RADIO EMISSION BY PARALLEL ACCELERATION MECHANISM

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Abstract. A generalized version of the linear (or parallel) acceleration mechanism for emission of radiation is presented. This facilitates harmonic generation and includes the effects due to the relaxation of the dipole-approximation. The results are applied to the solar radio bursts of type III and IIIb.

1. Introduction

In the polar-cap models of pulsars, electrons, positrons and protons are believed to be accelerated by electric fields parallel to the magnetic field. The emission by these accelerated particles is then responsible for the pulsar radio radiation. Coherence in emission is achieved either by antenna mechanism or by the maser effect. The linear acceleration mechanism essentially amounts to the calculation of the trajectory of the particle under the action of an electric field parallel to the magnetic field. In the case of uniform magnetic field, only the direction of the magnetic field appears since the transverse motion of the particles is neglected. While considering this mechanism (Melrose, 1980b), the external electric field is assumed to have only the time dependence, which means the treatment is valid within the dipole approximation.

The present treatment includes spatial dependence of the electric field. The spatial dependence is chosen to be of the form $e^{i\mathbf{q}\cdot\mathbf{r}}$ since the idea is to consider the acceleration of the particles by electromagnetic or electrostatic waves, the situation likely to be present, in many astrophysical contexts. Here, we give one application from the sporadic solar radio emission in the form of type III and IIIb radio bursts. Briefly, type III radio bursts are believed to be generated by the Rayleigh scattering of electrostatic waves produced via electron, beam-plasma instability. Type IIIb, with their frequency structure result from the trapping of the particles in the finite amplitude electrostatic waves at the plasma frequency, Smith and de la Noe (1976). An alternative way of looking at the system giving rise to type IIIb was presented where the electron beam was replaced by the electric field, it generates and the plasma dielectric function was obtained in the combined field of oscillating electric and parallel magnetic field, Krishan (1980), Krishan *et al.* (1980). Here, we calculate the radiation from a single particle in the presence of the electric field generated by the beam-plasma instability during its saturation phase. The validity of the constancy of the magnitude of the electric field during the saturation stage has been discussed earlier in the solar atmosphere conditions, Krishan (1980). In addition to relaxing the dipole approximation, the particle trajectory is determined to a higher approximation, which admits harmonic generation.

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In the first section, we derive a general expresion for the power emitted by an electron in the spatially and temporally periodic electric field parallel to the ambient magnetic field. Application to solar radio emission is discussed in Section 3.

2. Emission Spectrum

Let the electric field be of the form $\mathbf{E} = E_0 \mathbf{b} e^{-i(\omega_0 t - \mathbf{q}\cdot\mathbf{r})}$, where $\mathbf{b}$ is a unit vector pointing in the direction of the magnetic field, ω_0 and $\mathbf{q}$ are the frequency and the wave vector. The trajectory of the relativistic particle is given by

$$\ddot{\mathbf{r}} = \frac{e}{m\gamma^3} \mathbf{E}(\mathbf{r}, t), \tag{1}$$

$$\mathbf{r} = \mathbf{r}_0 + \beta c t \mathbf{b} + \mathbf{r}^{(1)}(t) + \mathbf{r}^{(2)}(t), \tag{2}$$

and

$$\mathbf{r}^{(1)} + \mathbf{r}^{(2)} = \mathbf{b} \frac{eE_0}{m\gamma^3} \sum_{l_1 l_2 l_3 l_4} A_{l_1 l_2 l_3 l_4}(z) \frac{e^{-il\omega' t}}{(-il\omega')^2}, \tag{3}$$

where

$$A_{l_1 l_2 l_3 l_4} = J_{l_1}(z) J_{l_3}(z) I_{l_2}(z) I_{l_4}(z) (-1)^{l_3} \times \\ \times i^{l_1 + l_2} e^{i\phi_q (l_1 - l_2 + 1)} e^{i\pi/2 (l_2 + l_4)}.$$

J's and I's are Bessel functions, l's are integers,

$$z = eE_0 q / m\gamma^3 (i\omega')^2, \qquad \phi_q = \mathbf{q}\cdot\mathbf{r}_0,$$

$$l = l_3 + l_4 + 1, \qquad \omega' = \omega_0 - \mathbf{q}\cdot\mathbf{b}\beta c,$$

and γ is the Lorentz factor.

The current can be calculated from

$$J(\mathbf{K}, \omega) = e \int \dot{\mathbf{r}}(t) e^{-i\mathbf{K}\cdot\mathbf{r} + i\omega t} \, \mathrm{d}t \tag{4}$$

$$= \sum_{l_1 l_2 l_3 l_4} \frac{iE_0 e^2}{m\gamma^3} \frac{\omega}{l^2 \omega'^2} e^{i(\mathbf{q} - \mathbf{K})\cdot\mathbf{r}_0} A_{l_1 l_2 l_3 l_4} \tag{5}$$

with $\omega = l\omega' + \beta c \mathbf{K}\cdot\mathbf{b}$.

The energy radiated per unit volume of K-space, (Melrose, 1980a) after summing over the polarization states is give by

$$U(\mathbf{K}) = \sum_{l_1 l_2 l_3 l_4} \frac{2\pi e^4 \omega^2 E_0^2}{m^2 \gamma^6 (l\omega')^4} \sin^2\theta |A_{l_1 l_2 l_3 l_4}|^2, \tag{7}$$

where the polarization vectors and the wave vector of the emitted radiation are chosen as

$$\mathbf{e}^{(1)} = (\sin\theta, 0, \cos\theta)\,, \qquad \mathbf{b} = (0, 0, 1)\,,$$
$$\mathbf{e}^{(2)} = (0, 1, 0)\,, \qquad \frac{\mathbf{K}}{|K|} = (\sin\theta, 0, \cos\theta)\,; \tag{8}$$

the total power emitted by an electron is

$$U(\omega) = 2\pi \int \frac{k^2}{(2\pi)^3} \frac{\mathrm{d}k}{\mathrm{d}\omega} U(\mathbf{K}) \sin\theta \,\mathrm{d}\theta\,. \tag{9}$$

The noticeable feature of the frequency of emission, Equation (6), is that instead of the frequency ω_0 of the parallel electric field, a Doppler-shifted frequency $(\omega_0 - \mathbf{q}\cdot\mathbf{b}\beta c)$ multiplied by an integer $l = l_3 + l_4 + 1$ appears. One can retrieve the linear result of Melrose (1980b) by putting $l_3 = l_4 = l_1 = l_2 = 0$. If the external electric field is that of an electromagnetic wave, then $\mathbf{q}\cdot\mathbf{b} = 0$ since $E_0\mathbf{b} \perp \mathbf{q}$ and one gets a spectrum consisting of harmonics of the external frequency ω_0 plus the Doppler factor $\beta c\mathbf{K}\cdot\mathbf{b}$. In addition to a modified frequency spectrum, the power emitted has a nonlinear dependence on the energy density of the Bessel functions. This furnishes another possibility. One can selectively generate a particular harmonic with the highest power associated with it. This is possible because of the property of the Bessel function $J_l(z)$ that for large z, $J_l(z)$ is maximum at $l \sim z$. Since Z is known, the harmonic number can be fixed. In order to evaluate Equation (9), we use for the wave vector $|K| = n(\omega)(\omega/c)$, where $n(w)$ is the refractive index; substituting for $\mathrm{d}k/\mathrm{d}\omega$ we find

$$U(\omega) = \left[\frac{E_0 e^2}{m\gamma^3}\right]^2 \frac{1}{c^3} \sum_{l_1 l_2 l_3 l_4} |A_{l_1 l_2 l_3 l_4}|^2 \times$$
$$\times \int \frac{\omega^4 n^2(\omega)}{l^4 \omega'^4} \left[n(\omega) + \omega \frac{\mathrm{d}n(\omega)}{\mathrm{d}\omega}\right] \sin^3\theta \,\mathrm{d}\theta\,, \tag{10}$$

where w is to be determined from Equation (6); we find the two roots to be

$$\omega_+ = \frac{l\omega'}{1 - \beta\cos\theta} - \frac{\beta\omega_p^2 \cos\theta}{2l\omega'} \tag{11}$$

and

$$\omega_- = \frac{\beta\omega_p^2\cos\theta}{2l\omega'} \tag{12}$$

in the approximation

$$l^2\omega'^2 \gg 2\beta\omega_p^2\cos\theta(1 - \beta\cos\theta)\,. \tag{13}$$

One notices that the inclusion of the plasma effects leads to two sets of frequencies of emission. For the propagation of the low frequency mode ω_- one requires that $\omega_-^2 > \omega_p^2$. This combined with the assumption given by Equation (13), results into an inequality

$$2\beta\omega_p^2 \cos\theta(1 - \beta\cos\theta) \ll l^2\omega'^2 < \frac{\beta^2}{4}\,\omega_p^2\cos^2\theta\,. \tag{14}$$

Equation (14) defines the range of values of the integer l and angle θ. Substituting for ω_+ and ω_- in (10) we find

$$U(\omega_+) = \left(\frac{E_0 e^2}{m\gamma^3}\right)^2 \frac{1}{c^3} \sum_{l_1 l_2 l_3 l_4} |A_{l_1 l_2 l_3 l_4}|^2 \int_{-1}^{1} \frac{\mathrm{d}x(1 - x^2)}{(1 - \beta x)^4} \times$$

$$\times \left[1 - \frac{\omega_p^2}{l^2\omega'^2}\,(1 - \beta x)^2\right]\left[1 + \frac{\omega_p^2(1 - \beta x)^2}{2l^2\omega'^2}\right] \tag{15}$$

and

$$U(\omega_-) = \left[\frac{E_0 e^2\omega_p^2}{4m\gamma^3 l^2\omega'^2}\right]^2 \frac{1}{c^3} \sum_{l_1 l_2 l_3 l_4} |A_{l_1 l_2 l_3 l_4}|^2 \int_{-1}^{1} \mathrm{d}x \times$$

$$\times \left(x^2 + \frac{2l^2\omega'^2}{\beta^2\omega_p^2}\right)\left(x^2 - \frac{4l^2\omega'^2}{\beta^2\omega_p^2}\right)(1 - x^2)\,. \tag{16}$$

The integrations can be performed exactly and in the limit lead to the 'corrected' result of Melrose (1980b). (There are some minor errors in Equation (13.88) of Melrose, which can be easily corrected.)

3. Application to an Electron-Beam Plasma System

It has already been mentioned in the introduction that an energetic electron beam propagating parallel to the magnetic field in the ambient coronal plasma is believed to be responsible for the generation of solar radio bursts of type III and type IIIb. Although the electron beam suffers deceleration while exciting Langmuir field during quasi-linear relaxation, a number of proposals are invoked for the maintainence of the beam velocity. For example the recycling of the beam is achieved by the acceleration of the particles at the rear end of the beam through the absorption of the Langmuir waves produced by the front portion, Zaitsev *et al.* (1972). In this case, therefore it is reasonable to assume the presence of fast electrons, the electrostatic field and the ambient coronal plasma, the electrostatic field being parallel to the direction of beam propagation and hence the magnetic field. Thus the circumstances are just ideal for the linear acceleration mechanism to get into action.

One recalls that for an electron beam-plasma instability, the dispersion relation is

$$\omega_0 = \mathbf{q} \cdot \mathbf{u} - \omega_b \quad \text{and} \quad \mathbf{q} \cdot \mathbf{u} \sim \omega_p , \tag{17}$$

where $\omega_b^2 = (4\pi n_b e^2/m)$, n_b is the electron density and $\mathbf{u}$ is the velocity of the beam, $\mathbf{u} = u\mathbf{b}$. Equation (6) gives

$$\omega = -l\omega_b + \beta c \mathbf{K} \cdot \mathbf{b} . \tag{18}$$

Now, for $\mathbf{K} = \mathbf{q}$ we find

$$\omega = -l\omega_b + \mathbf{q} \cdot \mathbf{u} \simeq \omega_p - l\omega_b . \tag{19}$$

Thus one gets the emission at plasma frequency ω_p as well as at frequencies which are displaced from ω_p by an integral multiple of the beam plasma frequency ω_b. One identifies the first kind of emission as type III and the second kind as type IIIb. In other treatments one gets the electrostatic side band spectrum where the electron bounce frequency $\omega_{\mathrm{B}} = [eE_0 q/m]^{1/2}$ appears in the place of ω_b. We may point out here that ω_b and ω_{B} are related because the electric field E_0 is a function of the electron beam density n_b. Further the harmonic number l the electron bounce frequency ω_{B} and the beam plasma frequency ω_b all appear in the argument of the Bessel function as $z = \omega_{\mathrm{B}}^2/\omega_b^2$, $\gamma \simeq 1$. In order to determine ω_{B}, one knows that for saturation stage of the instability $\omega_{\mathrm{B}} \sim \gamma_0$, where $\gamma_0 \sim \omega_p(n_b/n_p)^{1/3}$ is the linear growth rate. The power in each mode can be calculated using Equations (7) and (19). Since Equation (7) gives the radiation from a single electron we do not aim to make any comparison with the observed intensities yet. The distribution of power in various upper frequency bands can be appreciated through the factor containing Bessel functions. For $l_3 = l_4 = 0$, $l = 1$, $\omega = \omega_p - \omega_b$ which will not propagate out. For fixed value of l_4, say $l_4 = 0$, we observe that the power is proportional to $J_{l_3}^2(z)$. The sum over l_1 and l_2 can be carried out exactly. For $l_3 = -2$, $l = -1$, $\omega = \omega_p + \omega_b$ and the power emitted is proportional to $J_2^2(z)/4$, etc. One must notice that the frequency separation between consecutive bands is $\sim \omega_b$ and not ω_{B}, the two however are simply related. In the above consideration, the electrostatic Langmuir field produced via beam-plasma instability has been assumed to be monochromatic. In actual excitation, Langmuir field has a finite spectral width given by $\Delta\omega_0 = (\Delta q)u + (\Delta u)q$ with $q(\Delta u) \sim u(\Delta q)$, where $\Delta\omega_0$, Δq are the spread in the frequency and the wave vector of the plasma wave and Δu is the spread in the beam velocity. Therefore the condition that the finite width of the plasma wave does not disturb the band splitting of the dynamic spectra is $(\Delta\omega_0) \ll \omega_b$ or $\Delta u/u \ll \omega_b/\omega_p$. This requires electron beams with small thermal spread in their velocity distribution function.

We emphasize here that the application of the linear acceleration mechanism has been shown to be likely for solar radio emission, since the frequency structure agrees quite well. The intensities have to be calculated either by invoking the antenna mechanism or considering a suitable distribution of beam particles, which will furnish negative absorption as for example has been shown to be possible for a monoenergetic electron distribution in a limited range of frequencies. After looking at this aspect of the problem one could make a numerical estimate of the intensities in various frequency branches.

4. Conclusion

Linear acceleration mechanism is often discussed in order to account for pulsar radio emission. In this paper, this mechanism is generalized to include the effects arsing from the relaxation of the dipole approximation. A possible application of this generalized version of the mechanism to solar radio emission is discussed.

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