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ROTATING NEUTRON STAR STRUCTURE: IMPLICATIONS OF THE MILLISECOND PULSAR PSR 1937+214

A. RAY
Tata Institute of Fundamental Research, Bombay, India
AND
B. DATTA
Indian Institute of Astrophysics, Bangalore, India
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ABSTRACT

Rapidly rotating fluid objects of a given mass can remain stable only up to a critical angular speed, beyond which they may undergo instabilities leading to disruption. A semi-Newtonian condition of rotational stability applied to the recently discovered millisecond pulsar PSR 1937+214 implies lower bounds on the mass and moment of inertia and an upper bound on the radius of the neutron star. These upper and lower bounds are dependent on the equation of state of high-density neutron matter. We construct critically rotating realistic neutron star models, using the prescription of Hartle and Thorne, for six representative equations of state. The lower bounds on mass are found to be substantially higher than previous estimates. Results for various equations of state of neutron matter at high densities are compared with observational data for neutron star masses.

Subject headings: dense matter — equation of state — pulsars — stars: neutron — stars: rotation

I. INTRODUCTION

Backer *et al.* (1982) have recently reported the discovery of a pulsar (PSR 1937+214) with an extremely short period of 1.5577 milliseconds. The period is approximately 20 times shorter than that of the Crab pulsar, the fastest rotating pulsar known until now. The characteristics of the radio pulses of PSR 1937+214 reinforce the central dogma of pulsar physics—namely, a pulsar is a rotating neutron star—but the origin of such a neutron star in a canonical hot supernova explosion has been questioned. In particular, several theoretical suggestions have been made regarding the possible genesis of a new class of pulsars (Radhakrishnan and Srinivasan 1982; Alpar *et al.* 1982; Fabian *et al.* 1983; Henrichs and van den Heuvel 1983) as well as characteristic of such a fast-rotating object (Arons 1983; Ray and Chitre 1983). The extremely rapid rotation rate of PSR 1937+214 poses the problem as to whether such a rapidly rotating neutron star can be stable against breakup under centrifugal forces. In this paper we investigate the implications of the short period of the millisecond pulsar in relation to the properties of a *stable* rotating neutron star. Our results are obtained through numerical calculations involving realistic equations of state and using the prescription of Hartle and Thorne (1968), and are valid for strong gravitational fields but in the limit of uniform, slow rotation (slow compared with the critical speed for centrifugal breakup). The stability requirement for this pulsar allows us to obtain *lower limits* on the neutron star's mass and moment of inertia and also *upper limits* on the radius and the extent of the solid outer crust. These limits depend on the equation of state of neutron star matter. We have chosen a representative set of equations of state to calculate these bounds. From the existing observational data (Joss and Rappaport 1984) on neutron star masses, the bounds can, in turn, place interesting restrictions on the equation of state for matter at high densities. A preliminary summary of this work has appeared elsewhere (Datta and Ray 1983).

Until now, the lightest possible neutron star mass has been taken to be $\sim 0.1\ M_\odot$. This is because masses less than 0.15 $M_\odot$ are bound less tightly than white dwarfs with the same number of baryons and are even unbound relative to the same matter dispersed as a dilute gas of helium. Therefore, to make an extremely light neutron star, net positive work must be done by the surrounding matter, which must still have enough energy to escape and leave behind only the loosely bound light star. No neutron star formation scenario is known that would leave behind remnants as light as 0.15 $M_\odot$. On the other hand, application of the rotational stability criterion as discussed below raises the lower limit to about 0.8 $M_\odot$ for certain realistic equations of state.

The moment of inertia (I) is an important quantity for estimating the energy loss rate from a pulsar once its period derivative ($\dot{P}$) is known. Earlier estimates of I (Ruderman 1972) spanned a wide range (7×10^{43} g cm$^2 \leq I \leq 7 \times 10^{44}$ g cm^2), largely because of the uncertainty in the equation of state at high densities. This uncertainty has recently been considerably reduced, in part because of a better understanding of nuclear matter calculations and also because certain important nuclear matter parameters (such as the incompressibility, see Friedman and Pandharipande 1981) have been experimentally determined. With improved equations of state, the application of the rotational stability requirement narrows down considerably the possible range of the moment of inertia from the limits quoted above.

It is well known that the condition of hydrostatic equilibrium and stability with respect to radial perturbations predicts an *upper* limit of mass and moment of inertia that are dependent on the equation of state. A substantial rotation of the star (comparable to its critical breakup speed) induces spherical and quadrupole deformations and changes these limits. Though these limits for nonrotating neutron star mass and moment of inertia are available in the literature, the corresponding limits incorporating the effects of substantial rotation have not been investigated before this work for the six equations of state used here.

In constructing rotating neutron star models we follow the general relativistic formalism of Hartle and Thorne (1968) in the limit of slow rotation. In § II, we describe the conditions under which a rotating fluid star would be stable with respect to rotational instabilities, and we apply these in the context of the millisecond pulsar. Section III outlines the (numerical) construction of rotating neutron star models on the verge of a secular instability (valid for homogeneous and uniformly rotating stars in Newtonian gravity). Section IV describes the high-density equations of state used in the computations and also the manner in which these have been extended to subnuclear densities. Results and discussion are presented in § V.

II. ROTATIONAL INSTABILITY IN A FLUID STAR

In general, the gravitational binding energy in a neutron star is the dominant factor in determining its structure. For a fast pulsar such as PSR 1937+214, the rotational energy can also be an appreciable fraction of the total energy of the system. In comparison, the elastic energy stored in the solid crust is much smaller. Hence, rotational instabilities in a fluid star are important restrictions relevant to rapidly rotating neutron stars such as the millisecond pulsar.

In Newtonian gravity, self-gravitating fluid bodies in uniform rotation are described by Maclaurin spheroids and Jacobi ellipsoids. The former are axially symmetric configurations of uniform-density rapidly rotating fluid, while the latter are nonaxisymmetric, having surfaces of constant pressure that are ellipsoids with all three axes unequal.

Bodenheimer and Ostriker (1973) have shown in connection with rotating white dwarfs that even in the case of differentially rotating, inviscid polytropes (i.e., compressible fluids), the constructed models with a specified angular momentum distribution closely resemble the Maclaurin sequence. With specified total mass, angular momentum, and the angular momentum distribution, the constructed sequence of such stars is characterized by the parameter τ = (kinetic energy of rotation)/(|gravitational potential energy|). In particular, for this sequence also, a secular instability develops near the point $\tau = 0.1375$, where for the Maclaurin sequence a bifurcation occurs. A slight change in the angular velocity can move the configuration either along the Maclaurin sequence or into the Jacobi sequence. Once this instability develops, the star may undergo disruption. Density profiles of neutron stars are remarkably flat, and although general relativity plays a role in determining their structure, recent calculation (Cowsik and Ghosh 1983) has shown that the general relativistic neutron star structures are adequately described by Maclaurin spheroids. For a uniformly rotating homogeneous Maclaurin spheroid, this instability corresponds to the angular velocity Ω_s given by

$$\frac{\Omega_s^2}{2\pi G\bar{\rho}} \approx 0.18 , \tag{1}$$

where $\bar{\rho}$ is the density of the Maclaurin spheroid, and, in the case of the Bodenheimer-Ostriker polytropic sequence, it corresponds to the mean density. Equation (1) for a period of 1.5577 milliseconds implies a mean stellar density $\bar{\rho} = 2.4 \times 10^{14}$ g cm^{-3}. For a given equation of state, this, in turn, gives a lower limit for the gravitational mass of the neutron star. Exactly how this limit is derived, by constructing critically rotating neutron star models, is shown in the next three sections. While our construction of rotating neutron star structures is valid for arbitrarily strong gravitational fields and for slow rotation, the limiting criterion implied by equation (1) is, strictly speaking, semi-Newtonian, valid for homogeneous stars in uniform rotation. Secular instability against radiation of gravitational waves has been shown to be generally true for *all* rotating stars (Friedman and Schutz 1975, 1978). Using a scalar theory of gravitation and a Newtonian homogeneous star model, Papaloizou and Pringle (1978) have shown that such instability will be important, on astronomically interesting time scales, for neutron stars with period $\lesssim$a few milliseconds (see also Comins 1978). While the secular instability for homogeneous stars in general relativity in arbitrarily fast but uniform rotation is not completely understood, and some authors have even questioned if this instability operates for a uniformly rotating, centrally condensed star (see Shapiro, Teukolsky, and Wasserman 1983), we use the condition (1) in this paper on the presumption that relativistic analogs of this criterion exist for realistic stars which are nearly uniform in density and may not always be in uniform rotation.

III. THE STRUCTURE EQUATIONS

The effect of rotation on the structure of a star is to produce both spherical and quadrupole deformations. For a fixed central density, the fractional change in the gravitational mass ($\delta M/M$) and radius ($\delta R/R$) due to spherical deformation is proportional to Ω^2 (Ω = angular velocity of the surface measured by an observer at infinity) and can be obtained from a knowledge of the radial distributions of the mass and the pressure perturbation factors. The mass M and radius R for the nonrotating configuration are obtained by integrating the relativistic equations for hydrostatic equilibrium. The relevant equations for the nonrotating star are

$$\frac{dP}{dr} = -\frac{G}{r^2}\frac{(\rho + P/c^2)(m + 4\pi r^3 P/c^2)}{(1 - 2Gm/rc^2)} , \tag{2}$$

$$\frac{dm}{dr} = 4\pi r^2 \rho , \tag{3}$$

where P is the pressure, ρ is the total mass-energy density, and $m(r)$ is the gravitational mass within a proper radius r.

The potential $\nu(r)$, relating the element of proper time to the element of time at $r = \infty$, is given by

$$\frac{d\nu}{dr} = \frac{G}{r^2}\frac{m + 4\pi r^3 P/c^2}{1 - 2Gm/rc^2} . \tag{4}$$

For a given equation of state, $P(\rho)$, and a given central density, $\rho(0) = \rho_c$, the above equations are numerically integrated, with the boundary condition $m(0) = 0$, to give R and M. The radius R is defined by the point where $P = 0$, or equivalently, $\rho = \rho_s$, where ρ_s is the surface density. The total gravitational mass is then

$$M = m(R) . \tag{5}$$

A relativistic effect of rotation is a dragging of inertial frames, so that

$$\bar{\omega}(r) \neq \Omega ,$$

where $\bar{\omega}(r)$ is the angular velocity of the fluid relative to the local inertial frame and is given by

$$\frac{d}{dr}\left(r^4 j \frac{d\bar{\omega}}{dr}\right) + 4r^3\bar{\omega}\frac{dj}{dr} = 0 , \tag{6}$$

where

$$j(r) = e^{-\nu(r)}(1 - 2Gm/rc^2)^{1/2} , \tag{7}$$

with the boundary conditions

$$\left(\frac{d\bar{\omega}}{dr}\right)_{r=0} = 0 , \quad \bar{\omega}(\infty) = \Omega . \tag{8}$$

For $r > R$ (i.e., outside the star),

$$\bar{\omega}(r) = \Omega - \frac{2GJ}{c^2 r^3} , \tag{9}$$

where J is the angular momentum of the star:

$$J = \frac{c^2}{6G} R^4 \left(\frac{d\bar{\omega}}{dr}\right)_{r=R} . \tag{10}$$

The mass perturbation factor $m_0(r)$ and the pressure perturbation factor $P_0(r)$ corresponding to spherical deformation are given by (Hartle and Thorne 1968)

$$\frac{dm_0}{dr} = 4\pi G r^2 P_0\left(\rho + \frac{P}{c^2}\right)\frac{d\rho}{dP} + \frac{e^{-2\nu}}{12c^2} r^3\left(r - \frac{2Gm}{c^2}\right)\left(\frac{d\bar{\omega}}{dr}\right)^2 + \frac{8\pi G}{3c^4} e^{-2\nu} r^4 \bar{\omega}^2\left(\rho + \frac{P}{c^2}\right) , \tag{11}$$

$$\frac{dP_0}{dr} = -\frac{m_0(1 + 8\pi r^2 GP/c^4)}{(r - 2Gm/c^2)^2} - \frac{G}{c^2}\frac{4\pi r^2 P_0(\rho + P/c^2)}{(r - 2Gm/c^2)} + \frac{e^{-2\nu}}{12c^2} r^3\left(r - \frac{2Gm}{c^2}\right)\left(\frac{d\bar{\omega}}{dr}\right)^2 + \frac{8\pi G}{3c^4} e^{-2\nu}\bar{\omega}^2\left(\rho + \frac{P}{c^2}\right) , \tag{12}$$

with boundary conditions

$$m_0(0) = 0 , \quad P_0(0) = 0 . \tag{13}$$

With these boundary conditions, the rotating star will have the same central density as the nonrotating one (Thorne 1971). The deformations δM and δR are given by

$$\delta M = \frac{c^2}{G} m_0(R) + \frac{G}{c^4}\frac{J^2}{R^3} , \tag{14}$$

$$\delta R = -\left.\frac{P_0(\rho c^2 + P)}{dP/dr}\right|_{r=R} . \tag{15}$$

The above prescription for calculating the mass and radius of a rotating star is valid only for rotations that are slow compared with the critical $\Omega_c = (GM/R^3)^{1/2}$. Hartle and Thorne (1968) have constructed "slowly" rotating neutron star models up to this critical angular speed. In our case, since the secular angular speed is

$$\Omega_s = (0.27)^{1/2}\Omega_c ,$$

the models constructed are well into the limits of slow rotation.

IV. CHOICE OF THE EQUATION OF STATE (EOS)

The structure of neutron stars depends sensitively on the equation of state at high densities, especially around the density region $\sim 10^{15}$ g cm^{-3}. For our purpose, we have chosen six equations of state based on representative neutron and nuclear matter interaction models:

1. The Reid-Pandharipande (RP) model is based on the lowest order constrained variation (LOCV) method using the Reid nucleon-nucleon potential (Pandharipande and Bethe 1973). Although this equation of state has been extensively used in neutron star calculations, it is now primarily of historical interest since the Reid potential has since been found to be inadequate for describing known nuclear matter properties.

2. The Bethe-Johnson (BJ) model is based on phenomenological nucleon-nucleon potentials (Bethe and Johnson 1974) which have a more realistic short-range behavior than the Reid potentials. We have chosen BJ model I as a representative equation of state.

3. The tensor interaction (TI) model is based on the LOCV method with the assumption that the intermediate-range attraction between nucleons can be attributed to higher order contributions of the pion-exchanged tensor interaction (Pandharipande and Smith 1975). The short-range repulsive part of the interaction is not taken into account properly, so that this model is inadequate for describing known nuclear matter properties (Pandharipande, Pines, and Smith 1976).

4. The Brown-Weise (BW) model takes into account the presence of a negative pion condensate in high-density matter (Brown and Weise 1976). The calculation is based on the σ-model and includes the s- and p-wave pion-nucleon interaction and the effect of isobars.

5. The Friedman-Pandharipande (FP) model is based on a nuclear Hamiltonian that contains two- and three-nucleon interactions and fits the nucleon-nucleon scattering data for s-, p-, d-, and f-waves (Friedman and Pandharipande 1981). This interaction model reproduces the ground-state energy, density, and incompressibility of nuclear matter via variational calculations. Nuclear matter calculations using phenomenological potentials invariably give too large a value for the incompressibility parameter K. The FP model is fitted to the recently available experimental value $K = 240$ MeV. In view of this, the FP model provides the most realistic equation of state for dense neutron matter.

6. The Canuto-Datta-Kalman (CDK) model includes the short-range attraction due to nucleon-nucleon f^0-meson exchange, in addition to the usual ω-meson and σ-meson exchange forces. The equation of state is derived using relativistic mean field approximation. Here, we take the case corresponding to the spin-2 coupling constant $f^2 = 2.91$. Calculations for nonrotating neutron star structure based on this model are reported in Canuto, Datta, and Kalman (1978).

The composite equations of state to determine the neutron star structure are set up in the following way. For each of the first five models, for 10^{13} g cm^{-3} $< \rho < 10^{14}$ g cm^{-3}, $P(\rho)$ is taken from Pandharipande, Pines, and Smith, which corresponds to a gradual averaging of P for that model with that given by Negele and Vautherin (1973). For the inner crust region (3.7×10^{11} g cm^{-3} $< \rho < 10^{13}$ g cm^{-3}), $P(\rho)$ is taken from Negele and Vautherin, while for the outer crust and the surface ($\rho < 3.7 \times 10^{11}$ g cm^{-3}), $P(\rho)$ is taken as given by Baym, Pethick, and Sutherland (1971) and Feynman, Metropolis, and Teller (1949). The $P(\rho)$ for the CDK model has been joined to (i) BJ model V (Malone, Johnson, and Bethe 1975) for 5.2×10^{14} g cm^{-3} $> \rho > 1.7 \times 10^{14}$ g cm^{-3}, (ii) the Baym, Bethe, and Pethick (1971) model for 1.7×10^{14} g cm^{-3} $> \rho > 4.46 \times 10^{11}$ g cm^{-3}, and (iii) the Baym, Pethick, and Sutherland (1971) and Feynman, Metropolis, and Teller (1949) models for $\rho < 4.46 \times 10^{11}$ g cm^{-3}.

Nonrotating neutron star masses calculated using the BW model have been reported by Maxwell and Weise (1976). The corresponding values that we obtain are slightly different from those of Maxwell and Weise because the values for the axial vector coupling constant adopted by them differ slightly from the one we used (as given by BW). Also, the pressure below the condensation threshold has been taken by BW as given by the Reid potential, whereas in our case it is given by a gradual averaging with that given by Negele and Vautherin, as described by Pandharipande, Pines, and Smith.

V. RESULTS AND DISCUSSION

Equations (2), (3), (4), (6), (11), and (12) were integrated by a two-point modified Euler method using a variable step size. The step size employed in a particular region depended on the pressure scale height in that region. To obtain R and M, it is sufficient to start with an arbitrary value for $\nu(0)$, which is then rescaled to obey the surface condition

$$\nu(R) = \tfrac{1}{2} \ln (1 - 2GM/Rc^2) ,$$

so that $\nu(\infty) = 0$, which ensures the correct evaluation of $m_0(r)$ and $P_0(r)$. Likewise, $\bar{\omega}(0)$ is initially chosen to be an arbitrary constant; once Ω is obtained, a new starting value $\bar{\omega}_{\rm new}(0)$ corresponding to an observed $\Omega_0 = \Omega_{\rm new}$ is given by

$$\bar{\omega}_{\rm new}(0) = \Omega_{\rm new}\, \bar{\omega}(0)/\Omega .$$

For each equation of state, the interpolation for pressure was done using a three-point spline fit to the supplied data points of P and ρ. The numerical integration for each model was terminated with the last step before $\rho \leq \rho_s = 7.86$ g cm^{-3}. The code was tested by comparing the results with those reported by Hartle and Thorne (1968) for the Harrison-Walker-Wheeler EOS. The agreement was found to be within 0.6%.

Results of our computations for the six equations of state are presented in Tables 1 and 2 and Figures 1–7. Table 1 gives the masses and radii (M, R) of the nonrotating configurations, the secular angular velocity (Ω_s), the fractional changes in M and R corresponding to Ω_s, the moment of inertia (I), and the extent of the solid outer crust (Δ_c) of the nonrotating star (the region having $\rho < 2.8 \times 10^{14}$ g cm^{-3}). Figure 1 shows the secular limit mass M_r (in units of $M_\odot$) as a function of Ω_s^2. The lower bounds on M_r and the moment of inertia ($I = J/\Omega$) are the

TABLE 1

THE BULK PROPERTIES OF NEUTRON STARS (NONROTATING AND ROTATING CONFIGURATIONS) FOR VARIOUS EQUATIONS OF STATE

Model	ρ_c (g cm^{-3})	$M/M_\odot$	R (km)	Ω_s (rad s^{-1})	$\delta M/M$ $\alpha\Omega^2$	$\delta R/R$ $\alpha\Omega^2$	I (g cm^{-2})	Δ_c (km)
BW	2.00E15	0.83	8.25	7.26E3	0.081	0.032	3.48E44	1.20
	1.50E15	0.60	8.54	5.87E3	0.078	0.038	2.27E44	1.84
	1.30E15	0.48	8.79	5.05E3	0.070	0.041	1.70E44	2.39
	1.00E15	0.30	10.01	3.26E3	0.047	0.048	9.47E43	4.42
RP	7.00E15	1.60	7.72	1.11E4	0.046	0.007	8.56E44	0.36
	3.00E15	1.60	8.97	8.90E3	0.059	0.015	1.07E45	0.59
	2.00E15	1.42	9.72	7.43E3	0.071	0.023	9.91E44	0.92
	1.50E15	1.20	10.19	6.36E3	0.078	0.029	8.34E44	1.32
	1.00E15	0.83	10.77	4.89E3	0.083	0.037	5.48E44	2.31
	7.00E14	0.55	11.40	3.66E3	0.076	0.042	3.35E44	3.82
FP	4.00E15	1.98	8.83	1.01E4	0.048	0.003	1.56E45	0.33
	3.00E15	1.98	9.24	9.48E3	0.053	0.006	1.65E45	0.40
	2.00E15	1.81	9.94	8.11E3	0.063	0.014	1.54E45	0.62
	1.50E15	1.48	10.57	6.68E3	0.076	0.025	1.22E45	1.06
	1.00E15	1.10	11.13	5.33E3	0.088	0.033	8.77E44	1.80
	7.00E14	0.72	11.44	4.15E3	0.087	0.040	5.09E44	3.02
	4.00E14	0.30	12.96	2.24E3	0.053	0.048	1.67E44	7.52
CDK	4.00E15	1.71	10.39	7.39E3	0.066	0.019	1.48E45	0.79
	3.00E15	1.74	10.55	7.28E3	0.069	0.019	1.56E45	0.79
	2.00E15	1.74	10.88	6.96E3	0.073	0.020	1.67E45	0.87
	1.00E15	1.37	11.39	5.77E3	0.089	0.029	1.28E45	1.42
	8.00E14	1.04	11.42	5.00E3	0.093	0.035	8.60E44	2.04
	6.00E14	0.60	11.60	3.68E3	0.070	0.042	3.88E44	3.74
BJ	4.00E15	1.86	9.47	8.84E3	0.047	0.010	1.41E45	0.81
	3.00E15	1.87	10.02	8.16E3	0.051	0.012	1.54E45	0.95
	2.00E15	1.79	10.96	6.96E3	0.059	0.018	1.63E45	1.35
	1.00E15	1.32	12.76	4.76E3	0.075	0.032	1.31E45	3.04
	8.00E14	1.13	13.34	4.13E3	0.078	0.036	1.14E45	4.00
	6.00E14	0.91	14.10	3.41E3	0.079	0.040	9.37E44	5.65
TI	4.00E15	1.74	10.33	7.51E3	0.045	0.018	1.31E45	1.63
	3.00E15	1.77	11.08	6.82E3	0.050	0.020	1.50E45	1.91
	2.00E15	1.77	12.32	5.81E3	0.058	0.024	1.79E45	2.55
	1.00E15	1.63	14.73	4.27E3	0.077	0.032	2.27E45	4.52
	8.00E14	1.58	15.44	3.91E3	0.083	0.033	2.42E45	5.36
	4.00E14	1.29	16.27	3.28E3	0.094	0.038	2.09E45	7.99

NOTE.—Nonrotating gravitational mass M (in units of $M_\odot$), radius R, the secular angular velocity Ω_s, the corresponding fractional changes in mass and radius $\delta M/M$ and $\delta R/R$ for the particular angular velocity Ω_s, the moment of inertia I, and the extent of the solid outer crust of neutron stars Δ_c, as functions of the central density ρ_c, for different equations of state. The powers of 10 by which each entry must be multiplied are represented by the numbers following the letter E.

TABLE 2

PARAMETERS OF NEUTRON STARS ROTATING ON THE POINT OF SECULAR ROTATIONAL INSTABILITY IMPLIED BY PSR 1937+214 FOR VARIOUS EQUATIONS OF STATE

Equation of State	$M_{r,\min}/M_\odot$	$R_{r,\max}$ (km)	$I_{\min}$ (g cm^2)	$I_{\max}$ (g cm^2)	$\Delta_c(M_{\min})$ (km)
BW	0.40	9.8	1.2×10^{44}	...	3.4
RP	0.70	11.7	4.0×10^{44}	1.1×10^{45}	3.2
FP	0.76	12.0	5.0×10^{44}	1.6×10^{45}	3.1
CDK	0.79	12.0	4.7×10^{44}	1.6×10^{45}	3.0
BJ	1.20	13.9	1.1×10^{45}	1.5×10^{45}	4.2
TI	1.72	15.6	1.5×10^{45}	2.4×10^{45}	5.0

values at $\Omega_s^2 = \Omega_0^2 = 1.63 \times 10^7$ rad^2 s^{-2} (indicated by an arrow on the horizontal axis in Figs. 1 and 2), which corresponds to the millisecond pulsar. The upper bounds on M_r and I are obtained in the standard way, by finding the points of maxima in the curves $M_r(\rho_c)$ and $I(\rho_c)$, where ρ_c = central density. These bounds are summarized in Table 2. Ignoring for the moment the BW model, all the "normal" neutron matter equations of state give a lower bound for the mass of PSR 1937+214 of $\gtrsim 0.7\ M_\odot$. One particularly stiff equation of state, the TI model, gives a value that is quite large (1.72 $M_\odot$). The spread in the range of I is small if one ignores the first and the last rows. Referring to Table 2, for the first five equations of state, the $I_{\min}$ is obtained from the condition of rotational instability, and $I_{\max}$ from the turnover point in the I versus Ω_s^2 curve; the $I_{\min}$ and $I_{\max}$ for the TI model are obtained in just the opposite way (see Fig. 2).

The mass-radius relationship for rotating neutron star configurations is shown in Figure 3. Figures 4 and 5 illustrate the dragging of inertial frames induced by rotation. The spherical stretching due to rotation and the density profiles of rotating neutron stars are shown in Figures 6 and 7.

The parameter Δ_c determines the total amount of elastic energy stored in the crust of the (nonrotating) neutron star. If, as suggested by Baym and Pines (1971), pulsar glitches are due to giant starquakes, then the amount of mechanical energy released serves as a constraint on the size of the glitches and also the mass of the neutron star. Additionally, Δ_c is of interest in determining the thermal and mechanical communication time scales between the liquid core and the surface of the star

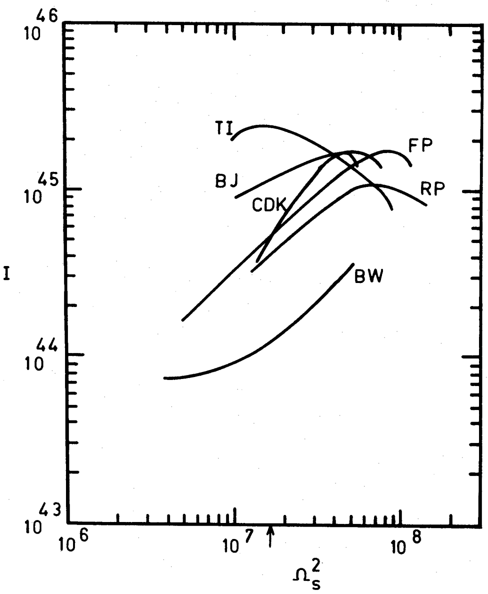

FIG. 2.—The moment of inertia corresponding to the secular angular velocity Ω_s vs. Ω_s^2 for different equations of state.

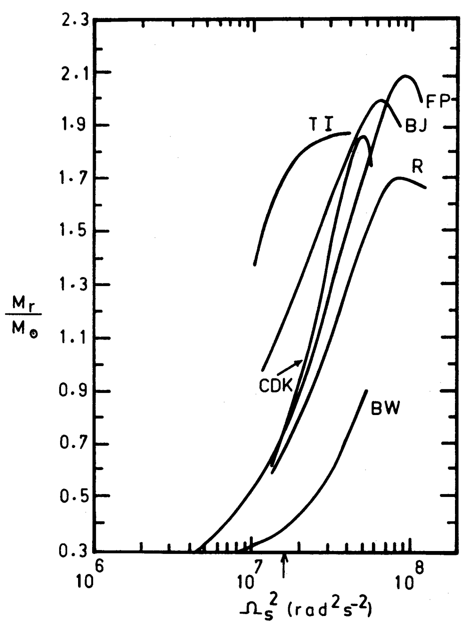

FIG. 1

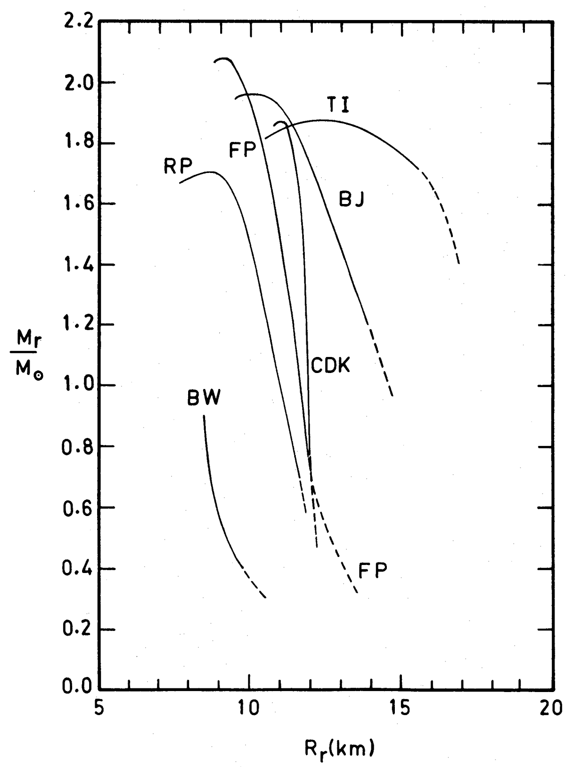

FIG. 3

FIG. 1.—The gravitational mass (in units of solar mass) corresponding to the secular angular velocity Ω_s vs. Ω_s^2 for different equations of state.

FIG. 3.—The gravitational mass (in units of solar mass) vs. radius (for rotating configurations corresponding to the secular angular velocity) for different equations of state.

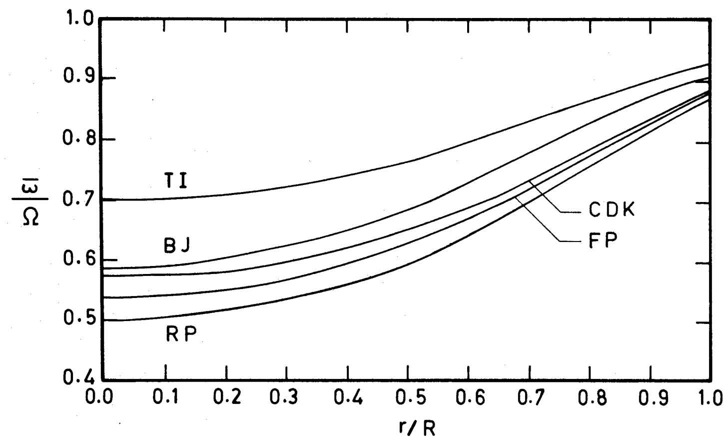

FIG. 4.—Fluid angular velocity at radial distance r (in units of the radius R) relative to the local inertial frame there, as measured by a distant observer, divided by the angular velocity of the fluid with respect to distant stars, for a neutron star with $M_r = 1.4\,M_\odot$ for different equations of state.

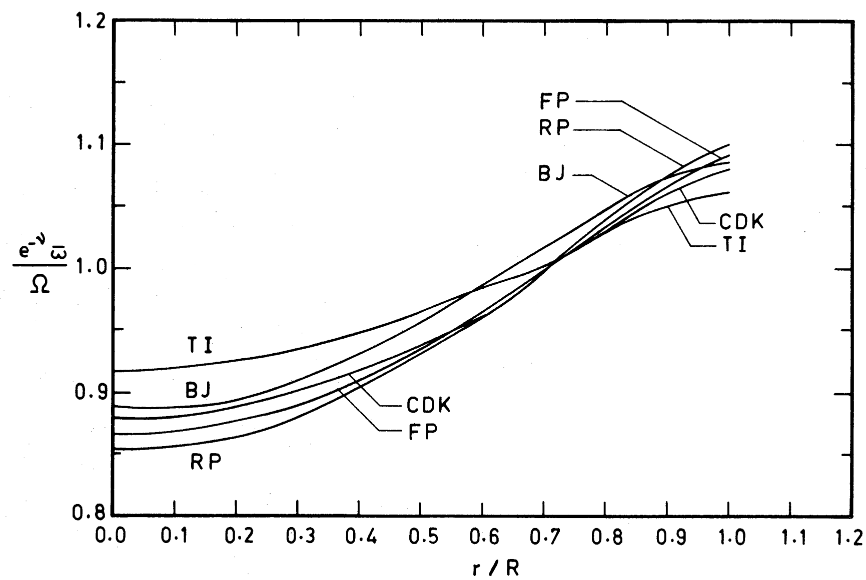

FIG. 5.—Fluid angular velocity at radial distance r (in units of the radius R) relative to the local inertial frame there, as measured by an observer in the fluid at r ($e^{-\nu}$ is the time dilation factor), for a neutron star with $M_r = 1.4\,M_\odot$ for different equations of state.

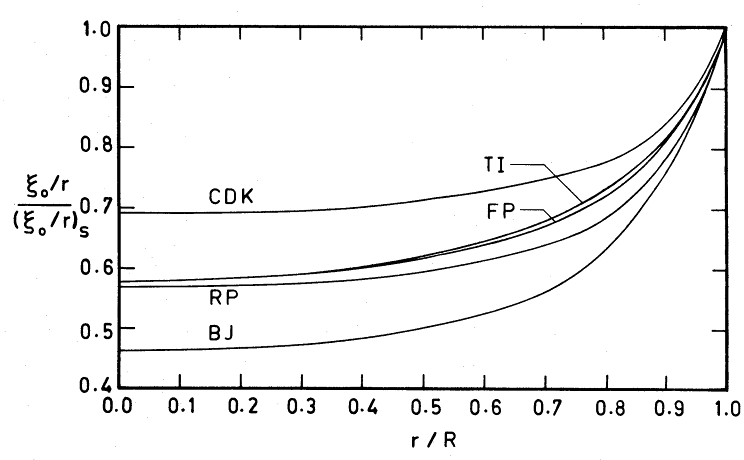

FIG. 6.—Spherical stretching due to rotation as a function of the radial distance r (in units of the radius R) for a neutron star with $M_r = 1.4\,M_\odot$ for different equations of state. The vertical axis represents the fractional change in coordinate radius ξ_0/r of the surfaces of constant density at r, divided by the fractional change $(\xi_0/r)_s = \delta R/R$ at the star's surface.

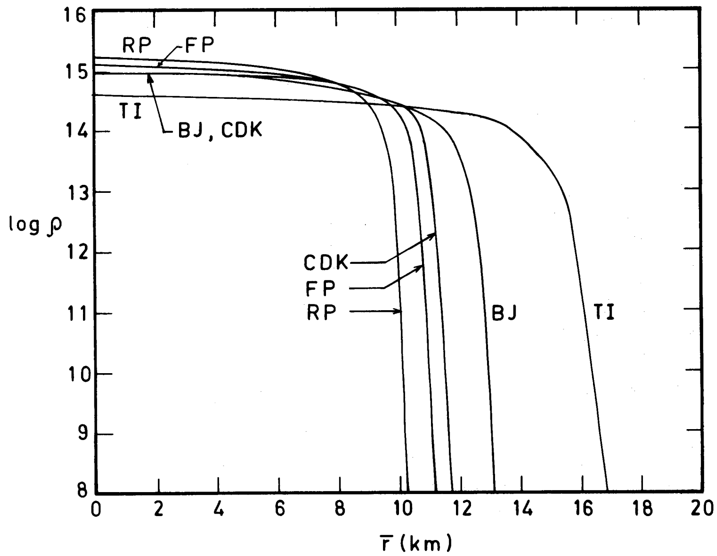

FIG. 7.—Density profiles of neutron stars ($M_r = 1.4\ M_\odot$) rotating at secular angular velocity (ρ is in g cm^{-3}) for different equations of state. The horizontal axis is the radial distance from the center of the star, incorporating spherical stretching due to rotation.

(Ray 1979, 1981) and can influence surface temperatures of a young neutron star (Nomoto and Tsuruta 1981). Since both radius (R_r) and crustal thickness (Δ_c) increase with decreasing mass, the third and the sixth columns in Table 2 represent the maximum values of R_r and Δ_c allowed by the six equations of state. Again, excluding the BW and TI models, $(R_r)_{max}$ and $(\Delta_c)_{max}$ span a rather narrow range.

Estimates of neutron star masses, mostly from binary X-ray sources, cover a wide range (Kelley and Rappaport 1981). In the case of the binary pulsar PSR 1936+16, however, the pulsar's mass has been accurately determined to be $(1.43 \pm 0.07)\ M_\odot$ from a measurement of periastron advance rate and time dilation arising from transverse Doppler shift and gravitational redshift (Taylor 1981). Joss and Rappaport (1984) have made a detailed study of the probability distribution of masses of six neutron stars which are part of X-ray binary systems and of the binary pulsar itself, using a Monte Carlo technique. The results of their analysis give masses that are consistent with a range of $(1.4 \pm 0.2)\ M_\odot$, expected, for example, in the collapse of accreting degenerate stars in a close binary system. For the five equations of state quoted in Joss and Rappaport, the *maximum* allowed masses for nonextreme equations of state lie between 1.4 $M_\odot$ and 2.7 $M_\odot$, and the authors conclude that the presently available observational mass estimates are marginally sufficient to constrain the equation of state of matter at high densities. In the current work we find that the *lower* limit given on grounds of rotational stability for one particular equation of state, namely, the TI model, is 1.72 $M_\odot$. This can be interpreted in two ways: either the millisecond pulsar (and objects of similar class) has a mass above 1.72 $M_\odot$, or the TI model is a somewhat unrealistic equation of state (assuming a mass of $[1.4 \pm 0.2]\ M_\odot$ to be characteristic of this class of objects). As pointed out earlier, nuclear physics arguments suggest that the TI model may indeed be somewhat unrealistic, and the current work on the basis of lower limits of neutron star masses may also constrain the validity of the TI model. For the more recent equations of state which may be taken to be more realistic, the predicted lower limits are consistent with observational data on neutron star masses. Thus the millisecond pulsar is not required to have a mass and a moment of inertia much in excess of 1.2 $M_\odot$ and 1.6×10^{45} g cm^2, respectively, on the basis of the five other equations of state reported here.

The entries in Table 2 have been arranged approximately in order of increasing stiffness of equation of state of dense neutron matter. It is clear that stiffer equations of state give larger lower bounds on mass and moment of inertia.

Though the masses reported in this work are for neutron stars on the verge of secular rotational instability, the same quantities for the dynamical instability (corresponding to $\tau = 0.2738$, or equivalently, $\Omega^2/2\pi G\bar{\rho} = 0.22$) can easily be constructed from the data reported in Table 1 by noting that $\delta M/M$ and $\delta R/R$ scale as Ω^2 for a given nonrotating (M, R). The secular limit, rather than the dynamical one, is reported here because the former is a stronger restriction on the rotational speed, and if the millisecond pulsar is envisaged as a neutron star spun up, the secular instability intervenes earlier than the dynamical.

Mention may be made here of very recent work on a subject similar to that considered in this paper. Harding (1983), using the same stability criterion as in equation (1) and a set of realistic *nonrotating* published models for neutron stars, gives lower limits of neutron star masses. However, because nonrotating models are used, these values are somewhat smaller than the corresponding limits obtained by us. Harding concludes (as we do) that the TI model may indeed be a somewhat unrealistic equation of state. Shapiro, Teukolsky, and Wasserman (1983), on the other hand, have considered published models of equilibrium *spherical nonrotating* stars that can be uniformly spun up to $P = 1.558$ ms and will still remain in hydrostatic equilibrium. Their treatment is general relativistic and valid for arbitrarily fast rotations of (1) a uniform density and (2) a centrally condensed Roche model. These considerations are *equilibrium* ones in contrast to the stability requirements. They conclude that the existing millisecond pulsar PSR 1937+214 does not constrain any of the currently available equations of state of neutron star matter, and searches for pulsars down to $P = 0.5$ ms should be made in order to put meaningful restrictions on the existing equations of state. While shorter period pulsars will certainly be useful in constraining the more realistic and interesting equations of state, we note that even the equilibrium Roche model of Shapiro, Teukolsky, and Wasserman (1983) predicts, for the TI equation of state, a mass $\gtrsim 1.6\ M_{\odot}$, which is on the borderline for the allowed range of masses implied by Joss and Rappaport (1984).

From an analysis of the unique position in the P-$\dot{P}$ (period and its time derivative) diagram occupied by PSR 1937+214 and five other pulsars, Alpar *et al.* (1982) have suggested that this class of radio pulsars may have had a different genesis and evolution from all other long-period pulsars. They and other authors (Radhakrishnan and Srinivasan 1982; Fabian *et al.* 1983) have argued that such objects could have been spun up during an accretion phase, and they have related their periods to neutron star parameters like the mass and magnetic field as well as the accretion rate. A new millisecond pulsar in a binary (PSR 1953+29) has been discovered by Boriakoff, Buccheri, and Fauci (1983), the spin-up scenario for which has been argued by several authors to be essentially the same as for PSR 1937+214 (see, for example, Helfand, Ruderman, and Shaham 1983 and references therein and other articles in the same issue). Since these pulsars form a separate class, their structural parameters are expected to be similar. In the case of the millisecond pulsar PSR 1937+214, we found the lower limits of mass, moment of inertia, etc., for given neutron matter equations of state. Thus, these lower limits should also apply to the other low (P, $\dot{P}$) pulsars, since from independent considerations, these should have similar parameters. Hence, the theoretical limits on mass and moment of inertia reported here will have a more general validity for at least a class of neutron stars.

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B. DATTA: Indian Institute of Astrophysics, Sarjapur Road, Bangalore 560034, India

A. RAY: Theoretical Astrophysics Group, Tata Institute of Fundamental Research, Homi Bhabha Road, Bombay 400005, India