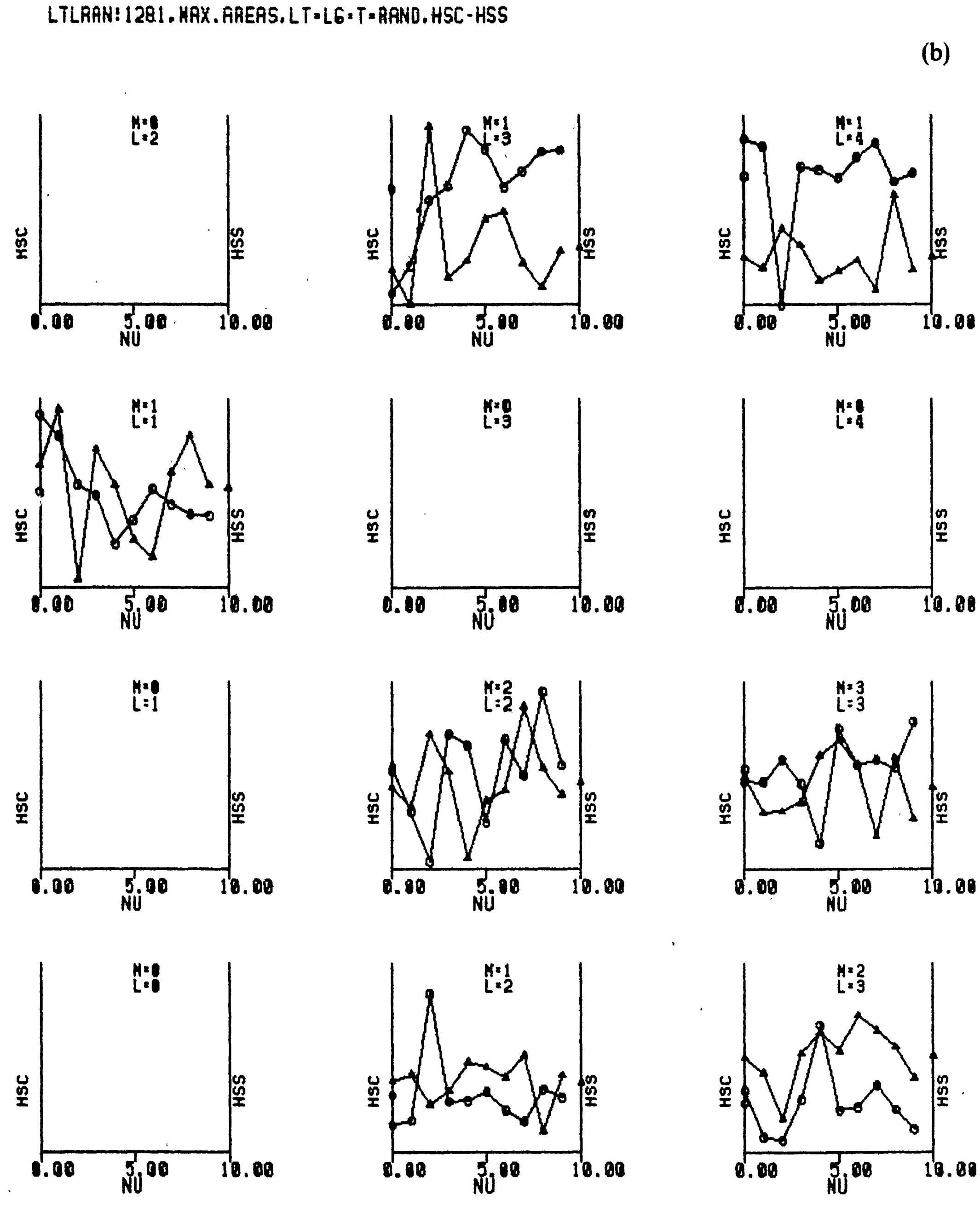


Figure 2. Results of analysis of 'imaginary' data as generated from independent random series, within appropriate limits, for θ_i, ϕ_i and t_i. Compare with the real data in Fig. 1.



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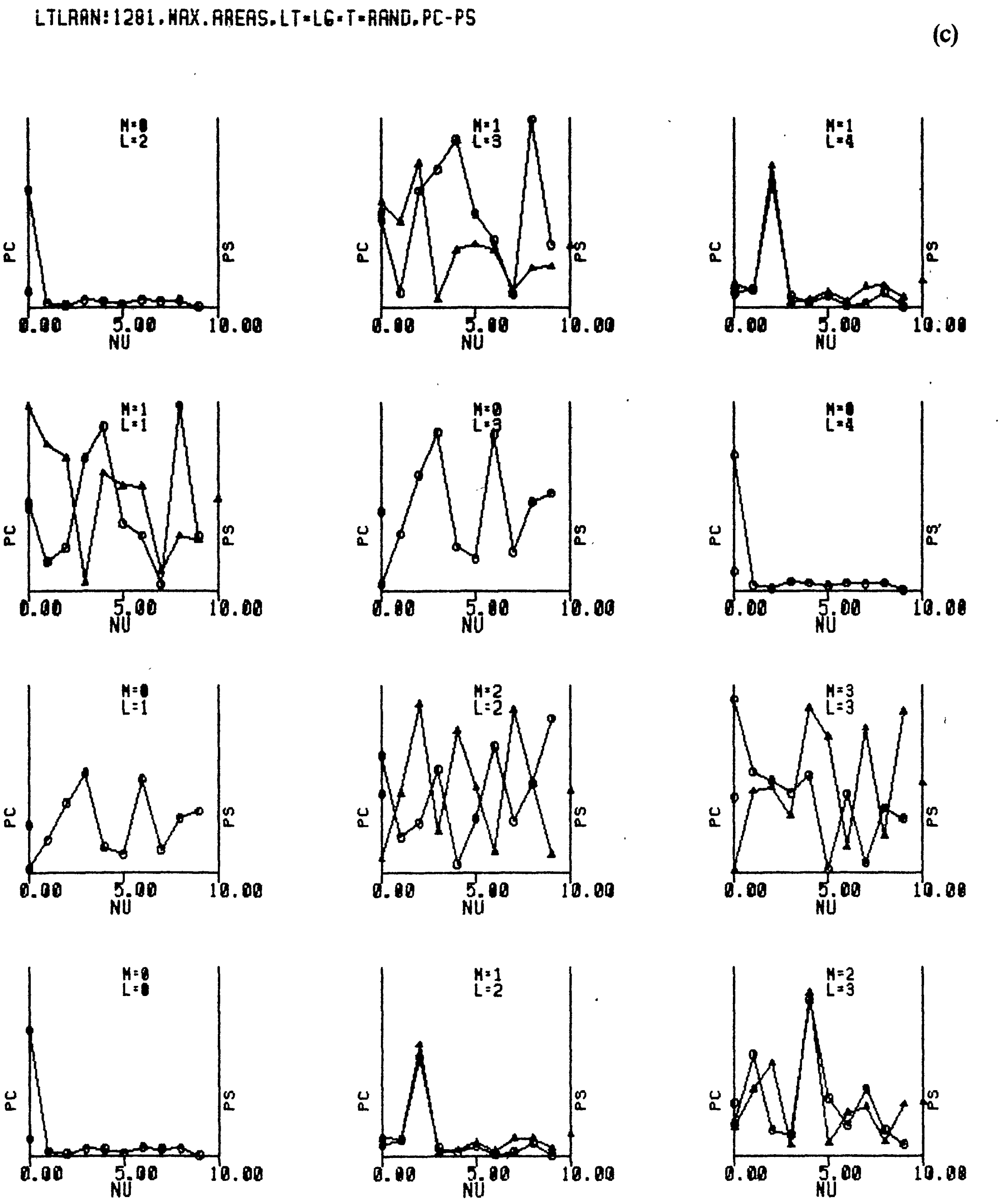

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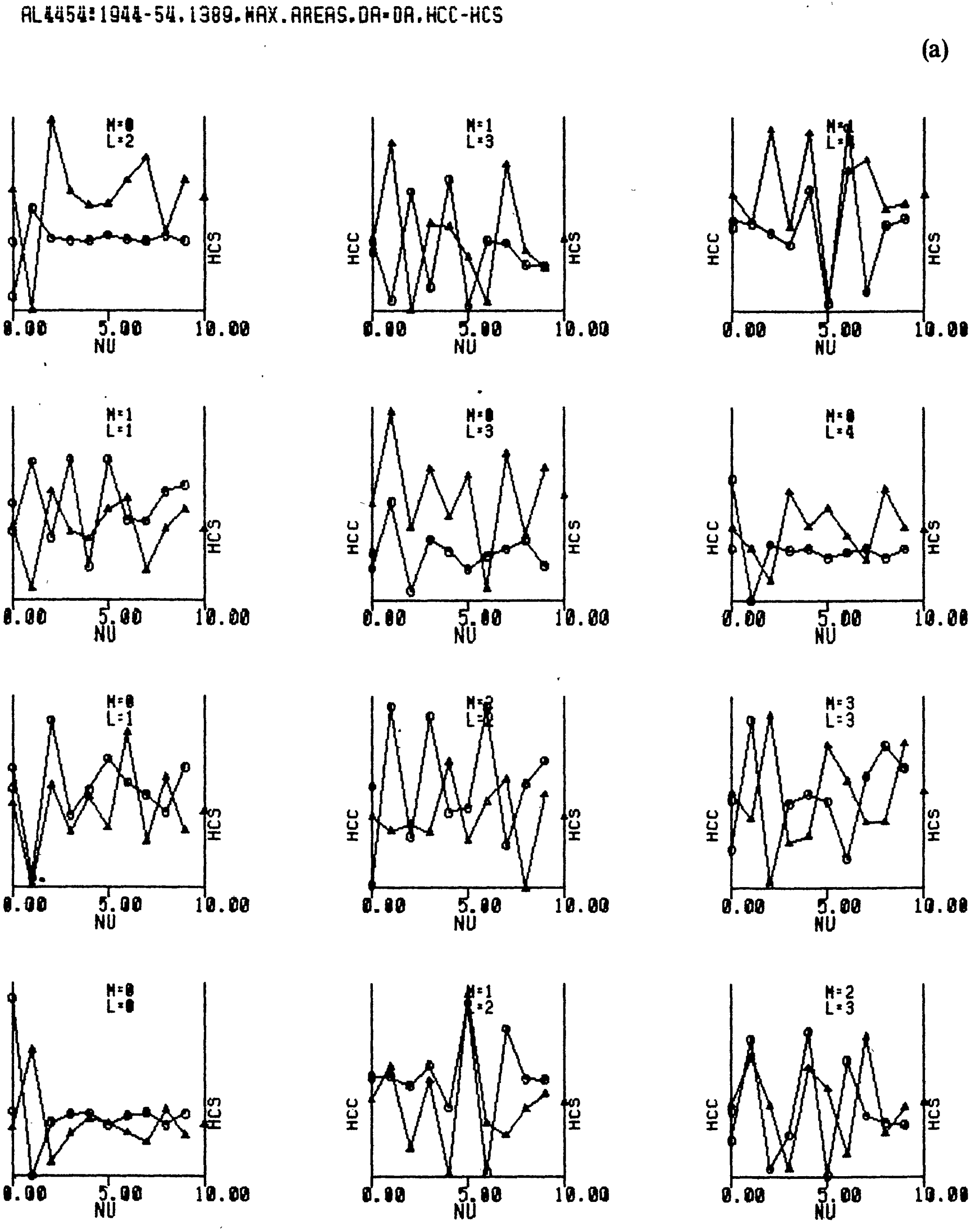
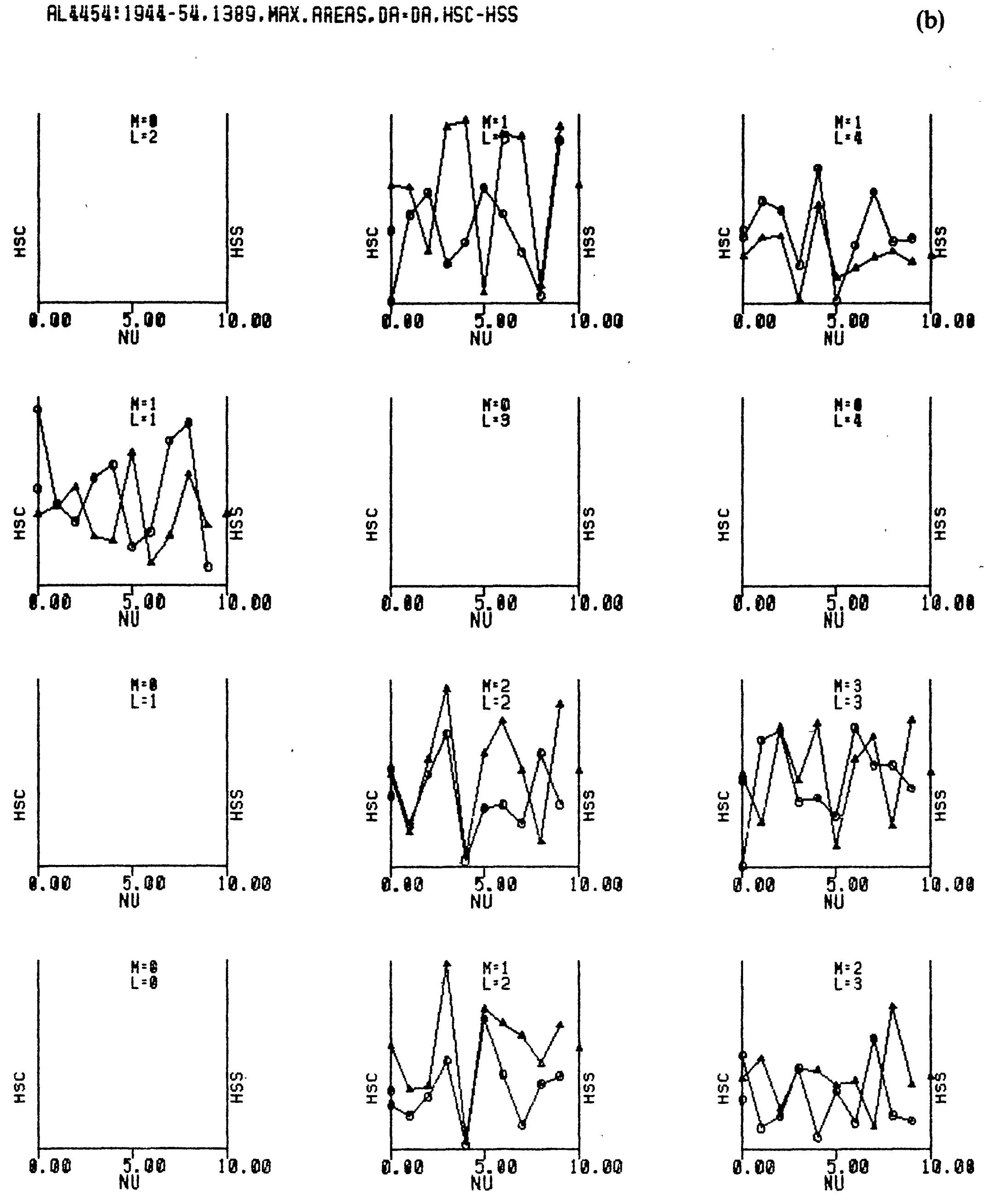


Figure 3. Results of analysis for the data on 1389 spot groups during 1944–1954. See Fig. 1 for explanations.



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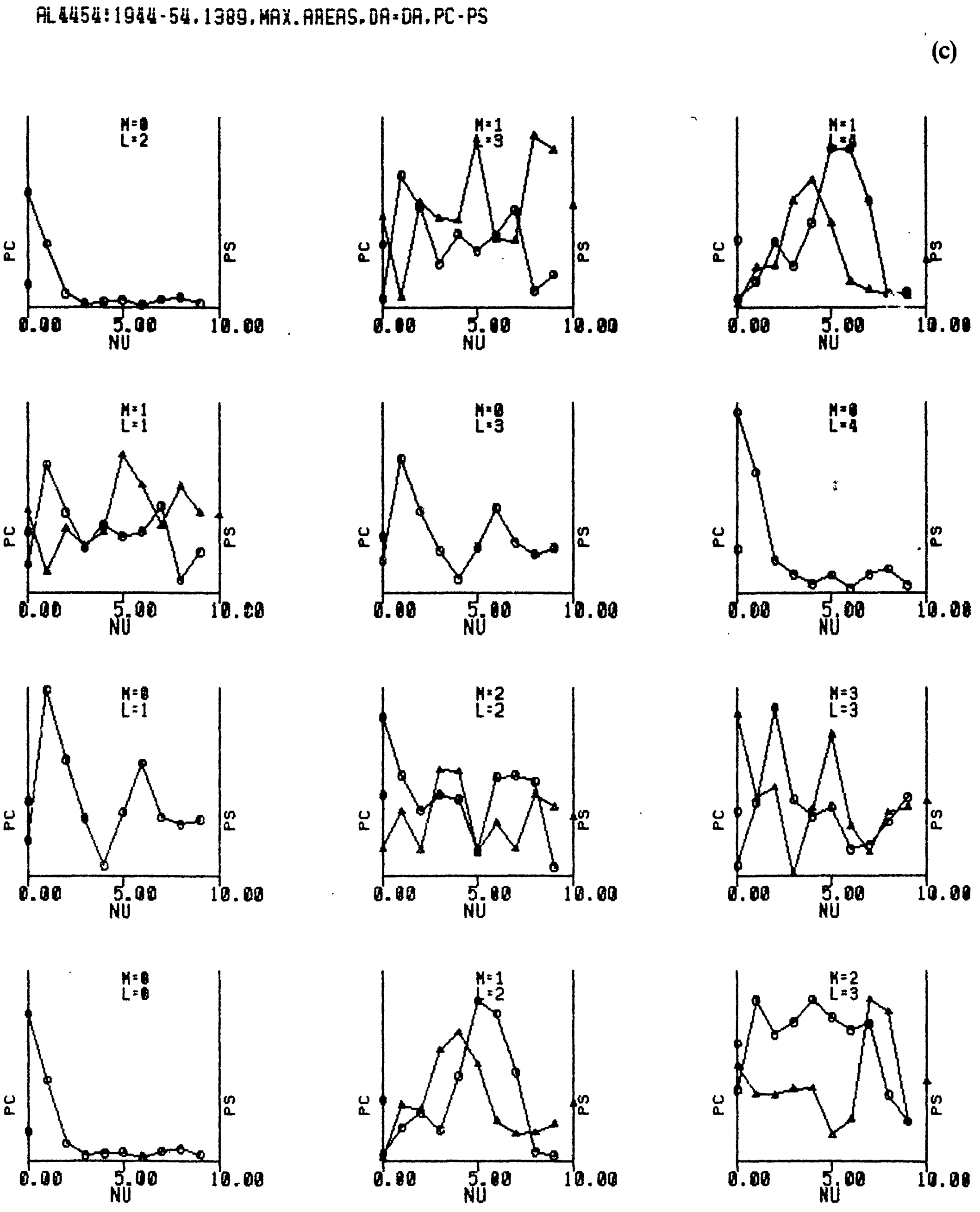

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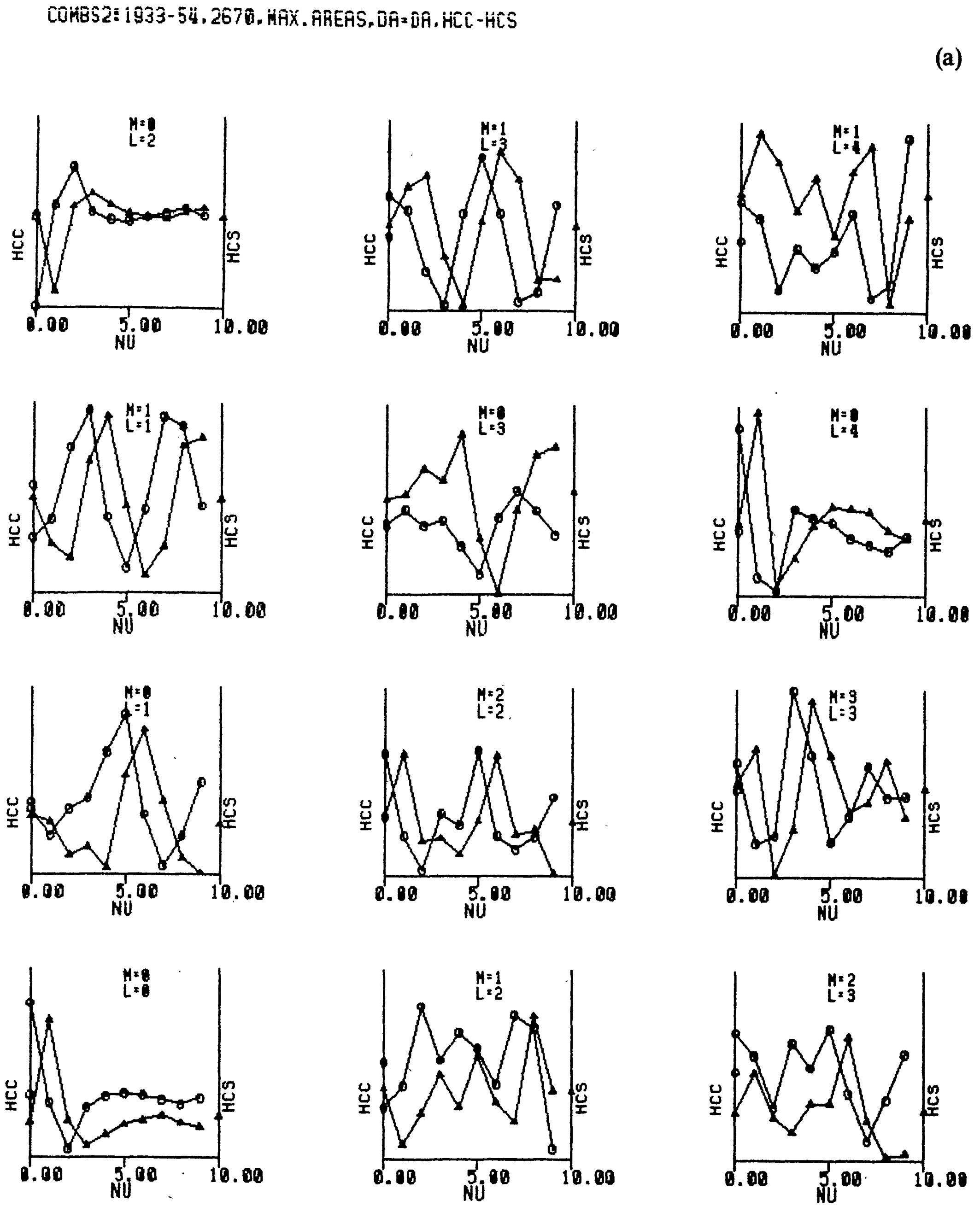
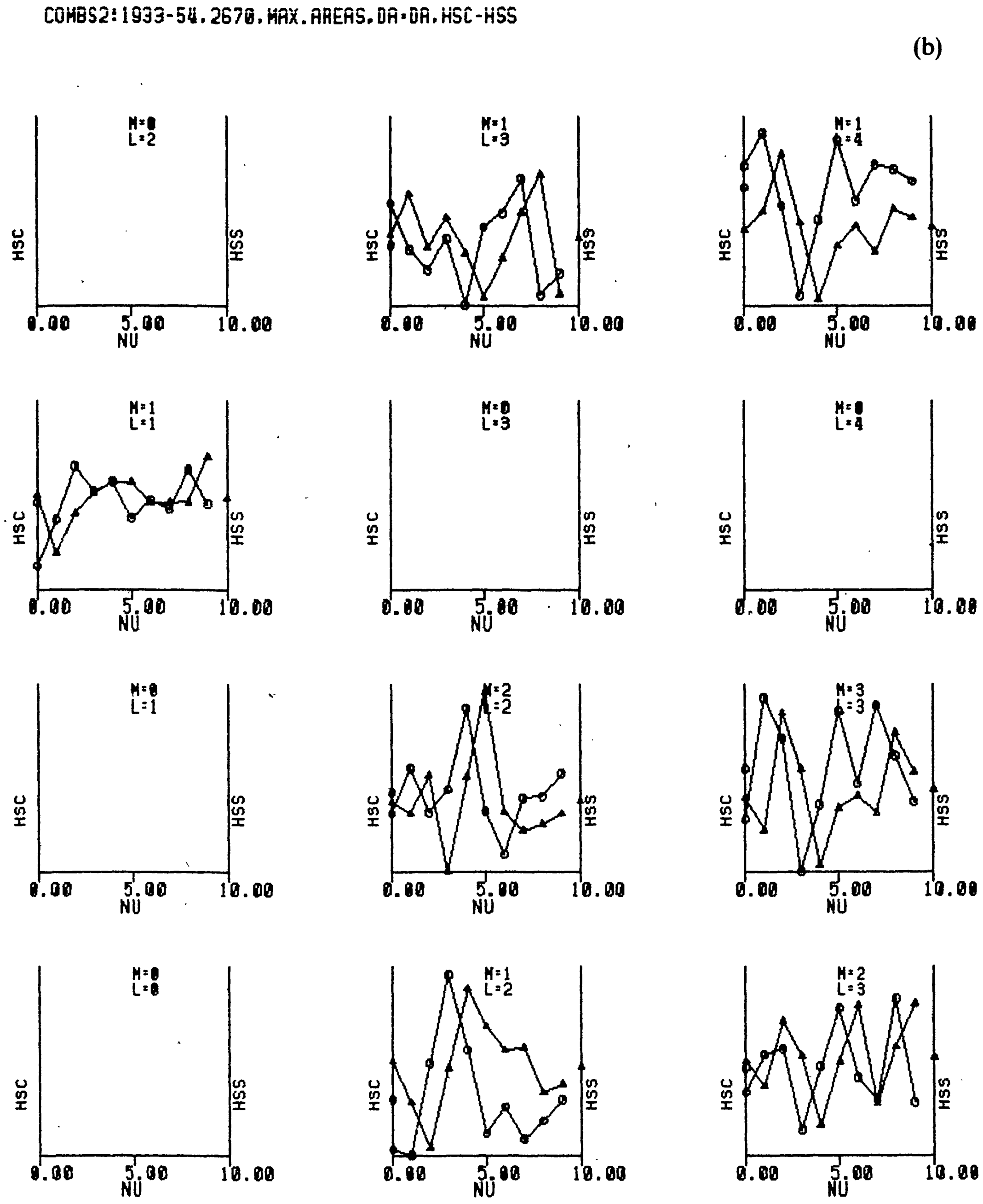


Figure 4. Results for the data on the 2670 spot groups during 1933–1954, the frequency unit being $0.000125\,\mathrm{d}^{-1}$, *i.e.* $\sim 1/22\,\mathrm{yr}^{-1}$. Explanations are similar to Fig. 1.



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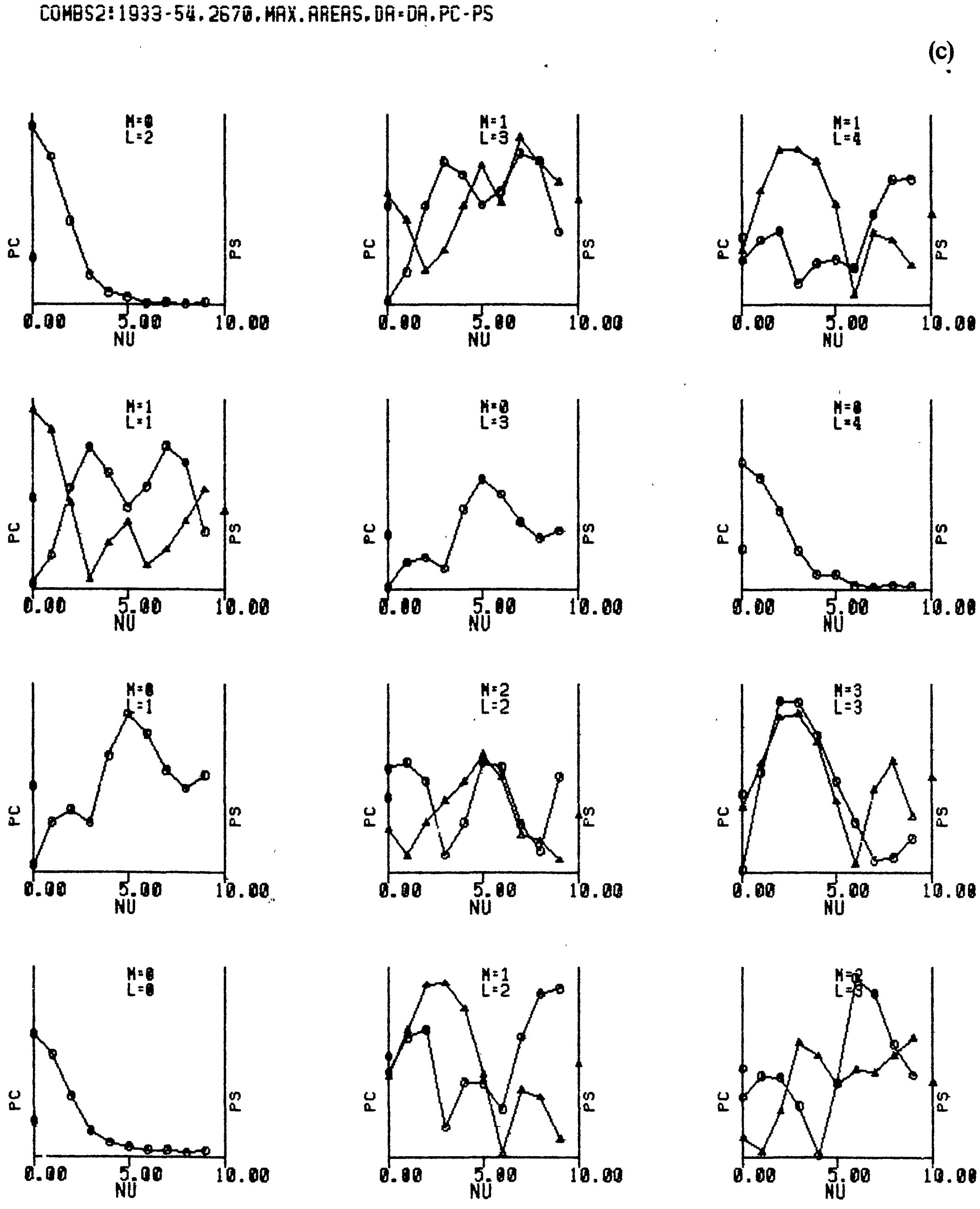

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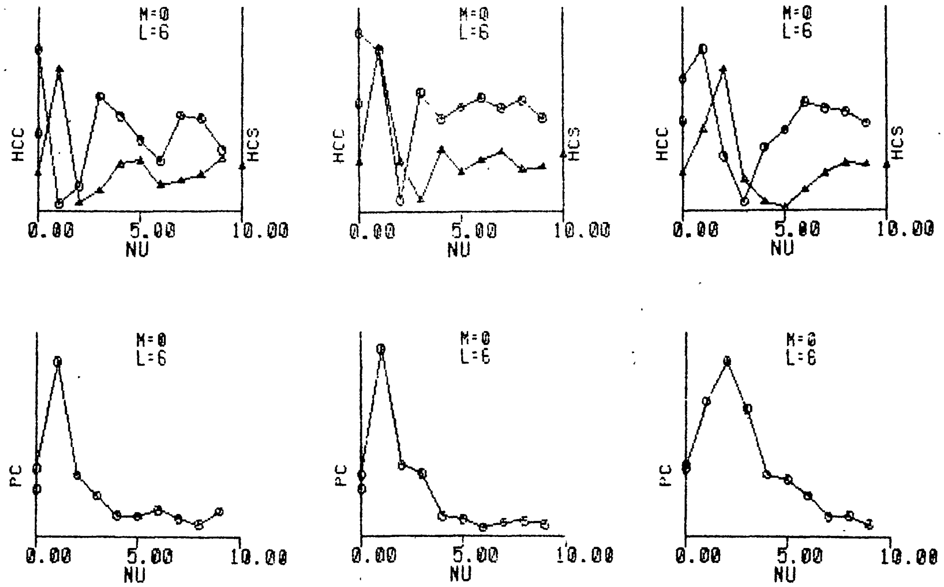

Figure 5. Plots of H_{cc}, H_{cs} (circle and triangle, respectively, in the upper panels) and P_c (circle in the lower panels) for the mode (6,0), obtained from the data during (a) 1933–43, (b) 1944–54, and (c) 1933–54.

Table 1. List of axisymmetric ($m = 0$) modes showing significant peaks at ~11 yr or ~22 yr periodicity in the Fourier amplitudes $|H_{cc}|$, $|H_{cs}|$ and/or in Fourier power P_c.

Data length	Fig.	Periodicity yr	$\|H_{cc}\|$	$\|H_{cs}\|$	$\|P_c\|$
1933–43 (cycle 17)	1	~11	(0,0)		—
			(2,0)		—
			(4,0)	(4,0)	—
			(6,0)	(6,0)	(6,0)
			(8,0)	(8,0)	—
1944–54 (cycle 18)	3	~11	(0,0)	(0,0)	—
			—	—	(1,0)
			(2,0)	(2,0)	—
			(3,0)	(3,0)?	(3,0)
			(4,0)	—	—
			(5,0)	(5,0)?	(5,0)
			(6,0)	(6,0)	(6,0)
			(7,0)	—	(7,0)
			(8,0)	(8,0)	—
			(9,0)	—	—
1933–54 (combined)	4	~11	(0,0)	—	—
			(2,0)?	—	—
			(4,0)	(4,0)	—
			—	(6,0)	(6,0)
			(8,0)	—	—
1933–54 (combined)	4	~22	(2,0)	(0,0)	—
			—	(1,0)	—
			—	(4,0)	—
			(6,0)	—	—
			—	(8,0)	—

power of the mode $l = 6$, $m = 0$ show a significant peak at ~ 11 yr periodicity during each of the cycles 1933–43 and 1944–54. In the analysis of the combined data, this mode shows ~ 11 yr peaks in $|H_{cs}|$ and P_c, and a ~ 22 yr peak in $|H_{cc}|$ (Fig. 5).

From Table 1 it is clear that significant peaks are present at the 11 yr periodicity in both the Fourier amplitudes of all the axisymmetric modes of *even* parity up to (8,0) during 1933–43. On the contrary, during 1944–54 the ~ 11 yr peak is present in a few of the *even* parity modes and most of the *odd* parity modes. During this sunspot cycle the power P_c shows ~ 11 yr periodicity in *all odd parity* modes up to $l = 7$ and in only *one even parity* mode *viz.* $l = 6$. During the *whole* 22 yr period 1933–54, the ~ 22 yr and the ~ 11 yr periodicity peaks are present in the amplitudes of all *even* parity modes up to (8,0) and the power P_c shows a peak at ~ 11 yr only in the (6,0) mode.

Thus if the sunspot cycle really originates in global modes of ~ 11 yr periodicity then the set of such modes responsible for sunspot activity is different from one sunspot cycle to another. On the contrary, as found by Stenflo & Vogel (1986), axisymmetric modes of only *odd* parity exhibit 22 yr periodicity in the evolution of the largescale photospheric field. Whether the difference between the results obtained from the sunspot activity and those from the evolution of the largescale photospheric field is due to the difference in the time intervals analyzed or due to an independence of the two phenomena remains to be seen.

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