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Lower bounds on neutron star mass and moment of inertia implied by the millisecond pulsar

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Summary. The short period of the millisecond pulsar PSR 1937 + 214 implies that it could be close to the onset of rotational instabilities. Conditions of rotational stability imply lower bounds on the mass and moment of inertia and upper bounds on the radius of neutron stars belonging to this new class of radio pulsars. For six representative high density equations of state, we construct critically rotating neutron star models using the prescription of Hartle & Thorne and obtain these bounds. The lower bounds on mass are found to be substantially higher than previous estimates.

1 Introduction

The recent discovery of a pulsar (PSR 1937 + 214) with an extremely short period of 1.5577 millisec (Backer *et al.* 1982) has led to several theoretical suggestions regarding the genesis of a new class of pulsars (Radhakrishnan & Srinivasan 1982; Alpar *et al.* 1982; Fabian *et al.* 1983; Henrichs & van den Heuvel 1983). In addition, radiation characteristics from this pulsar have been discussed by others (Becker & Helfand 1983; Ray & Chitre 1983). The extremely rapid rotation rate of PSR 1937 + 214 poses the problem as to whether such a rapidly rotating neutron star can be stable against break-up under centrifugal forces. In this paper we investigate the implications of the short period of PSR 1937 + 214 in relation to the properties of a *stable* rotating neutron star. Stability requirement for this pulsar allows us to obtain lower limits on the neutron star's mass and moment of inertia and also upper limits on the radius and the extent of solid outer crust. These limits, which depend on the equation of state (EOS) of neutron star matter, are expected to be relevant to this new class of pulsars. We use a representative set of EOS to calculate these bounds.

The moment of inertia is an important quantity to estimate the energy loss rate from a pulsar. Earlier estimates of this quantity (Ruderman 1972) spanned a wide range (7×10^{43} g cm$^2 \leqslant I \leqslant 7\times10^{44}$ g cm^2). Application of the rotational stability requirement allows us to reduce this gap.

2 Rotational instability in a fluid star

In Newtonian gravity, self-gravitating fluid bodies in uniform rotation are described by Maclaurin spheroids and Jacobi ellipsoids. The former are axially symmetric configurations of uniform-density rapidly-rotating fluid, while the latter are non-axially symmetric, having surfaces of constant pressure that are ellipsoids with all three axes unequal.

Bodenheimer & Ostriker (1973) have shown in connection with rotating white dwarfs that even in the case of differentially rotating, inviscid polytropes (i.e. compressible fluids), the constructed models with a specified angular momentum distribution closely resemble the Maclaurin sequence. With specified total mass, angular momentum and the angular momentum distribution, the constructed sequence of such stars is characterized by the parameter τ = (kinetic energy of rotation)/(|gravitational potential energy |). In particular, for this sequence also, a secular instability develops near the point $\tau = 0.1375$, where for the Maclaurin sequence a bifurcation occurs. A slight change in the angular velocity can move the configuration either along the Maclaurin sequence or into the Jacobi sequence. Once this instability develops, the star may undergo disruption. Density profiles of neutron stars are remarkably flat and, although general relativity plays a role in determining their structure, the secular instability in the context of Maclaurin spheroids are relevant to their rotational stability considerations as well. For a uniformly rotating homogeneous spheroid, this instability corresponds to the angular velocity Ω_s given by

$$\frac{\Omega_s^2}{2\pi G\bar{\rho}} \simeq 0.18, \tag{1}$$

where $\bar{\rho}$ is the density of the Maclaurin spheroid and, in the case of the Bodenheimer–Ostriker polytropic sequence, it corresponds to the mean density. Equation (1) for a period of 1.5577 millisec implies a mean stellar density $\bar{\rho} = 2.4 \times 10^{14}\,\mathrm{g\,cm^{-3}}$. For a given EOS, this in turn, gives a lower limit for the gravitational mass of the neutron star.

3 The structure equations

The effect of rotation on the structure of a star is to produce both spherical and quadrupole deformations. For a fixed central density, the fractional change in the gravitational mass ($\delta M/M$) and radius ($\delta R/R$) due to spherical deformation are proportional to Ω^2 (Ω = angular velocity) and can be obtained from a knowledge of the radial distributions of the mass and the pressure perturbation factors. The non-rotating mass M and radius R are obtained by integrating the relativistic equations for hydrostatic equilibrium (see, e.g. Arnett & Bowers 1977).

The mass perturbation factor $m_0(r)$ and the pressure perturbation factor $P_0(r)$ corresponding to spherical deformation are calculated using the prescription of Hartle & Thorne (1968). The deformations δM and δR are then obtained as

$$\delta M = \frac{c^2}{G} m_0(R) + \frac{G}{c^4}\frac{J^2}{R^3} \tag{2}$$

$$\delta R = -\left.\frac{P_0(\rho c^2 + P)}{dP/dr}\right|_{r=R} \tag{3}$$

where J and R are respectively the angular momentum and radius of the star, and $P(r)$ and $\rho(r)$ are the pressure and total mass-energy density at the point r.

The above prescription for calculating the mass and radius of a rotating star is valid only for rotations slow compared to the critical $\Omega_c = (GM/R^3)^{1/2}$. Hartle & Thorne (1968) have constructed 'slowly' rotating neutron star models up to this critical angular speed Ω_c. In our case, since the secular angular speed is

$$\Omega_s = (0.27)^{1/2}\,\Omega_c,$$

the models constructed are well into the limits of slow rotation.

4 Choice of the equation of state (EOS)

The structure of neutron stars depends sensitively on the EOS at high densities, especially around the density region $\rho \sim 10^{15}\,\mathrm{g\,cm^{-3}}$. In this work, we have chosen the following six EOS based on representative neutron and nuclear matter interaction models: (1) Reid–Pandharipande (RP) model, (2) Bethe–Johnson (BJ) model I, (3) the tensor interaction (TI) model (for discussion on these three models see Pandharipande, Pines & Smith 1976, PPS), (4) Brown–Weise (BW) model, which takes into account the presence of a negative pion condensate in high density matter (Brown & Weise 1976), (5) Canuto–Datta–Kalman (CDK) model which includes the short-range attraction due to nucleon–nucleon f^0 meson exchange ($f^2 = 2.91$ case) (Canuto, Datta & Kalman 1978) and (6) Friedman–Pandharipande (FP) model, which is based on an improved nuclear Hamiltonian that fits adequately the nucleon–nucleon scattering data and known nuclear matter properties (Friedman & Pandharipande 1981).

The composite equation of state to determine the neutron star structure are set up in the following way. For all except the CDK model, $P(10^{13} < \rho < 10^{14}\,\mathrm{g\,cm^{-3}})$ is taken as from (PPS 1976). In the range $3.7 \times 10^{11} < \rho < 10^{13}\,\mathrm{g\,cm^{-3}}$ $P(\rho)$ is taken from Negele & Vautherin (1973) (NV) and below $\rho < 3.7 \times 10^{11}\,\mathrm{g\,cm^{-3}}$ it is taken from Baym, Pethick & Sutherland (1971) (BPS). The $P(\rho)$ for CDK model has been joined to (i) BJ model V (Malone, Johnson & Bethe 1975) for $5.2 \times 10^{14}\,\mathrm{g\,cm^{-3}} > \rho > 1.7 \times 10^{14}\,\mathrm{g\,cm^{-3}}$, (ii) Baym Bethe & Pethick (1971) model for $1.7 \times 10^{14}\,\mathrm{g\,cm^{-3}} > \rho > 4.46 \times 10^{11}\,\mathrm{g\,cm^{-3}}$ and (iii) Baym *et al.* (1971) and Feynman, Metropolis & Teller (1949) models for $\rho < 4.45 \times 10^{11}\,\mathrm{g\,cm^{-3}}$.

5 Results and discussion

For each EOS, the interpolation for pressure was done using a three-point Spline fit, taking a variable step-size that depended on the pressure scale height. The numerical integration was terminated with the last step before $\rho \leqslant 7.86\,\mathrm{g\,cm^{-3}}$, which corresponds to the neutron star surface. The code was tested by comparing the results with those reported by Hartle & Thorne (1968) for the Harrison–Walker–Wheeler EOS. The agreement was found to be within 0.6 per cent.

Results of our computations for the six EOS are presented in Tables 1 and 2 and Fig. 1. The lower bounds on the secular limit mass (M_r) and the moment of inertia ($I = J/R$) are the values of M_r and I at $\Omega_s^2 = 1.63 \times 10^7\,\mathrm{rad^2\,s^{-2}}$, corresponding to PSR 1937 + 214. The upper bounds on M_r and I are obtained in the standard way, by finding the points of maxima in the curves $M_r(\rho_c)$ and $I(\rho_c)$, where ρ_c = central density. For the TI model, however, $I_{\min}$ and $I_{\max}$ are obtained in exactly the reverse way. Ignoring for the moment the BW model, all the 'normal' neutron matter EOS give a lower bound for the mass of PSR 1937 + 214 to be $\gtrsim 0.7\,M_\odot$. One particularly stiff EOS, the TI model, gives this value to be quite large ($1.72\,M_\odot$). The spread in the range of I is small if one ignores the first and the last rows.

Table 1. Non-rotating gravitational mass (in units of solar mass), radius, the secular angular velocity, the corresponding fractional changes in mass and radius, the moment of inertia and the extent of the solid outer crust of neutron stars as functions of the central density for different equations of state.

Model	ρ_c (g cm^{-3})	$M/M_\odot$	R (km)	Ω_s (rad s^{-1})	$\delta M/M$ $\alpha\Omega^2$	$\delta R/R$ $\alpha\Omega^2$	I (g cm^2)	Δ_c (km)
BW	2.00E15	0.83	8.25	7.26E3	0.081	0.032	3.48E44	1.20
	1.50E15	0.60	8.54	5.87E3	0.078	0.038	2.27E44	1.84
	1.30E15	0.48	8.79	5.05E3	0.070	0.041	1.70E44	2.39
	1.00E15	0.30	10.01	3.26E3	0.047	0.048	9.47E43	4.42
RP	7.00E15	1.60	7.72	1.11E4	0.046	0.007	8.56E44	0.36
	3.00E15	1.60	8.97	8.90E3	0.059	0.015	1.07E45	0.59
	2.00E15	1.42	9.72	7.43E3	0.071	0.023	9.91E44	0.92
	1.50E15	1.20	10.19	6.36E3	0.078	0.029	8.34E44	1.32
	1.00E15	0.83	10.77	4.89E3	0.083	0.037	5.48E44	2.31
	7.00E14	0.55	11.40	3.66E3	0.076	0.042	3.35E44	3.82
FP	4.00E15	1.98	8.83	1.01E4	0.048	0.003	1.56E45	0.33
	3.00E15	1.98	9.24	9.48E3	0.053	0.006	1.65E45	0.40
	2.00E15	1.81	9.94	8.11E3	0.063	0.014	1.54E45	0.62
	1.50E15	1.48	10.57	6.68E3	0.076	0.025	1.22E45	1.06
	1.00E15	1.10	11.13	5.33E3	0.088	0.033	8.77E44	1.80
	7.00E14	0.72	11.44	4.15E3	0.087	0.040	5.09E44	3.02
	4.00E14	0.30	12.96	2.24E3	0.053	0.048	1.67E44	7.52
CDK	4.00E15	1.71	10.39	7.39E3	0.066	0.019	1.48E45	0.79
	3.00E15	1.74	10.55	7.28E3	0.069	0.019	1.56E45	0.79
	2.00E15	1.74	10.88	6.96E3	0.073	0.020	1.67E45	0.87
	1.00E15	1.37	11.39	5.77E3	0.089	0.029	1.28E45	1.42
	8.00E14	1.04	11.42	5.00E3	0.093	0.035	8.60E44	2.04
	6.00E14	0.60	11.60	3.68E3	0.070	0.042	3.88E44	3.74
BJ	4.00E15	1.86	9.47	8.84E3	0.047	0.010	1.41E45	0.81
	3.00E15	1.87	10.02	8.16E3	0.051	0.012	1.54E45	0.95
	2.00E15	1.79	10.96	6.96E3	0.059	0.018	1.63E45	1.35
	1.00E15	1.32	12.76	4.76E3	0.075	0.032	1.31E45	3.04
	8.00E14	1.13	13.34	4.13E3	0.078	0.036	1.14E45	4.00
	6.00E14	0.91	14.10	3.41E3	0.079	0.040	9.37E44	5.65
TI	4.00E15	1.74	10.33	7.51E3	0.045	0.018	1.31E45	1.63
	3.00E15	1.77	11.08	6.82E3	0.050	0.020	1.50E45	1.91
	2.00E15	1.77	12.32	5.81E3	0.058	0.024	1.79E45	2.55
	1.00E15	1.63	14.73	4.27E3	0.077	0.032	2.27E45	4.52
	8.00E14	1.58	15.44	3.91E3	0.083	0.033	2.42E45	5.36
	4.00E14	1.29	16.27	3.28E3	0.094	0.038	2.09E45	7.99

Estimates of neutron star masses cover a wider range (Kelley & Rappaport 1981). Their analysis of experimental data is consistent with a range of neutron star masses expected, for example, in the collapse of accreting degenerate stars in a close binary system, i.e. to a neutron star mass in the range $1.4\,M_\odot \pm 0.2\,M_\odot$. For PSR 1936 + 16 (the binary pulsar), however, the pulsar's mass has been accurately determined to be $(1.43 \pm 0.07)\,M_\odot$ (Taylor 1981). This value is well below the lower limit given by one case here, the TI model. This can be interpreted in two ways: either the millisecond pulsar (and objects of similar class) has a mass above $1.72\,M_\odot$, or the TI model is an unrealistic EOS, assuming the mass of PSR 1936 + 16 to be characteristic of this class of objects. Nuclear physics arguments suggest that the TI model may indeed be somewhat unrealistic (FP 1981). Thus, the millisecond pulsar need not have mass and moment of inertia much in excess of $1.2\,M_\odot$ and 1.6×10^{45} g cm^2 respectively.

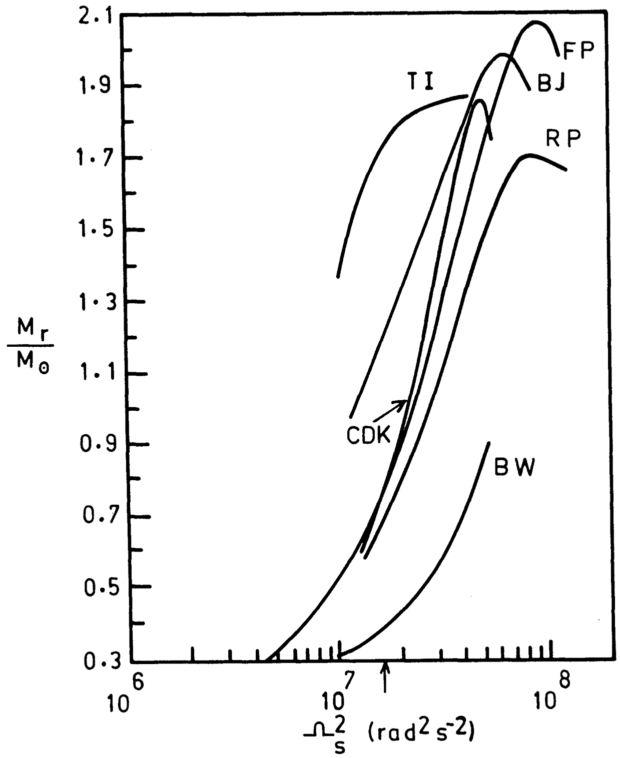

Figure 1. The gravitational mass (in units of solar mass) corresponding to the secular angular velocity Ω_s versus Ω_s^2 for different equations of state.

Table 2. Parameters of neutron stars on the point of secular rotational instability implied by PSR 1937 + 214 for various equations of state.

Equation of state	$M_{r,\,min}/M_\odot$	$R_{r,\,max}$ (km)	I_{min} (g cm^2)	I_{max} (g cm^2)	$\Delta_c\,(M_{min})$ (km)
BW	0.40	9.8	1.2×10^{44}	–	3.4
RP	0.70	11.7	4.0×10^{44}	1.1×10^{45}	3.2
FP	0.76	12.0	5.0×10^{44}	1.6×10^{45}	3.1
CDK	0.79	12.0	4.7×10^{44}	1.6×10^{45}	3.0
BJ	1.20	13.9	1.1×10^{45}	1.6×10^{45}	4.2
TI	1.72	15.6	1.5×10^{45}	2.4×10^{45}	5.0

The entries in Table 2 have been arranged approximately in the increasing order of stiffness in the EOS of dense neutron matter. It is clear that stiffer EOS give larger lower bounds on mass and moment of inertia.

Though the masses reported in this work are for neutron stars on the verge of secular rotational instability the same quantities for the dynamical instability (corresponding to $\tau = 0.2738$, or equivalently, $\Omega^2/2\pi G\bar{\rho} \simeq 0.22$) can easily be constructed from the data reported in Table 1 by noting that $\delta M/M$ and $\delta R/R$ scale as Ω^2. The secular limit, rather than the dynamical one, is reported here because the former is a stronger restriction on the rotational speed, and if the millisecond pulsar is envisaged as a neutron star spun up, the secular instability intervenes earlier than the dynamical.

Until now, there has been no good estimate for the lower limit of neutron star mass, which is generally taken to be ~ 0.1. $M_\odot$ (Ruderman 1972). Using realistic EOS and rotational stability arguments we have found this limit to be ~ 0.7 $M_\odot$. From an analysis of the unique position in the P–$\dot{P}$ (period and its time derivative) diagram occupied by the PSR

1937 + 214 and five other pulsars, Alpar *et al.* (1982) have suggested that this class of radio pulsars may have had a different genesis and evolution from all other long-period pulsars. They have argued that such objects could have been spun up during an accretion phase and have related their periods to neutron star parameters like the mass and magnetic field as well as the accretion rate. Since these pulsars form a separate class, their structure parameters also have to be similar to some extent. In the case of the millisecond PSR 1937 + 214 we found the lower limits of mass, moment of inertia etc. for given neutron matter equations of state. Thus, these lower limits should also apply to the other low $(P, \dot{P})$ pulsars, since from independent considerations these should have similar parameters. We note in this connection, that even in the type II supernova explosion scenarios leading to neutron star formation, the current numerical models (though incomplete in some respects) suggest that the neutron star remnant in these explosions would have a mass $\sim 0.8\, M_\odot$ (Hillebrandt 1982). Such a mass is close to the values reported here for the more realistic equations of state. Thus, from different considerations, the theoretical lower limits on mass and moment of inertia reported here are expected to be of general validity for at least a class of neutron stars.

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