ond paragraph above follows from the observation that in all the three cases the energy relations can be cast in the same form in terms of ψ:

$$\text{potential (magnetic) energy density} = \tfrac{1}{2}a\left(\frac{\partial\psi}{\partial x}\right)^2, \quad (1)$$

$$\text{kinetic (electric) energy density} = \tfrac{1}{2}b\left(\frac{\partial\psi}{\partial t}\right)^2, \quad (2)$$

power transmission rate (Poynting vector)

$$= -a\left(\frac{\partial\psi}{\partial x}\right)\left(\frac{\partial\psi}{\partial t}\right), \quad (3)$$

longitudinal momentum density

$$= -b\left(\frac{\partial\psi}{\partial x}\right)\left(\frac{\partial\psi}{\partial t}\right). \quad (4)$$

(The meaning of the symbols is given in Table I.)

Since in a progressive wave $\psi = f(x \mp ct)$, we note that

$$\left(\frac{\partial\psi}{\partial x}\right)^2 = \frac{1}{c^2}\left(\frac{\partial\psi}{\partial t}\right)^2. \quad (5)$$

From Table I we find that in each case

$$a/b = c^2. \quad (6)$$

It follows from Eqs. (1) and (2) together with Eqs. (5) and (6) that the potential (magnetic) energy density in a progressive wave is equal to the kinetic (electric) energy density. Further, from Eqs. (3), (4), and (6) we note that the ratio of the power transmission rate and the momentum density in each case is equal to the square of the wave velocity (c^2).

In addition, in the radiation gauge, the Lagrangian and Hamiltonian densities of a light wave are also formally identical to the corresponding expressions for a string wave or a sound wave. Finally, just as Benumof cautions that the energy distribution in a continuous mechanical wave is very much different from that in a single discrete spring–mass system, we wish to remark that the energy distribution in a continuous electromagnetic wave is exactly as much different from that in a lumped LC circuit (which is the electrical analog of the spring–mass system).

[1]Reuben Benumof, Am. J. Phys. **48**, 387 (1980).
[2]See the problem no. 7-9 in Brono Rossi; *Optics* (Addison-Wesley, Reading, MA, 1957), p. 344.

Cosmological constraint on black hole temperatures

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In a recent paper[1] a curious relation to which Weinberg[2] drew attention in his book [Eq. (16.4.2)] was sought to be understood as an operational requirement that the mass of an elementary particle be such that its gravitational self-energy be at least measurable over a Hubble period. The relation is[2]

$$m = (\hbar^2 H_0/Gc)^{1/3} \simeq m_\pi . \quad (1)$$

It connects the mass m of a typical elementary particle and the fundamental constants c, G, and $\hbar$ to a single cosmological parameter H_0, the Hubble constant. We point out in this note that Eq. (1), arises in a natural manner while considering the evaporation of black holes due to Hawking radiation,[3] a typical quantum gravity phenomenon. On the basis of analogies between thermodynamic quantities and the parameters of black holes dynamics, Bekenstein[4] pointed out that a black hole should be assigned a temperature (proportional to its surface gravity) given by

$$T = \hbar c^3/GMk_B , \quad (2)$$

M being the black hole mass and k_B is the Boltzmann constant. Direct calculations by Hawking[3] showed that a black hole does radiate like a blackbody with temperature given by Eq. (2). This would give a black hole of mass M a finite lifetime t given by

$$t = G^2M^3/\hbar c^4 . \quad (3)$$

The only manner in which black holes can form in the universe at the present epoch is by the collapse of stellar bodies greater than a solar mass. For black holes of a solar mass or more which are the only ones forming now, the temperature is only $\sim 10^{-7}$ °K and the Hawking evaporation is ridiculously small. The only black holes for which the Hawking effect is significant are the "mini" ones formed in the early stages of the big bang due to pressure and density fluctuations.[5] The lightest among these would have already evaporated. The hottest black holes in the universe at the present epoch would be those with masses corresponding to lifetimes of the order of a Hubble age ($1/H_0$), which means, from Eq. (3), their mass would be given by

$$M = (\hbar c^4/G^2H_0)^{1/3} . \quad (4)$$

The corresponding temperature would be, from Eq. (2),

$$T = \frac{1}{k_B}\left(\frac{\hbar^2 H_0}{Gc}\right)^{1/3} c^2 \simeq \frac{m_\pi c^2}{k_B} ; \quad (5)$$

the quantity within parentheses having the dimensions of mass. Comparing Eq. (5) with Eq. (1), we see that the relation for m naturally arises in Eq. (5); the black hole temperature being constrained by its lifetime corresponding to that of a Hubble age. Thermodynamic bootstrap models of hadrons such as the Hagedorn model give a limiting temperature for hadronic matter[6] (known as the Hagedorn temperature) which is $\sim k_B T_{max} \sim m_\pi c^2 \sim 10^{12}$ °K.

Thus by a curious coincidence, blackholes with an evaporation time of the Hubble age have a temperature which is the same as the Hagedorn temperature.

[1]C. Sivaram, Am. J. Phys. **50**, 279 (1982).
[2]S. Weinberg, *Gravitation and Cosmology* (Wiley, New York, 1972).
[3]S. W. Hawking, Commun. Math. Phys. **43**, 199 (1975).
[4]J. D. Bekenstein, Phys. Rev. D **I**, 2333 (1973).

[5]Ya. B. Zeldovich and I. D. Novikov, *Relativistic Astrophysics* (University of Chicago, Chicago, 1971).

[6]D. C. Kelly, *Thermodynamics and Statistical Physics* (Academic, New York, 1973), p. 404.

The discovery of synchrotron radiation

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Thirty-five years ago the electromagnetic radiation that results from the acceleration of electrons in a circular accelerator was observed for the first time in a 70-MeV synchrotron at the General Electric Research Laboratory in Schenectady, NY. In May 1981, an entire issue of *Physics Today* was devoted to synchrotron radiation, which is widely recognized as an important research tool for physicists, chemists, and biologists and perhaps in medicine as higher-energy synchrotrons and electron storage rings have been constructed. It seems timely to review the background of its discovery at this laboratory and to record the exact circumstances of the first visual observation and measurements of the radiation.

Before discussing the first observation of synchrotron radiation from a laboratory machine it should be noted that for centuries man had been seeing synchrotron radiation from stars or galaxies without knowing that some of their light resulted from the acceleration of elementary particles in the large magnetic fields associated with astronomical objects.

In 1898 Liénard[1] first pointed out that an electric charge moving in a circular path should radiate energy and he calculated the rate of radiation from the centripetal acceleration of an electron. The theory was extended subsequently by Schott,[2] who received the Adams Prize in 1908 at Cambridge University for his essay, "The Radiation from Electric Systems or Ions in Accelerated Motion and the Mechanical Reactions on their Motion which Arise from It." Schott, attempting to provide the background for an electron theory of matter, calculated the amount and the angular distribution of radiation from relativistic electrons grouped in various ways in orbits of proposed atomic models.

Three decades later, when the building of multimillion volt accelerators began, the classical radiation loss of accelerated electrons again received attention. Circular electron accelerators of various designs were proposed, by Slepian (1922) at Westinghouse, by Wideroe (1928) in Norway, and by Kerst and Serber[3] (1941) at the University of Illinois. The first such machine which was successful was the 2.3-MeV betatron which Kerst built at Illinois. In this machine radiation loss from the electrons was so small that it could be neglected. With the building of larger electron accelerators the increase of radiation loss, as the fourth power of energy for relativistic electrons, became a serious matter. Two Russians, Ivanenko and Pomeranchuk,[4] pointed out in a letter to the *Physical Review* in 1944 that radiation loss would indeed place an energy limit on betatron design.

At the time William D. Coolidge, the eminent x-ray-tube pioneer and inventor of ductile tungsten, who was the Director of this laboratory, had initiated the construction of a 100-MeV betatron in Schenectady. This large induction accelerator for x-ray and nuclear research was designed by Westendorp and Charlton.[5] A GE physicist, J. P. Blewett, who had seen the Russians' paper, urged that an experimental test be made of their predictions. When the machine came into operation Blewett,[6] believing that a total radiation power of about 1 W might be available for detection, searched the radio spectrum from 50 to 1000 megacycles with receivers capable of detecting less than 10 μW. No radiation was detected. It was known that near the peak energy in the betatron the beam orbit began to shrink and the electrons impinged on a target inside their stable orbit. Blewett showed the orbit contraction was consistent with the radiation loss predicted by Ivanenko and Pomeranchuk. He also showed that the deflection current in orbit contraction coils on the machine pole faces was consistent with an orbit size reduced by classical radiation. At about this time Schwinger[7] of Harvard worked out in great detail the theory of the classical radiation of accelerated electrons. The calculations, made available to Blewett and others, but not published until 1949, made it clear that the radiated energy would not peak in the low harmonics of the orbit frequency where Blewett had searched but in the near infrared or in the visible spectrum. If the 100-MeV betatron had been built with a transparent glass vacuum tube, as was a 70-MeV synchrotron in 1946, synchrotron radiation today would be called betatron radiation.

Why was a 70-MeV synchrotron built in 1946 in a laboratory which already had a 100-MeV betatron in successful operation? Several GE physicists and engineers had been assigned by Coolidge in 1943 to work at Berkeley on the Manhattan Project research directed by Ernest O. Lawrence. After the war, in late 1945, Lawrence made one of his frequent Schenectady visits and at a seminar with these physicists and others discussed the principle of synchrotron acceleration, recently proposed by McMillan[8] at Berkeley. McMillan and Lawrence were beginning to plan construction of a 300-MeV synchrotron for nuclear research. The magnetic guide field of a synchrotron would be similar to that of a betatron but the electrons would be accelerated by rf voltage between dees, or in cavity resonators, rather than by magnetic induction. McMillan believed the electrons would accept energy from the rf system so as to maintain a stable orbit, its size defined by the frequency, and he also thought the synchrotron phase stability principle would compensate for classical radiation losses. During a brief discussion of ways to inject electrons into the proposed machine, Pollock[9] suggested to Lawrence that induction acceleration up to 2 MeV in each magnetic cycle could bring the electron velocity to approximately 98% that of light, at which time in the cycle a fixed frequency oscillator might bunch the beam and continue the acceleration. After the seminar Willem Westendorp

