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LIMB DARKENING DUE TO THE INCIDENCE OF A PARALLEL BEAM OF RADIATION

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ABSTRACT

We have investigated how the radiation is distributed from the centre to the limb when the atmosphere receives a parallel beam of radiation from infinity. We have considered a purely reflecting surface at the bottom of the atmosphere when the outer layers are illuminated uniformly from all directions. We have considered an electron scattering atmosphere with decreasing density from inner radius towards the outer radius. The atmosphere under question is half the radius of the star itself. The density of the electrons at the bottom of the atmosphere is set to be equal to 10^{13} cm^{-3}. The nature of limb darkening is very similar when $N_e = 10^{12}$ cm^{-3} whereas when $N_e = 10^{14}$ cm^{-3} there is considerable amount of difference between spherical and plane parallel cases. Although the extension of the atmosphere is quite small the differences in the laws of limb darkening are considerable. the differences are further accentuated when the radiation is assumed to be totally reflected from the bottom of the atmosphere. the situation can be very similar to that of a planetary atmosphere.

Key words : law of limb darkening, parallel beam, reflecting surface

1. Introduction

In a series of papers we have investigated how the radiation incident from external sources is reflected from the atmosphere of a binary component. (Peraiah 1982, 1983). There we have considered the radiation coming either from a point source or from an extended surface. This was mainly meant to represent the components of a close binary system. However we know from our experience that the planetary atmosphere receives radiation from distant sun and this radiation is incident in parallel beams. The planetary atmospheres are thin compared to the radius of the planet itself and therefore people have employed plane parallel approximations in obtaining the variation of radiation from centre to limb. However the curvature due to the small atmosphere or due to the geometrically thin atmosphere should be calculated accurately by employing accurate methods of solving the radiative transfer equation. Although it is geometrically thin, as this is nearer to the centre of curvature of the planet this will certainly influence the emergent radiation from the atmosphere because of high curvature. In this paper we shall calculate the radiation observed at infinity by employing both plane parallel and spherically symmetric approximations and make a comparison of the results of these two approximations. We shall also investigate the effects of density changes on the variation of radiation from centre to limb.

2. The Computational Procedure

In Fig. 1 we have given the schematic diagram of how a parallel beam of radiation from infinity is incident on the atmosphere, we will calculate the ray PQ at infinity. One of the parallel rays enters the atmosphere at point T making an angle α with the radius vector OT. The ray traverses the path TPS. We have divided the atmosphere into several spherical shells and P is a point where the ray intersects the boundary of such a spherical shell. We will calculate a ray originating from point P and reaching the observer at infinity along PQ. In doing so we will have to calculate the ray path TP where TP is given by

$$TP = (OT^2 - OQ^2 - OP^2)^{\frac{1}{2}} - OQ \quad (1)$$

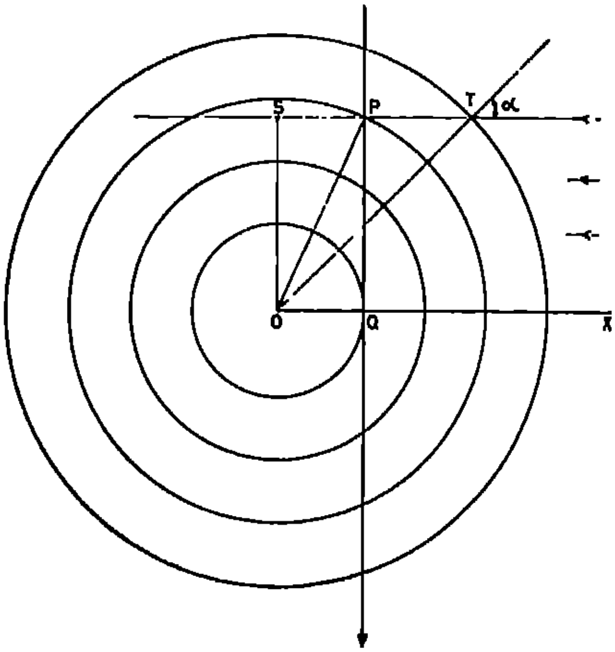

Fig 1. Schematic diagram of the model

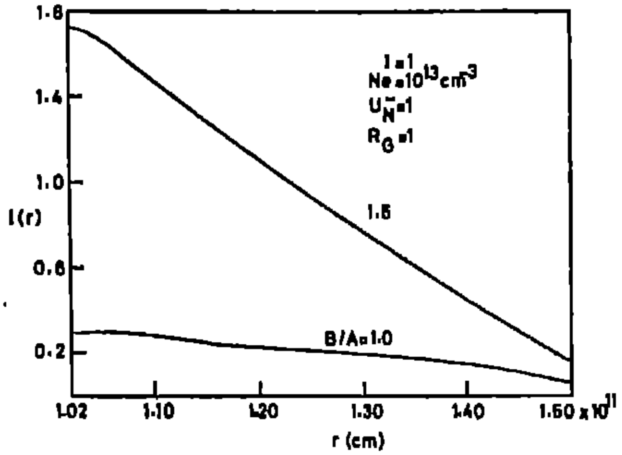

Fig 2. Law of limb darkening for the parameters shown

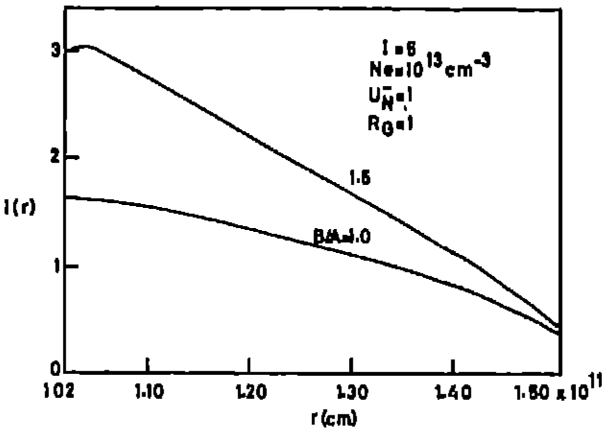

Fig 3. Law of limb darkening for the parameters shown

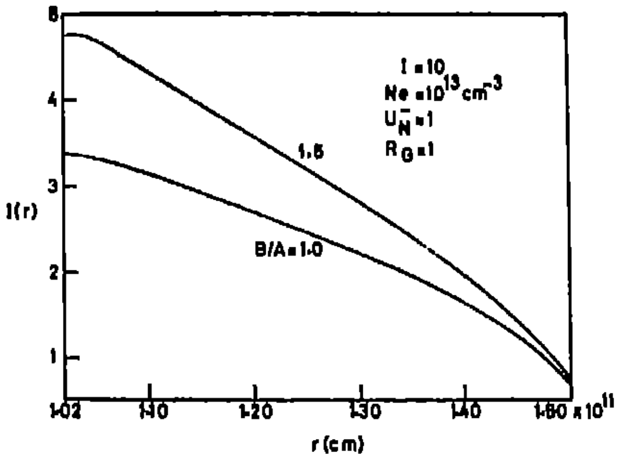

Fig 4. Law of limb darkening for the parameters shown

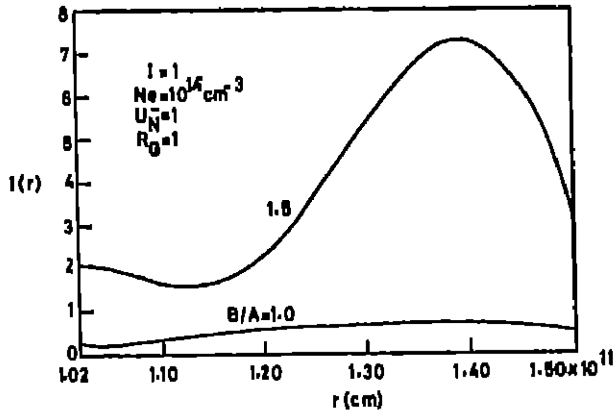

Fig 5. Law of limb darkening for the parameters shown

where OQ is parallel to SP and OT is the outer radius of the atmosphere. We calculated the segments such as TP for all the rays of the beam and calculated their contribution to the source function at the points where the rays along the line of sight intersect the shell boundaries. To calculate the contribution to the total source function due to the incident parallel beam, we have employed the Rod Model (see Peraiah 1982). However we have to calculate the contributions to the total source function from the atmosphere itself. This is calculated by employing the radiative transfer equation in spherical symmetry given by

$$\mu \frac{\partial u\ (r,\mu)}{\partial r} + \frac{1}{r}\frac{\partial}{\partial \mu}[(1-\mu^2)\ u\ (r,\mu)] + \sigma(r)\ u\ (r,\mu)$$

$$= \sigma(r)\ \{[1-\omega(r)]\ B\ (r) + \tfrac{1}{2}\omega\ (r)\int_{-1}^{+1} p(r,\mu,\mu)\ u\ (r,\mu')\ d\mu'\} \quad (2)$$

where

$$u = 4\pi r^2\ I(r,\mu) \quad (3)$$

with $I\ (r,\mu)$ is the specific intensity of the ray making an angle $\cos^{-1}\mu$ with the radius vector r. Here P represents the phase function with isotropic scattering, ω is the albedo for single scattering, B is the Planck function and σ (r) is the absorption coefficient and as we are considering only scattering by electrons the quantity σ (r) becomes scattering coefficient. The equation (2) is solved along the lines described in Peraiah and Grant (1973). The total source function is given by

$$S_T = S_i + S_I \quad (4)$$

By using the total source function S_T at point where the line of sight intersects with the shell boundaries we obtain the distribution of radiation at infinity from centre to limb.

We have assumed a reflecting surface at the inner radius of the atmosphere with operator R_G in which case the incident radiation at the point A would become

$$u_{N+1} = R_G\ U^{+}_{N+1} \quad (5)$$

and

$$U^{+}_{N+1} = [I - r\ (1, N+1)\ R_G]^{-1}\ V^{+}_{N+1} \quad (6)$$

for the derivations and meaning of the symbols, see Peraiah & Grant (1973). Here R_G is the reflecting matrix and for a perfectly reflecting surface we put

$$R_G = I \quad (7)$$

where I is the unit matrix.

3. Results and Discussion

We have presented results in Figs 2 to 15. We have set the inner radius as 10^{11} cms and the outer radius at 1.5×10^{11} cms. In cases where we do not illuminate the outer layers isotropically we have given an incident radiation of unit intensity for all μ's at the inner boundary. The incident radiation from outside is represented by the parameter I [not to be confused with I in equation (7)] which is given values 1, 5 & 10times the radiation incident on the inner boundary. In Fig 1 we have presented the distribution of intensities from centre to limb. For the two cases B/A = 1 and 1.5 where B and A represent outer and inner radii of the atmosphere respectively. The plane parallel calculations show almost a flat variation whereas in the

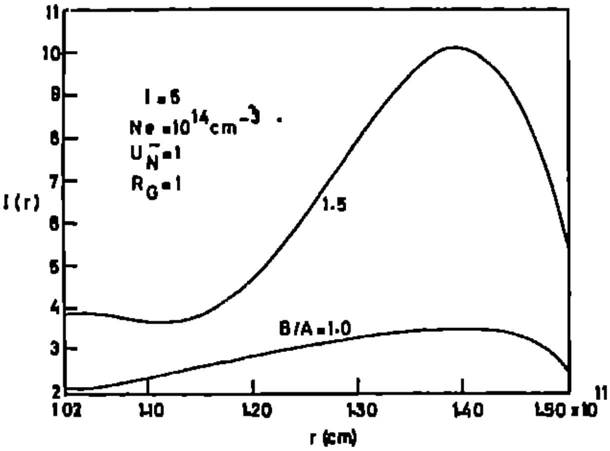

Fig 6. Law of limb darkening for the parameters shown

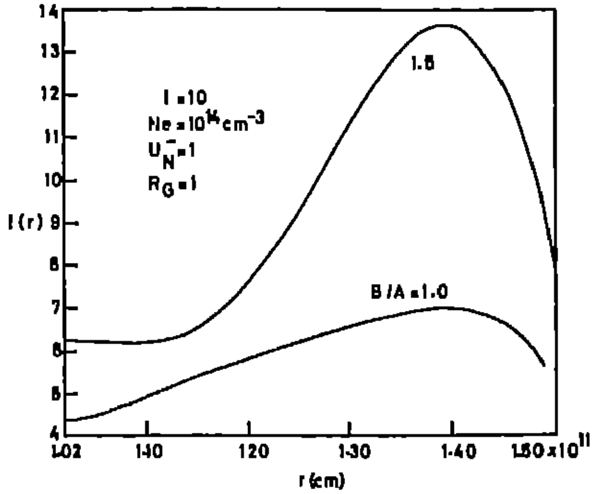

Fig 7. Law of limb darkening for the parameters shown

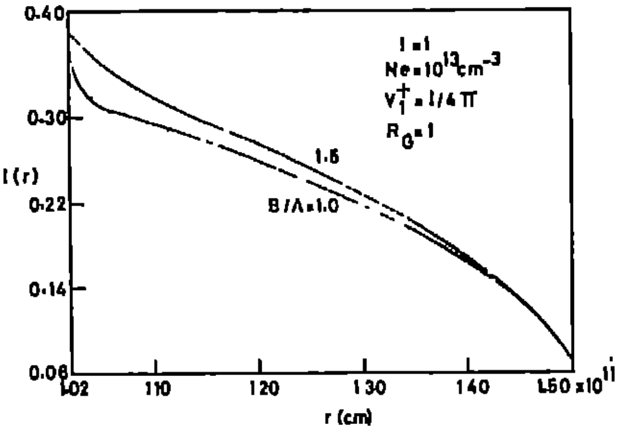

Fig 8. Law of limb darkening for the parameters shown

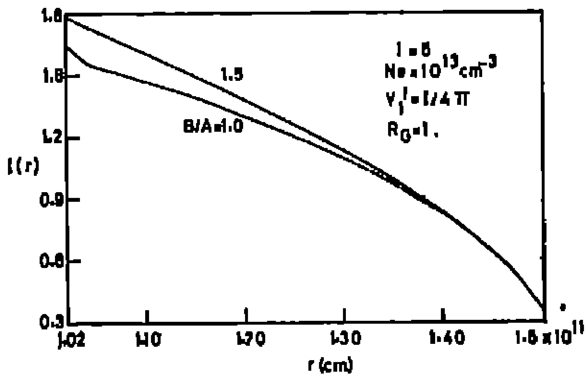

Fig 9. Law of limb darkening for the parameters shown

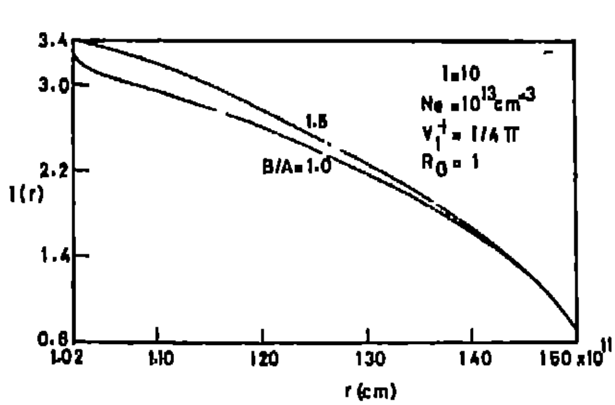

Fig 10. Law of limb darkening for the parameters shown

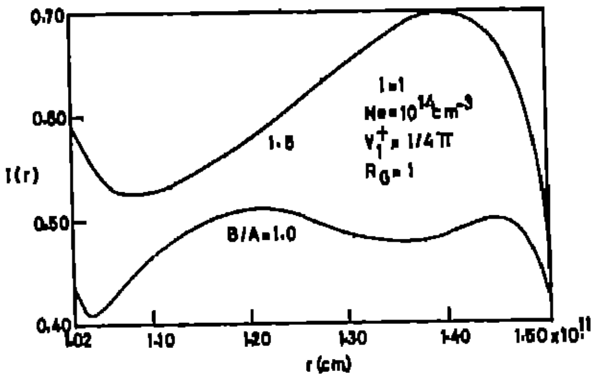

Fig 11. Law of limb darkening for the parameters shown

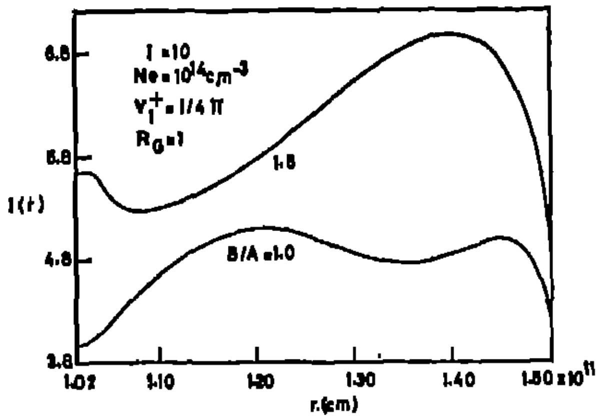

Fig 12. Law of limb darkening for the parameters shown

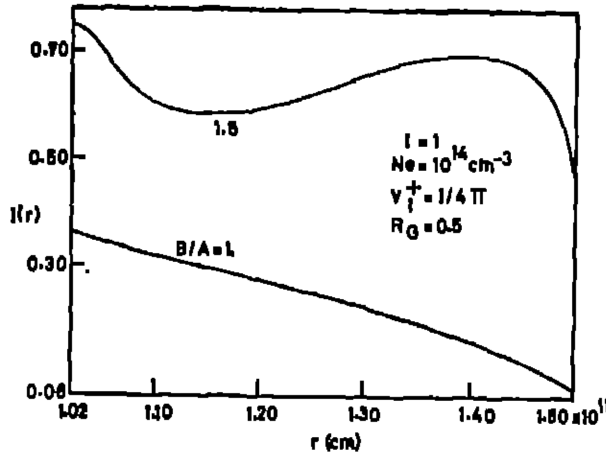

Fig 13. Law of limb darkening for the parameters shown

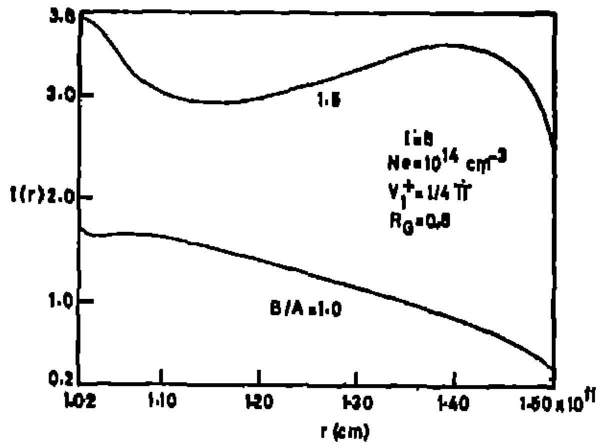

Fig 14 Law of limb darkening for the parameters shown

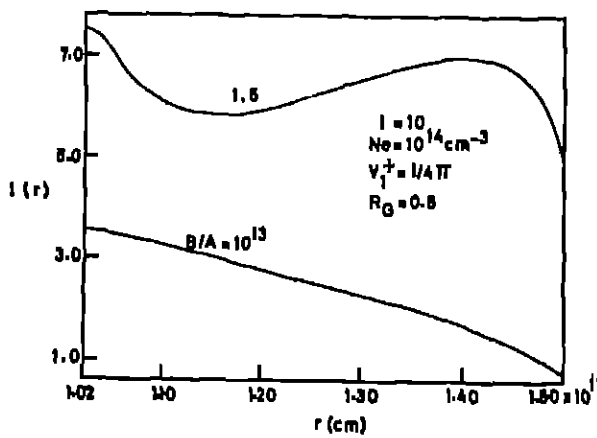

Fig 15. Law of limb darkening for the parameters shown

spherical case there is a steep rise in the intensities towards the centre. Although the differences in the outer layers are quite nominal but these differences increase towards the centre where the spherically symmetric solution gives an intensity nearly six times as large as that given by plane parallel solution.

In Fig 3 we have increased this incident radiation five fold. The difference between a plane parallel and spherical symmetry solution are very similar to those shown in the previous figure but the difference at the outermost layers are small and the differences towards the centre increase only by a factor of two in the case of the solution given by spherical symmetry. In Fig. 4 we have increased the incident beam further by factor of 10 and this reduces the differences between the plane parallel and spherical symmetry quite considerably. This shows that when the atmosphere is flooded by a parallel beam which is of much higher intensity compared to the self radiation of the atmosphere itself the results of plane parallel approximation and spherical symmetry approximation tend to be the same. However when the incident radiation is not as powerful as the self radiation system the differences between the results of plane parallel and spherical symmetry approximation is very considerable. These results should be tested in the case of the planetary atmospheres.

In Figs 5, 6 and 7 we have increased the density from $10^{13} cm^{-3}$ to $10^{14} cm^{-3}$. The characteristics of these results are very different from what we have seen in Figs 2, 3, & 4. The main differences occur in the spherical case. The results of the spherical symmetry show more variation than the result of plane parallel calculation. The plane parallel results remain almost flat from centre to limb whereas the results due to spherical symmetry show a minimum and a maximum between the centre and the limb. The minimum being nearer to the centre and maximum shown towards the outer surface of the atmosphere. When the density is increased the difference between the results of the plane parallel and spherical symmetry calculation persist whether the intensity of the incident beam is increased or reduced.

In Figs 8, 9, and 10 we have shown the results due to a perfectly reflecting surface at the bottom whereas the outer layers are illuminated isotropically. Here the difference between the results of plane parallel and spherical symmetry are not as conspicuous as in the previous cases. Although some differences between those two cases exists towards the centre and here we have taken $R_a = 1$ and $N_e = 10^{13} cm^{-3}$. In figs 11 and 12 we have plotted the law of limb darkening with $N_e = 10^{14} cm^{-3}$ with $R_a = 1$ and here we see strange behaviour by the plane parallel solution which cannot be explained whereas the results of spherical symmetry solution show the same characteristic as seen in Figs 5, 6 and 7. In Figs 13, 14 and 15 we have employed $R_a = .5$ and drawn the curves for $\frac{B}{A} = 1.5$ and $1 = 1, 5$ and 10.

We can clearly see how the electron density changes the law of limb darkening with $N_e = 10^{13} cm^{-3}$, we get almost a linear variation but with $N_e = 10^{14} cm$ we obtain a law of limb darkening with minimum and maximum variations from centre to limb.

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