NONLINEAR RESPONSE OF SLENDER MAGNETIC FLUX TUBES TO EXTERNAL PRESSURE FLUCTUATIONS

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(Received 8 May, 1985; in revised form 31 January, 1986)

Abstract. The effect of applying external pressure fluctuations on slender flux tubes is studied as a nonlinear initial value problem. Large amplitude velocity oscillations are seen to be produced when the frequency of the imposed fluctuations matches the natural frequency of the tube. Radiative cooling does not significantly damp these resonantly built-up oscillations. The absence of observational evidence for such a resonant response of the tubes is used to put a constraint on the length of tubes.

1. Introduction

The solar photosphere supports a wide variety of dynamical phenomena, e.g., granulation, oscillations and waves. There are also intense magnetic flux tubes which will be acted upon by the pressure fluctuations caused by these various kinds of dynamical processes. Several questions arise in connection with the understanding of the actual nonlinear response of the tubes to such external pressure fluctuations. The first question concerns the efficiency of this process for generating fluid motions. The second concerns the nature of the resulting flow, viz., whether it is oscillatory or not, and if not, whether the flow is predominantly upwards or downwards into the Sun. A third question devolves on the role of radiative processes in damping the flow.

Answers to these questions exist in the literature mostly on the basis of linear theory. These will be enumerated in Section 2.

2. Results from Linear Theory

A rigorous analysis of the response of a flux tube to external pressure fluctuations requires the separate solutions of the wave modes within the tube and in its environment. Boundary conditions must then be applied at the interface of the tube and its surroundings to evaluate the degree of coupling of the tube's motions to the external pressure perturbations. However, for periods of these fluctuations larger than the time taken for a fast magnetosonic wave to cross the tube, one can conveniently consider the external pressure fluctuations to act as forcing terms in the differential equation representing the vertical motions of gas within the tube. Using such a forcing term, Roberts (1983) showed that the frequency response of the resulting motion tends to infinity when the period of the external fluctuation matches the time taken for a tube wave to traverse one wavelength of the fluctuation. As a result, one can expect that if a broad spectrum of waves squeeze the tube, then the resulting motion of the gas in the tube would contain only the 'resonant' or matching frequencies. If the tube is further constrained by

Solar Physics **104** (1986) 347–357.

boundary conditions at its base and top, then a further screening of frequencies would result.

The above result of a resonance was obtained under adiabatic conditions. However, for short period waves, radiative relaxation becomes important near the temperature minimum. Webb and Roberts (1980) considered the propagation of non-adiabatic waves within slender flux tubes and showed that non-adiabaticity leads to spatial damping of these waves. Thus one would expect a decrease of the relative amplitude of such waves with increasing height. However, application of external pressure fluctuations to such waves modulate the spatial form of their amplitude leading to an increase of relative amplitude with height (Roberts, 1983). What is not known from such linear analysis is the absolute amplitude of the motions. In what follows we shall attempt to estimate what amplitudes would result from a given amount of pressure fluctuation, the nonlinear response to different frequencies of the fluctuation, as well as the effect of radiative losses on the finite amplitude motions produced by the fluctuations.

It could be argued that since the pressure fluctuations in the photosphere arise out of subsonic velocity fields, nonlinear effects could be neglected. This is not true if one considered the long time behaviour of the tubes. Nonlinear effects sometimes generate persistent, albeit small, unidirectional accelerations which do not cancel out if integrated over one wave period (Hasan and Venkatakrishnan, 1980). Furthermore, resonant interactions could lead to large amplitude motions. In a recent numerical study of the time-dependent nonlinear response of tubes to external turbulence (Venkatakrishnan, 1984) it was shown that resonant response exists and only non-resonant interactions lead to non-oscillatory flow in the tubes. The present paper describes these results as well some additional results of extending the previous calculations to longer times and of including the radiative exchange of heat by the tubes with their surroundings.

3. The Basic Equations

The equations for a slender magnetic flux tube were first rigorously derived by Roberts and Webb (1978). These were later on extended for inclusion of radiative heat transport in optically thin tubes (Webb and Roberts, 1980) as well as in optically thick tubes (Venkatakrishnan, 1985; Hasan, 1985). In the present case, we shall confine our attention to optically thin tubes assuming small optical thickness for the upper photospheric tubes. The complete set of equations then is:

$$\frac{\partial}{\partial t}(\rho/B) + \frac{\partial}{\partial z}(\rho v/B) = 0\,, \tag{1}$$

$$\frac{\partial}{\partial t}v + v\frac{\partial}{\partial z}v + \frac{1}{\rho}\frac{\partial}{\partial z}p + g = 0\,, \tag{2}$$

$$B^2 = 8\pi(p_e - p)\,, \tag{3}$$

$$\left(\frac{\partial}{\partial t} + v\frac{\partial}{\partial z}\right)p - \left(\frac{\gamma p}{\rho}\right)\left(\frac{\partial}{\partial t} + v\frac{\partial}{\partial z}\right)\rho = (\gamma - 1)\rho\frac{C_v}{\tau}(T_e - T), \tag{4}$$

$$p_e(z, t) = p_{e_0}(z) + p_{e_1}(z, t), \tag{5}$$

where p_{e_1} is assumed to be one of the two following forms:

$$p_{e_1} = \hat{p}_{e_1} \exp(-z/H) \sin \omega t \tag{6a}$$

and

$$p_{e_1} = \hat{p}_{e_1} \sin kz \sin \omega t. \tag{6b}$$

In the above equations ρ is the density; p, the pressure; v, the velocity; B, the magnetic field; and T, the temperature within the tube. Outside the tube p_e is the gas pressure, composed of a static part p_{e_0} and a fluctuating part p_{e_1}. This fluctuating part is assumed to be either the two forms in Equation (6) representing spatially evanescent and oscillatory disturbances respectively. T_e is the ambient temperature of the gas surrounding the tube and τ is the timescale for radiative relaxation.

These equations form a system of hyperbolic partial differential equations and can, therefore, be cast into characteristic form. The characteristic equations are:

$$\mathrm{d}p \pm \rho C_T \,\mathrm{d}v = (\varepsilon \pm \mu)\,\mathrm{d}t, \quad \text{along} \quad \frac{\mathrm{d}z}{\mathrm{d}t} = v \pm C_T \tag{7}$$

and

$$\mathrm{d}p - S^2\,\mathrm{d}\rho = Q\,\mathrm{d}t, \quad \text{along} \quad \frac{\mathrm{d}z}{\mathrm{d}t} = v, \tag{8}$$

where

$$C_T^2 = S^2A^2/(S^2 + A^2); \qquad A^2 = B^2/4\pi\rho; \qquad S^2 = \gamma p/\rho;$$

$$\varepsilon = \frac{C_T^2}{A^2}\left(\frac{\partial}{\partial t} + v\frac{\partial}{\partial z}\right)p_e + \frac{C_T^2}{S^2}Q; \qquad \mu = -g;$$

$$Q = \frac{(\gamma - 1)}{\tau}\rho C_v(T_e - T).$$

These equations can now be integrated using a suitable algorithm. In this paper a backward marching algorithm was used (cf., e.g., Zucrow and Hoffman, 1976).

4. The Initial and Boundary Conditions

To start the integration of Equations (7) and (8) one must specify the variables at some reference time $t = 0$, say. For this we assumed an initial polytropic state in hydrostatic equilibrium and also in thermal equilibrium with the surroundings. In this case the only parameters describing the state are Γ, the polytropic index, and β, the ratio of gas pressure to magnetic pressure. The initial state at $t = 0$ is thus

$$T = T_b\left(1 - \frac{\Gamma - 1}{\Gamma}\frac{z}{\Lambda}\right), \qquad (\Lambda = \mathcal{R} T_b/g), \tag{9a}$$

$$\rho = \rho_b(T/T_b)^{1/(\Gamma - 1)}, \tag{9b}$$

$$p = \mathcal{R} \rho_b T_b(\rho/\rho_b)^{\Gamma}, \tag{9c}$$

$$B = (8\pi p/\beta)^{1/2}, \tag{9d}$$

$$\rho_e = \rho(1 + 1/\beta), \tag{9e}$$

$$p_e = p(1 + 1/\beta), \tag{9f}$$

$$T_e = T, \tag{9g}$$

where T_b and ρ_b are the temperature and density at the base of the tube.

For $t > 0$ one requires boundary conditions to be applied at the bottom and top of the tube. However, the number of boundary conditions is determined by the number of characteristics communicating from the interior to the boundary. In the present calculations we choose the following boundary conditions:

$$p(z = 0; t) = \mathcal{R} \rho_b T_b$$

and

$$p(z = d; t) = 2p(z = d - \Delta z; t) - p(z = d - 2\Delta z; t).$$

Whenever auxilliary boundary conditions become necessary due to failure of the v-characteristics to reach the boundary then the following density boundary condition was used:

$$\rho(z = 0; t) = \rho_b$$

or

$$\rho(z = d; t) = 2\rho(z = d - \Delta z; t) - \rho(z = d - 2\Delta z; t),$$

where Δz is the spatial step size.

5. Results

As mentioned in the previous section, we assumed an initial hydrostatic equilibrium with a polytropic stratification and space independent β. The time dependent calculations were performed only for a single value of $\beta = 2.0$. In the solar photosphere, this value of β corresponds to ≈ 1800 G which is approximately the value of the field within intense magnetic flux tubes. Since the magnetic field imparts a kind of 'rigidity' to the associated fluid, weaker fields would imply a 'softer' equation of state for the tube and, therefore, one would expect smaller values for the resulting longitudinal flow as compared to the response for a tube with stronger fields. We chose the value of the polytropic index $\Gamma = 1.1$ for matching with the Harvard–Smithsonian reference atmosphere (Gingerich *et al.*, 1971) for the first 500 km above the photosphere.

The form of the external pressure fluctuations given in Equations (6) was chosen to simulate both spatially evanescent fluctuations characteristic of the granulation as well as spatially oscillatory fluctuations characteristic of waves and oscillations. The amplitude of the pressure fluctuations was chosen to be 1% representing 4% variations in the intensity caused by the corresponding changes in temperature.

The time dependence of the longitudinal velocity near the midpoint of the tube ($z = 0.48$) is shown in Figure 1 for evanescent perturbations with three values of the

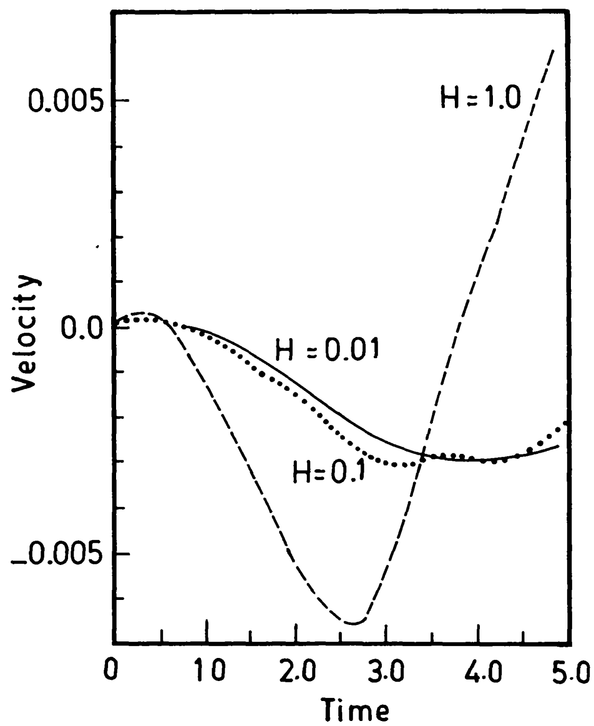

Fig. 1. Time-dependence of gas velocity along the tube seen at $z = 0.48$ for exponentially decaying external pressure fluctuations. The time unit in this and other figures corresponds to one free fall time over one scale height which is the unit of length. The velocity is given in terms of the sound velocity at the base of the tube.

scale length H (in units of the pressure scale height at the base of the tube). It is evident from the figure that the response is more vigorous for the case of $H = 1.0$, than for the other two cases. It is worth mentioning here that $H = 1.0$ corresponds to that case where the decay length of the fluctuations is closest to the distance travelled by a tube weve during one period of the fluctuation.

It is also worth noting that the response for $H = 0.01$ and $H = 0.1$ has a tendency to lag behind the forcing frequency. A consequence of this lagging is seen in the spatial velocity profiles for the case of $H = 0.01$ (Figure 2). We see that the node in the profile at $t = 1.00$ does not exist for $t = 2.95$ and $t = 4.90$. Thus the possibility of maintaining

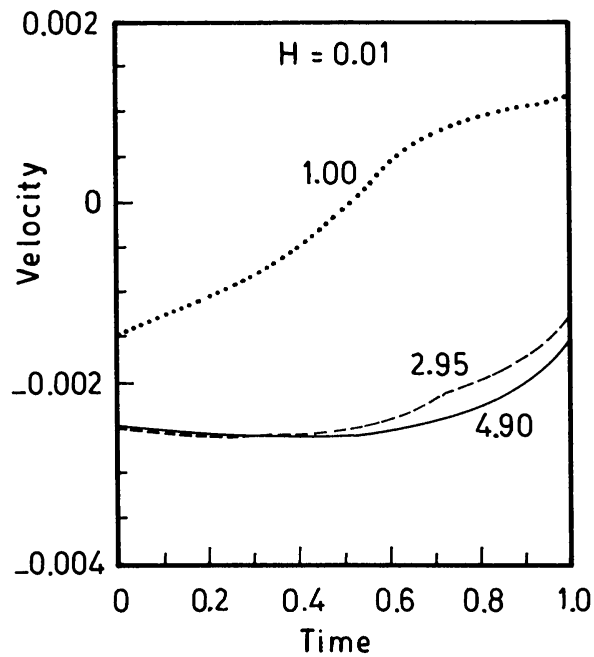

Fig. 2. Spatial profiles of the velocity for $H = 0.01$ at various intervals of time as marked in the figure.

unidirectional flows for longer times exists in the case of pressure fluctuations confined to the base of the tube. However, the magnitude of such flows is seen to be rather small.

The maximum response to a spatially oscillatory fluctuation also occurs when the wavelength of the fluctuation matches the distance travelled by a 'tube wave' within one

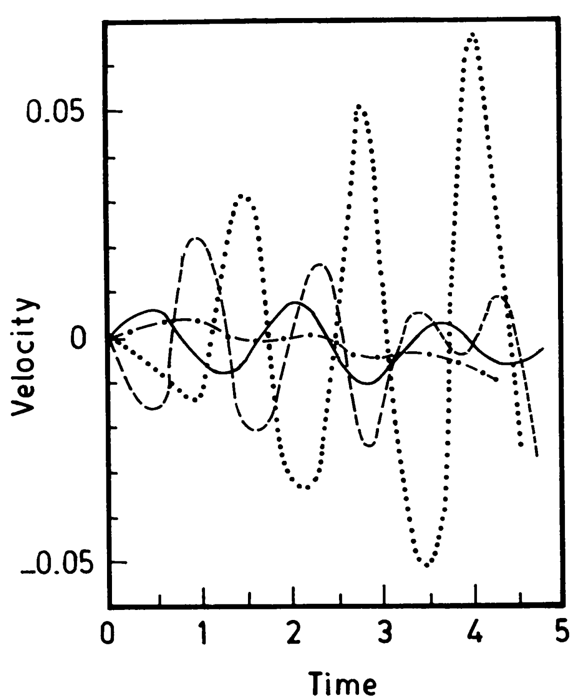

Fig. 3. Time-dependence of velocity for spatially periodic external pressure fluctuations, for unit length of tube and the spatial periodicity double this length. The different values of ω/kC_T are: 10.0 (full line), 1.25 (dashes), 1.0 (dots), and 0.2 (dash dot).

period of the fluctuation. This is seen in Figure 3 where the time-dependence of longitudinal velocity is plotted for different values of ω/kC_T where ω is the frequency and k the wavenumber of the external perturbations, C_T being the phase velocity of tube waves. A clear increase in the response is seen for $\omega/kC_T = 1$ as compared for $\omega/kC_T = 0.1$ and $\omega/kC_T = 10.0$, respectively.

The question that next comes to mind is whether boundary conditions affect this resonance. In Figure 3, the wavenumber was chosen as π/d where d is the length of the tube. To settle this question, further computations were performed where the wavenumber was chosen to be $2\pi/d$. Figures 4(a–c) show the spatial elocity profiles for $\omega/kC_T = 0.1$, 1.0, and 10.0, respectively. Again, it is clearly seen that maximum response occurs for $\omega/kC_T = 1.0$. Furthermore the computations were extended to larger durations than those depicted in Figure 3. Thus the resonance seems to persist for long times.

Finally, let us consider the role of radiative processes in damping the resonance. Figure 5 represents the time-dependence of the velocity for zero damping ($1/\tau = 0$ in Equation (5)) at three spatial points ($z = 0.0$, 1.5, and 3.0) for a tube of length $d = 3.0$

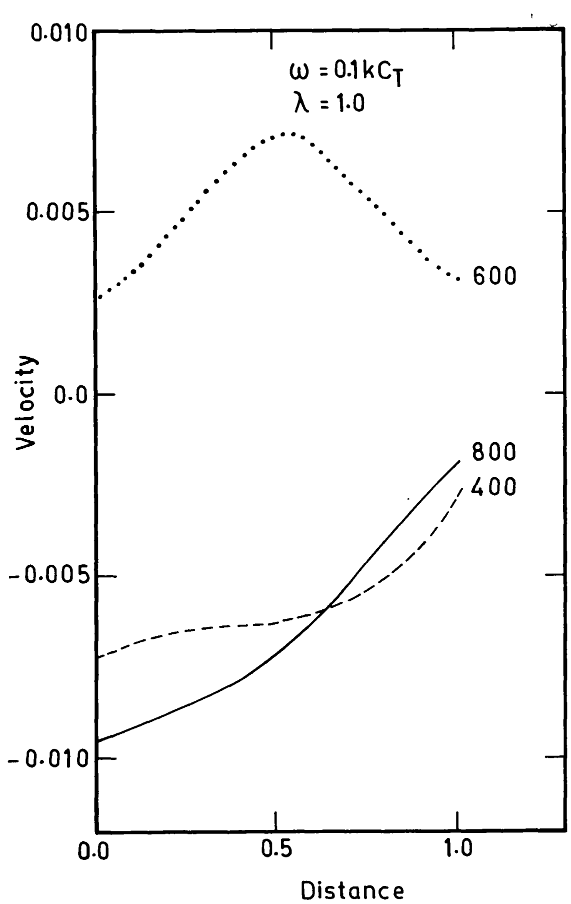

Fig. 4a.

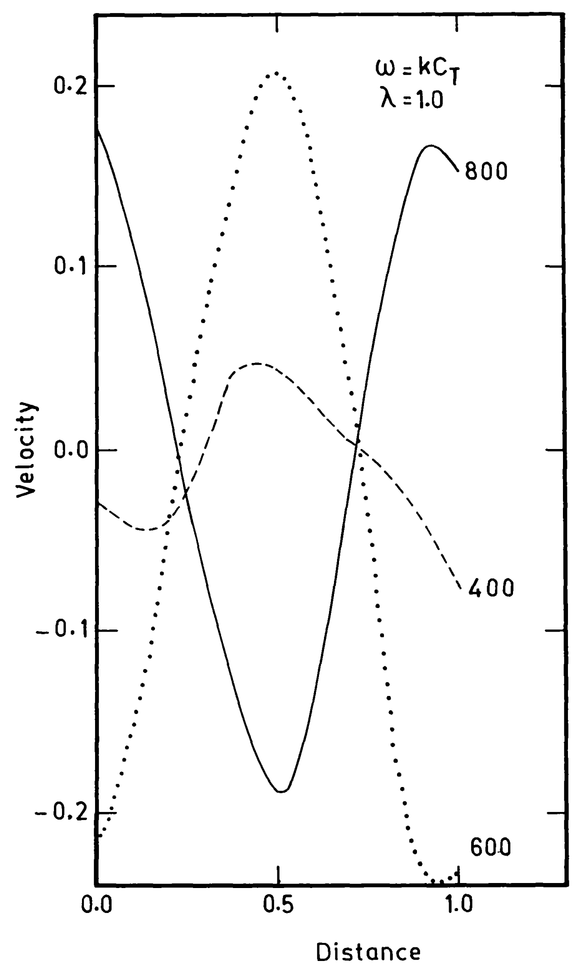

Fig. 4b.

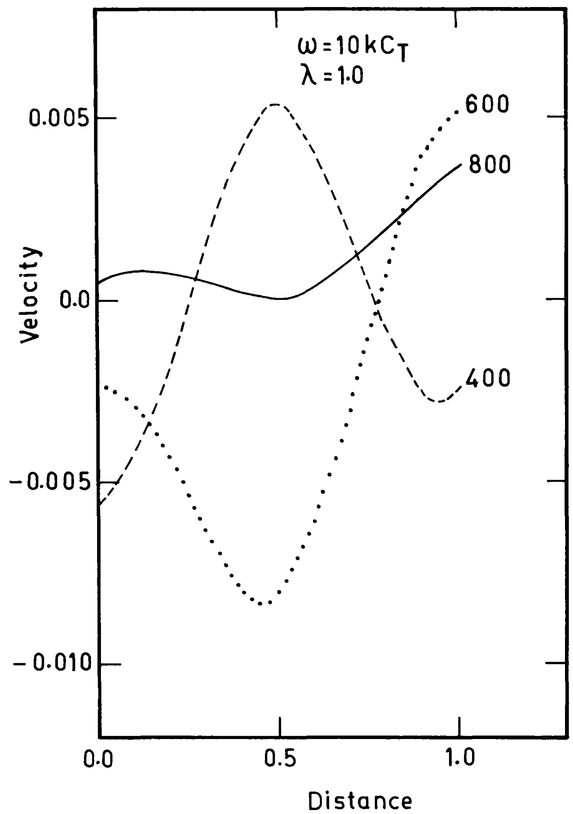

Fig. 4c.

Fig. 4(a–c). Spatial profiles of the velocity for unit length of tube and equal length of spatial periodicity for the cases (a) $\omega/kC_T = 0.1$, (b) $\omega/kC_T = 1.0$, and (c) $\omega/kC_T = 10.0$.

(base pressure scale heights) with wavenumber of external fluctuations = $2\pi/d$ and frequency = $2\pi C_T/d$. It is seen that there is an increase of amplitude with height which is probably related to the decrease of gas density with height. Figure 6 shows a similar time-dependence, but now for a fixed spatial position $z = 1.5$ and for different values of cooling time $\tau = \frac{1}{3}, \frac{4}{3}$, and ∞, respectively. There is evidence for some damping of motion for $\tau = \frac{4}{3}$ as compared to $\tau = \infty$, while there is not much difference in amplitude between the motions for $\tau = \frac{4}{3}$ and $\tau = \frac{1}{3}$. Another effect produced by the radiative term is to introduce a phase lag between the forcing term and the response.

6. Discussion

The main results of this paper are that (a) resonance does lead to large amplitude motions within slender magnetic flux tubes and (b) radiation does not seem to significantly damp these large amplitude motions if the cooling time is shorter than the period of external fluctuations. On the other hand, the only direct observations of vertical velocities within magnetic elements (Giovanelli *et al.*, 1978) show neither such large amplitude motions nor any evidence for shocks at least within ≈500 km above the

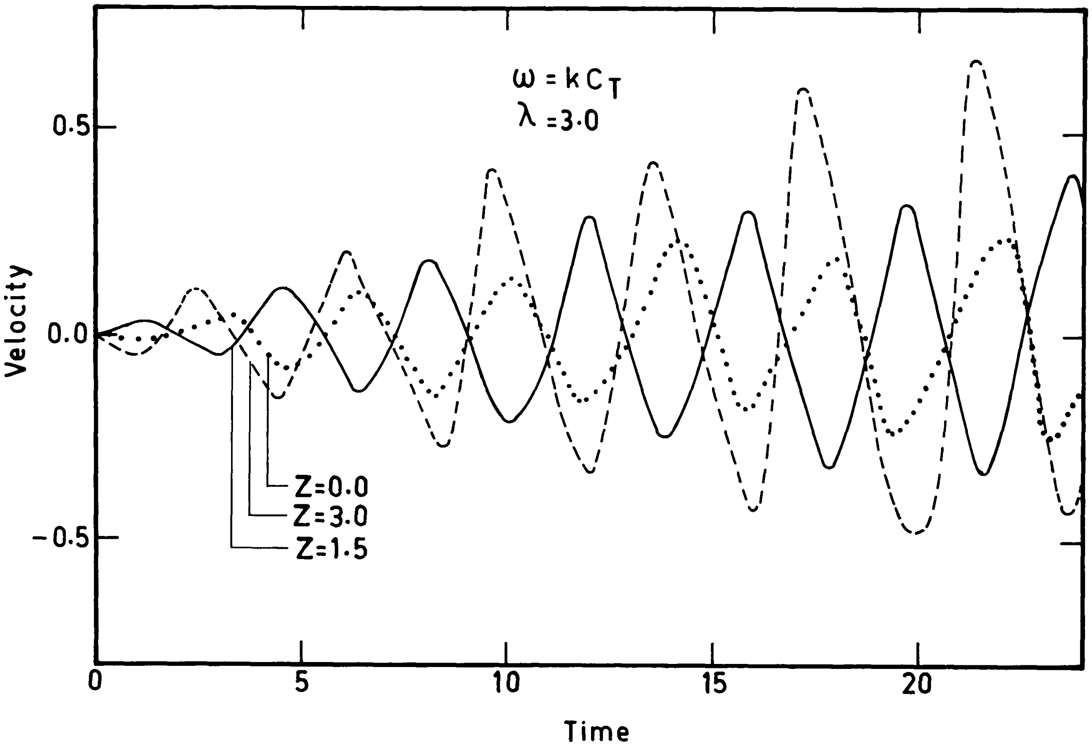

Fig. 5. Time-dependence of velocity at $z = 0.0$, 1.5, and 3.0, respectively, for length of tube $d = 3.0$ and wavelength = 3.0 at the resonant frequency.

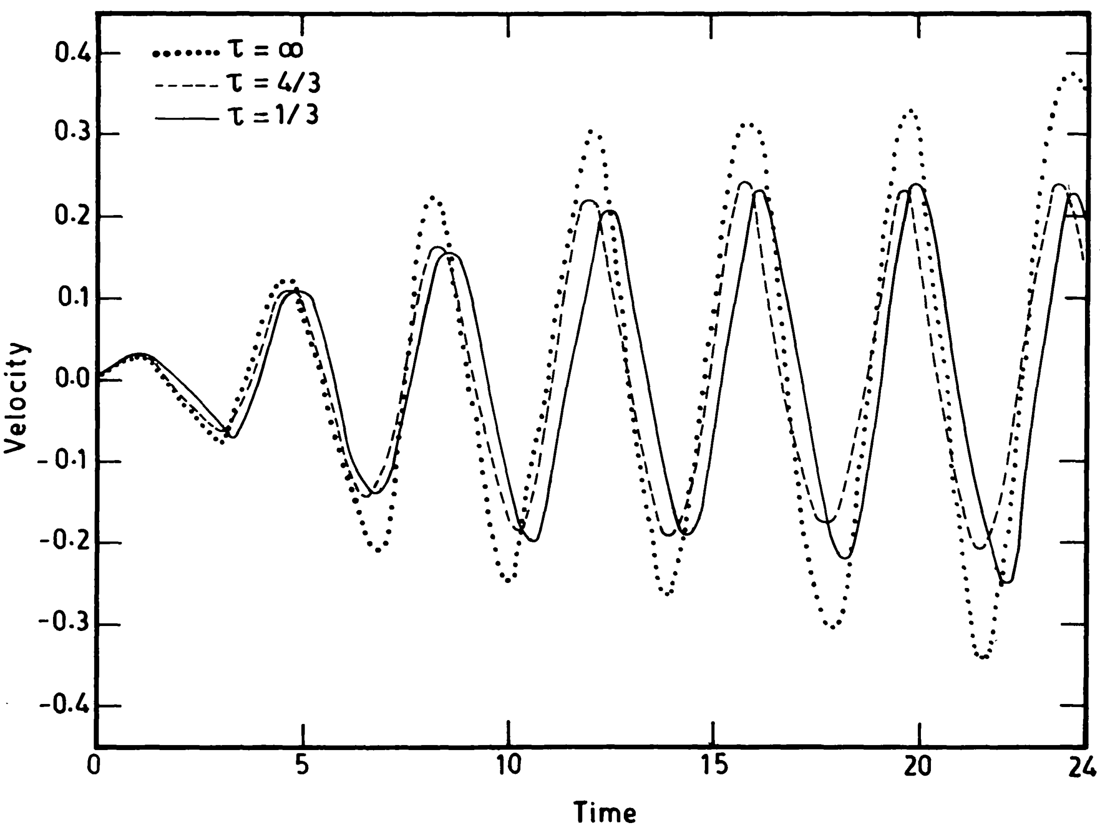

Fig. 6. Time-dependence of velocity at $z = 1.5$ for $d = 3.0$, wavelength = 3.0 and three values of cooling time = $\frac{1}{3}$, $\frac{4}{3}$, and ∞, respectively.

photosphere. The only dominant period of oscillations detected in these observations was 5 min. Clearly these observations represent, if not anything else, the response of tubes to the pressure fluctuations produced by those five minute oscillations whose horizontal wavelengths are comparable to the tube's cross-section. Evidently such oscillations would have to be of very high degree. If the tubes have an effective length larger than the halfwidth of the eigenfunction of the oscillation, then the interaction would be resonant. Otherwise the interaction would be nonresonant and weak. The fact that the observed amplitudes are not unduly large cannot be explained away as due to radiative damping since we have seen that radiation does not significantly damp the resonance when the cooling time is considerably shorter than the period of the fluctuations. Furthermore, the narrow bandwidth of the five minute oscillations precludes the dilution effects of larger bandwidths (Spruit, 1982) on the resonance. Thus, one can perhaps conclude that the observed interaction was nonresonant and, therefore, the tubes must be shorter than the scale length of the eigenfunction.

The absence of smaller periods in the observations of Giovanelli *et al.* (1978) (whose presence can be expected from resonant interaction of the tubes with shorter waves) could have been due to a variety of reasons. One such reason might have been the lack of sufficient spectral and temporal resolution and the consequent inclusion of the shorter scale velocities into the so-called microturbulence. Another reason could be that these short periodic acoustic fluctuations do not cause enough amplitude of pressure fluctuations on the tubes owing to their incoherent nature. A third reason could be the dilution of the resonance by a broad spectrum of waves being incident on the tubes (Spruit, 1982). In this connection, it may be relevant to mention that direct launching of short period waves along tubes (as opposed to the indirect creation of waves by lateral squeezing considered in this paper) leads to shocks only at higher levels in the atmosphere an account of the distendibility of tubes as well as radiative damping (Herbold *et al.*, 1985). Thus even such waves would have escaped detection in the Giovanelli *et al.* (1978) experiment.

In summary, the present paper shows that persistent nonlinear resonant response to pressure fluctuations is possible for slender magnetic flux tubes. Radiation does not seem to significantly damp the resonance for forcing periods much larger than the cooling time. Thus the absence of observational evidence for resonant interaction between the narrow band five minute oscillations and the tubes seems to indicate that the tubes are shorter than the extent of the eigenfunction of the oscillations.

Acknowledgements

The author gratefully acknowledges the help rendered by Dr Sunetra Giridhar in executing several jobs at the VAX 11/780 system of the Kavalur Observatory. Comments from the referee have considerably improved the author's thinking on this problem.

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