Astrophysical Letters, 1984, Vol. 24, pp. 21–25
0004-6388/84/2401-0021/$18.50/0

Electromagnetic Radiation in a Helical Magnetic Field

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(*Received October 13, 1982; in final form April 13, 1983*)

Electromagnetic radiation spectrum is studied in a helical magnetic field since such magnetic field configurations may be present in cosmic objects. It is shown that, by a proper choice of the spatial period of the magnetic field, one could get large amounts of power over a wide region of the electromagnetic spectrum. The peak in the radiation spectrum shifts towards higher frequencies as the spatial period of the magnetic helix is reduced. As an illustration, this mechanism is applied to the Crab Nebula pulsar NP 0532.

Synchrotron radiation has been proposed to be the emission mechanism for several cosmic objects. The measurements of the characteristics of the synchrotron radiation enable one to determine the physical conditions relating to the energy distribution of the relativistic electrons and the magnetic fields, Ginzburg and Syrovatskii (1965, 1969). The most commonly considered model of the synchrotron source consists of relativistic electrons spiralling in a uniform magnetic field. Synchrotron radiation in a dipole magnetic field has been studied in order to explain the observed decameter radiation from Jupiter Korchak (1963), Thorne (1963). Synchro-compton and inverse compton processes have been proposed for a host of objects at galactic and extragalactic scales Fabian and Rees (1979). The polarization measurements of the X-ray band in Crab Nebula are in favour of the synchrotron mechanism Novic *et al.* (1972). The same cannot be said about the extended extragalactic X-ray and radio sources. In addition the magnetic field configuration varies from object to object. For example in low luminosity radio galaxies, the magnetic field is observed to be mainly perpendicular to the jets, whereas in high luminosity sources, the polarization indicates a jet aligned magnetic field Miley (1980). In fact it is difficult to derive the three dimensional structure of the magnetic field in jets and other sources. Here, we assume that the three-dimensional magnetic field configurations may be of a helical form as has been recently suggested. A helical structure of the magnetic field can result in several situations. The Fourier analysis of space dependent random fields does project spatial periodicity in the field, Perola (1980). The ambient uniform field gets modified due to the inclusion of the field of magnetohydrodynamic waves as in the case of crab nebula, Melrose (1980). Force free magnetic fields are known to be present in many Astrophysical situations and have been measured in the case of the Sun. The currents following parallel to an axial magnetic field give rise to an azimuthal component and to a helical configuration. This has been discussed in an earlier paper Krishan (1982a). It is also possible to generate spatially periodic magnetic field through langmuir turbulence in which case the magnetic spatial period comparable to the wavelength of electron plasma waves can be produced, Belkov and Tsytovich (1982). And again the Langmuir turbulence is easily generated through currents and nonthermal particle distributions. Electron beams propagating in a static periodic magnetic field have been suggested as the source of sporadic solar radio emission Krishan (1980, 1982b).

The present paper describes the properties of electromagnetic radiation in a three-dimensional helical magnetic field. It is found that the power emitted and the extent of the frequency spectrum depend upon the period of the magnetic helix and this system is much more efficient than the one with a uniform magnetic field. We take the source to be composed of relativistic electrons streaming in a magnetic field of the form

$$\vec{B} = B_0 \sin k_0 z\hat{x} + B_0 \cos k_0 z\hat{y} + B_z\hat{z} \tag{1}$$

where $(\hat{x}, \hat{y}, \hat{z})$ are unit vectors. The trajectory of an electron in this magnetic field is found to be:

$$\vec{r}(t) = -r_0 \cos \omega_0 t\hat{x} + r_0 \sin \omega_0 t\hat{y} + \beta_z ct\hat{z} \tag{2}$$

and the velocity:

$$c\vec{\beta}(t) = \vec{V}(t) = V_0 \sin \omega_0 t\hat{x} + V_0 \cos \omega_0 t\hat{y} + \beta_z c\hat{z} \tag{3}$$

with the assumption

$$\dot{V}_z = 0, V_z = \beta_z c$$

where $r_0 = K/k_0$ is the radius of the helix described by the electron,

$$K = \frac{\Omega_0}{\omega_0 + \Omega_z}, \omega_0 = k_0 c\beta_z, V_z = \beta_z c$$

is the z component of the velocity,

$$\Omega_0 = \frac{eB_0}{mc\gamma_z}, \Omega_z = \frac{eB_z}{mc\gamma_z}, \gamma_z = [1 - V_z^2/c^2]^{-1/2} \quad \text{and} \quad V_0 = \frac{\Omega_0 \beta_z c}{\omega_0 + \Omega_z}$$

Knowing the trajectory of the particle, one can calculate the energy radiated per unit band width by one electron into a solid angle $\alpha\Omega$ by using the formula Jackson (1950), Pacholczyk (1970).

$$\frac{dI(\omega)}{d\Omega} = \frac{e^2\omega^2}{4\pi^2 c}\left|\int_{-\infty}^{\infty} \hat{n} \times (\hat{n} \times \vec{\beta}) \exp\left\{i\omega\left(t - \frac{\hat{n}\cdot\vec{r}(t)}{c}\right)\right\} dt\right|^2 \tag{4}$$

where $\hat{n}$ is a unit vector pointing from the origin to the observer:

$$\hat{n} = \sin\theta\hat{y} + \cos\theta\hat{z} \tag{5}$$

One can evaluate (4) to find:

$$\frac{dI(\omega)}{d\Omega} = \frac{e^2\omega^2 K^2}{\pi^2 c\omega_0^2\gamma_z^2} \sum_{n=1}^{\infty} \left[\frac{J_n'^2(x) + \left(\frac{\gamma_z\theta}{K} - \frac{n}{x}\right)^2 J_n^2(x)}{\left(\frac{\omega}{\omega_1} - n\right)^{+2}}\right]^2 \sin N\pi\left(\frac{\omega}{\omega_1} - n\right) \tag{6}$$

where

$$x = \frac{K\omega \sin\theta}{\gamma_z\omega_0}, \omega_1 = \frac{2\omega_0\gamma_z^2}{(1 + \theta^2\gamma_z^2 + K^2)}, J_n$$

and J_n' are Bessel function and its derivative, and N is the number of periods in the magnetic field of the source, $N\lambda_0 = 2\pi N/k_0$ is the length of the magnetic helix. The case with $B_z = 0$ has been treated in the context of free electron laser Kincaid (1977). We may note here that the inclusion of B_z changes the value of K, which changes the radiation pattern significantly. For example if $\omega_0 < \Omega_z$ and Ω_0 then K remains close to unity. When $\Omega_z \to 0$, and only transverse component survives, K could have very small to very large values. One can integrate Equation (6) over all frequencies in order to study the angular distribution of power $P(\Omega)$. A plot of $P_\Omega = \int (dI(\omega)/d\Omega) d\omega$ is shown in Figure (1),

where
$$P_\Omega = \frac{8e^2K^2N\gamma_z^4\omega_0}{c(1+K^2+\gamma_z^2\theta^2)^3}\sum_{n=1}^{\infty} n^2\left\{J_n'^2(x_n) + \left(\frac{\gamma_z\theta}{K} - \frac{n}{x_n}\right)^2 J_n^2(x_n)\right\}$$

The radiation spectrum consists of the harmonics of the frequency ω_1 which may be thought of as the fundamental frequency. This resembles the suggestion made in the literature (Bertotti *et al.* (1970)) for Crab Nebula pulsar that all the observed spectrum is associated with a basic frequency Ω_0 = angular rotation frequency of the star. Further the bunching length of the particles desired for coherent radiation, Bertotti *et al.* (1970) could be provided by the spatial period of the helical magnetic field proposed here, since this has been observed in the laboratory experiments Hopf *et al.* (1979). One can calculate the power emitted by N_0 electrons:

$$P(\omega) = 4\pi^2c^2e^2\gamma_z n_0 N^2 K\frac{\Delta\omega}{\omega_0^2} \times \sum\left[J_n'^2(x_n) + \left(\alpha_n - \frac{n}{x_n}\right)^2 J_n^2(x_n)\right] \times \theta(\alpha_n^2) \quad (7)$$

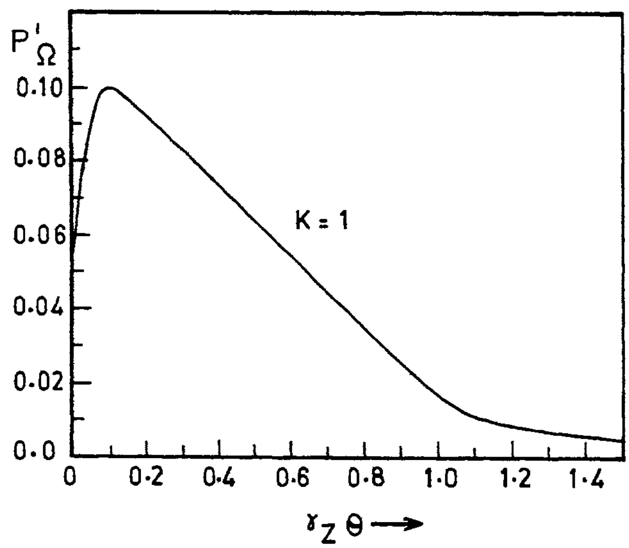

FIGURE 1 A Plot of $P'_\Omega = P_\Omega[8e^2K^2N(\gamma/c^4)\omega_0]^{-1}$ vs $\gamma\theta$ showing the angular distribution of radiation.

where

$$\alpha_n^2 = \frac{2n\gamma_z^2\omega_0}{\omega} - 1 - K^2, x_n = \frac{K\omega\alpha_n}{\gamma_z^2\omega_0}, \Delta\omega$$

is the band width, n_0 = electron density and $\theta(\alpha_n^2)$ is the step function. One observes that for large values of K, the maximum of radiation occurs at low frequencies. On the other hand the radiation intensity is low for low K. Thus a moderative value of K is desirable. Here we present a representative case with $K \sim 1$. The power $p(\omega)$ has been evaluated for several values of the spatial period of the magnetic field and is shown in Figure (2). The plot shows $\log P$ vs $\log \omega$ for several values of λ_0 such that the length of the source $N\lambda_0$ is a constant. The density n_0 is taken to be $10^{15}/\text{cm}^3$ and $K \sim 1$, with the assumption of $\omega_0 < \Omega_z$. The value of $N\lambda_0$ is fixed by fixing the power at $\omega = 10^9/\text{sec}$ to be $\sim 10^{31}/\text{ergs/sec}$, representative of the Crab Nebula pulsar[13]. One finds that as ω_0 increases, the power spectrum extends to higher frequencies. Thus by a proper choice of the spatial period of the magnetic field, one could get a large amount of power over a wide range of frequencies. In general, the spatial period may vary from radio region to X-ray regions, being large for extended radio sources and small for compact X-ray sources. The polarization of the radiation emitted is in general circular.

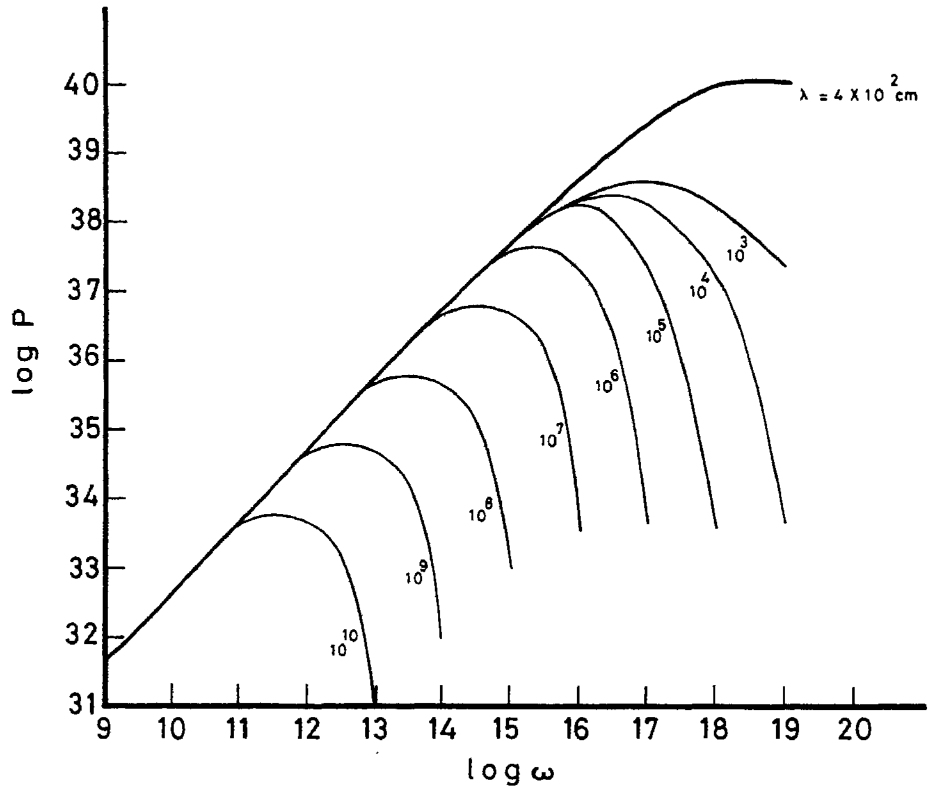

FIGURE 2 A plot of $\log P$ vs $\log \omega$ for several values of λ_0 (Equation (7)).

If the electron beam has a finite angular divergence i.e. the electrons have a pitch angle distribution, the radiation at a given frequency will be emitted over a finite range of viewing angles θ. In order to include this effect one should multiply the angular distribution of radiation Equation (6) with the angular divergence function of the electron beam. Let the distribution of the viewing angles be a Gaussian of the form

$$W(\theta) = \frac{1}{2\pi\sigma^2} e^{-\theta^2/2\sigma^2}$$

then the brightness function is determined by

$$\int \frac{dI(\omega)}{d\Omega} \frac{1}{\sigma^2} e^{-\theta^2/2\sigma^2} \, d\theta \sin\theta$$

The effects of the pitch angle distribution of the electron beam which results in a range of viewing angles for each frequency are to reduce the brightness by a factor $\sigma^2 N$ at a given frequency and in the overlapping of different harmonics.

In conclusion, we may point out that the attempt here has been to show the potential of the helical configuration of the magnetic field in exciting the radiation spectrum over a wide range of frequencies, since such magnetic field configurations may be expected to be present in cosmic objects. The treatment given here is rather idealized and will be extended to include the cases of the low magnetic fields, the non-circular helical configuration, and the non-monoenergetic nature of the electron beam. The detailed properties of the polarization of the radiation emitted in a helical magnetic field are under study.

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