J. Astrophys. Astr. (1985) **6**, 21–34

Nonlinear Development of Convective Instability within Slender Flux Tubes. II. The Effect of Radiative Heat Transport

P. Venkatakrishnan *Indian Institute of Astrophysics, Bangalore 560034*

Received 1984 March 5; accepted 1984 November 23

Abstract. Inclusion of radiative heat transport in the energy equation for a slender flux tube leads to oscillations of the tube. The amplitude of the oscillations depends on the radius of the tube when lateral heat exchange alone is considered. Longitudinal heat transport has a greater influence on the evolution of the instability than lateral heat exchange for the particular value of tube radius considered in the calculation. Heat transport is seen to reduce the efficiency of concentration of magnetic fields by convective collapse in the case of polytropic tubes.

Key words: Sun, magnetic field—Sun, convection—fluid dynamics, unsteady flow—heat transport

1. Introduction

The general behaviour of convectively unstable fluids in the presence of magnetic field and heat diffusion has been well studied in the past on the basis of linear theory. For a Boussinesq fluid Chandrasekhar (1961) showed that whenever the electrical resistivity η was greater than the heat diffusivity χ, convective instability sets in as a monotonically growing instability at a critical value of the Rayleigh number $R^{(e)}$. This value increases with magnetic field, thereby demonstrating the stabilizing influence of the field. In stellar interiors, the radiative heat diffusivity is generally much larger than the electrical resistivity of the fluid. In such a situation, Chandrasekhar (1961) proved that overstability could set in at a Rayleigh number $R^{(o)}$ which is less than $R^{(e)}$, the critical Rayleigh number for onset of overturning convection.

The effect of compressibility on the instability for an ideally conducting fluid was studied by Kato (1966). In the case of an inviscid Boussinesq fluid, Kato (1966) showed that any arbitrary adverse temperature gradient led to overstability, irrespective of the value of the magnetic field. When compressibility is included, a regime of damped oscillations exists whenever the magnetic field is greater than a critical value. Thus, for weak magnetic fields in non-resistive inviscid fluids, convective instability always sets in as an overstability. Kato's analysis, being a local analysis, could not take boundary conditions into account. The classification and behaviour of the linear modes of a polytropic fluid with vertical magnetic field and imposed boundary conditions have been explored in detail by Antia & Chitre (1979). They found that convective-slow modes would be overstable for weak magnetic fields, while at moderate values of the magnetic field, overstable fast modes would dominate the spectrum. Moreover, the growth rate of both series showed a maximum with respect to the horizontal wave number, thereby indicating the suppression of instability on small length-scales. For horizontally structured magnetic fields, Roberts (1976) has demonstrated the existence of overstable modes.

All the above results lead us to expect similar oscillatory convection for slender magnetic flux tubes as well. The linear stability of slender radiating flux tubes has not been explicitly studied although Webb & Roberts (1980a, b) have considered the spatial and temporal damping of optically thin disturbances in slender flux tubes. In this paper we first derive the energy equation for a slender optically thick flux tube (Section 2). We then investigate, albeit in a restricted sense, the linear stability of such a tube in Section 3. The initial and boundary conditions are spelt out in Section 4. We then describe the results of a few nonlinear calculations in Section 5 and discuss these results in Section 6.

2. The Basic Equations

The equations of continuity and motion for a slender magnetic flux tube are (Roberts & Webb 1978):

$$\frac{\partial}{\partial t}(\rho/B)+\frac{\partial}{\partial z}(\rho v/B)=0, \tag{1}$$

$$\frac{\partial}{\partial t}v+v\frac{\partial v}{\partial z}+\frac{1}{\rho}\frac{\partial p}{\partial z}+g=0, \tag{2}$$

$$B^2=8\pi(p_e-p). \tag{3}$$

The complete energy equation in the presence of non-adiabatic terms is given by

$$\left(\frac{\partial}{\partial t}+\mathbf{v}\cdot\nabla\right)p-\left(\frac{\gamma p}{\rho}\right)\left(\frac{\partial}{\partial t}+\mathbf{v}\cdot\nabla\right)\rho+(\gamma-1)\nabla\cdot\mathbf{F}=0, \tag{4}$$

where
$$F=-K\nabla T \tag{5}$$

with
$$K=\rho C_v\chi \tag{6}$$

and
$$\chi=16\sigma T^3/3\kappa\rho, \tag{7}$$

where p, ρ, v, B, T are the pressure, density, velocity, magnetic field and temperature inside the tube, p_e is the pressure outside the tube, $\mathbf{F}$ is the radiative heat flux, K is the radiative heat conductivity, χ is the heat diffusivity, κ is the Rosseland mean opacity and σ is the Stefan–Boltzmann constant. Equations (5) through (7) are valid for an optically thick tube in the 'diffusion' approximation.

For thin tubes the lateral exchange of heat is important as can be seen from the magnetostatic models of Spruit (1977). When such a lateral exchange of heat is considered, one can no longer neglect the radial dependence of the dynamical variables as was done previously for the adiabatic flow (Venkatakrishnan 1983; hereinafter referred to as Paper I). We shall now follow Roberts & Webb (1978) to further simplify Equation (4). If Λ is the scale length of variation of the tube radius along its axis, then we shall consider terms of zero order in (r/Λ) in Equation (4). The first two terms reduce to

$$\left(\frac{\partial}{\partial t}+v\frac{\partial}{\partial z}\right)p-\left(\frac{\gamma p}{\rho}\right)\left(\frac{\partial}{\partial t}+v\frac{\partial}{\partial z}\right)\rho+O(r/\Lambda), \tag{4a}$$

vide equation (A10) of Roberts & Webb (1978).

The evaluation of the zeroth order of the third term in (4a) needs some consideration. For this, let us assume the following profile for the radial dependence of temperature, *viz.*

$$T = T_0(z) + T_1(z)(r/\Lambda) + T_2(z)(r/\Lambda)^2 + \dots \tag{8}$$

After substituting Equation (8) in Equations (3) and (5), and multiplying the resulting equation by r/Λ, we have upto first order in r/Λ,

$$(\gamma-1)\nabla\cdot\mathbf{F} = -\left(\frac{\gamma-1}{\Lambda^2}\right)\left[\frac{KT_1}{(r/\Lambda)} + \left(\frac{\partial K}{\partial T} + \frac{\partial K}{\partial p}\frac{\partial p}{\partial T}\right)\left\{T_1^2 + \Lambda^2\left(\frac{\partial T_0}{\partial z}\right)^2\right\}\right.$$
$$\left. + K\left(4T_2 + \Lambda^2\frac{\partial^2 T_0}{\partial z^2}\right)\right] + O(r/\Lambda). \tag{9}$$

From Equations (4), (4a) and (9) we have

$$-\left(\frac{\gamma-1}{\Lambda^2}\right)\frac{KT_1}{r/\Lambda} + \left(\frac{\partial}{\partial t} + v\frac{\partial}{\partial z}\right)p - \left(\frac{\gamma p}{\rho}\right)\left(\frac{\partial}{\partial t} + v\frac{\partial}{\partial z}\right)\rho$$
$$-\left(\frac{\gamma-1}{\Lambda^2}\right)\left[\left(\frac{\partial K}{\partial T} + \frac{\partial K}{\partial p}\frac{\partial p}{\partial T}\right)\left\{T_1^2 + \Lambda^2\left(\frac{\partial T_0}{\partial z}\right)^2\right\}\right.$$
$$\left. + K\left(4T_2 + \Lambda^2\frac{\partial^2 T_0}{\partial z^2}\right)\right] + O(r/\Lambda) = 0. \tag{10}$$

Equating the coefficients of each power of r/Λ to zero, we have

$$(\gamma-1)KT_1/\Lambda^2 = 0, \tag{11}$$

and

$$\left(\frac{\partial}{\partial t} + v\frac{\partial}{\partial z}\right)p - \left(\frac{\gamma p}{\rho}\right)\left(\frac{\partial}{\partial t} + v\frac{\partial}{\partial z}\right)\rho - \left(\frac{\gamma-1}{\Lambda^2}\right)\left[K\left(4T_2 + \Lambda^2\frac{\partial^2 T_0}{\partial z^2}\right)\right.$$
$$\left. + \left(\frac{\partial K}{\partial T} + \frac{\partial K}{\partial p}\frac{\partial p}{\partial T}\right)\left\{T_1^2 + \Lambda^2\left(\frac{\partial T_0}{\partial z}\right)^2\right\}\right] = 0. \tag{12}$$

Finally, by setting $T = T_i$ at $r = 0$ we have

$$T_0 = T_i \tag{13}$$

and by setting $T = T_e$ at $r = r_0$, we have

$$T_2 = (T_e - T_i)(\Lambda/r_0)^2, \tag{14}$$

where r_0 is the radius of the tube and T_e and T_i are the temperatures outside and inside the tube respectively. Substituting Equations (13) and (14) in Equation (12) we have finally,

$$\left(\frac{\partial}{\partial t} + v\frac{\partial}{\partial z}\right)p - \left(\frac{\gamma p}{\rho}\right)\left(\frac{\partial}{\partial t} + v\frac{\partial}{\partial z}\right)\rho = (\gamma-1)\left[K\left\{4\left(\frac{T_e - T_i}{r_0^2}\right) + \frac{\partial^2 T_i}{\partial z^2}\right\}\right.$$
$$\left. + \left(\frac{\partial K}{\partial T} + \frac{\partial K}{\partial p}\frac{\partial p}{\partial T}\right)\left(\frac{\partial T_i}{\partial z}\right)^2\right]. \tag{15}$$

The first term on the right hand side of Equation (15) represents the lateral influx of heat from the surroundings. The second term represents the longitudinal diffusion of heat while the last term arises because of variations of the heat conductivity with temperature and pressure. In what follows we will present a restricted linear stability analysis of a slender flux tube in the presence of lateral heat exchange alone and with constant diffusivity χ. We shall then numerically study the effect of lateral heat exchange, with constant conductivity K, on the nonlinear evolution of convective instability. We will next include longitudinal heat transport with constant K and finally consider a case of variable heat conductivity as well.

3. Linear stability of slender radiating flux tubes

The linearized version of Equations (1), (2), (3) and (15) are

$$\frac{1}{B_0}\left(\frac{\partial}{\partial t}\delta\rho+\frac{\mathrm{d}\rho_0}{\mathrm{d}z}\delta v\right)-\frac{\rho_0}{B_0^2}\left(\frac{\partial}{\partial t}\delta B+\delta v_1\frac{\partial B_0}{\partial z}\right)=0, \tag{16}$$

$$\rho_0\frac{\partial}{\partial t}\delta v+\frac{\partial}{\partial z}\delta p+\delta\rho g=0 \tag{17}$$

and

$$\left(\frac{\partial}{\partial t}\delta p-\frac{\gamma p_0}{\rho_0}\frac{\partial}{\partial t}\delta\rho\right)+\delta v\left(\frac{\mathrm{d}p_0}{\mathrm{d}z}-\frac{\gamma p_0}{\rho_0}\frac{\mathrm{d}\rho_0}{\mathrm{d}z}\right)$$

$$=(\gamma-1)\left\{K\left(\frac{\partial^2}{\partial z^2}\delta T-4\frac{\delta T}{r_0^2}\right)+\left(\frac{\mathrm{d}T_0}{\mathrm{d}z}\right)\left(\frac{\partial K}{\partial p}\frac{\partial}{\partial z}\delta p\right.\right.$$

$$\left.\left.+\frac{\partial K}{\partial T}\frac{\partial}{\partial z}\delta T\right)+\frac{\partial K}{\partial z}\frac{\partial}{\partial z}\delta T\right\}, \tag{18}$$

where

$$\delta T=T_0(\delta p/p_0-\delta\rho/\rho_0),\quad \delta B=-4\pi\delta p/B_0$$

and δ represents a perturbation of a variable whose zero order is represented by a subscript '0'. In the presence of lateral heat transport alone, Equation (18) becomes

$$\left(\frac{\partial}{\partial t}\delta p-\frac{\gamma p_0}{\rho_0}\frac{\partial}{\partial t}\delta\rho\right)+\delta v\left(\frac{\mathrm{d}}{\mathrm{d}z}p_0-\frac{\gamma p_0}{\rho_0}\frac{\mathrm{d}\rho_0}{\mathrm{d}z}\right)$$
$$=-4(\gamma-1)K\delta T/r_0^2. \tag{19}$$

We further write $K=\rho_0C_v\chi$ where χ is the radiative diffusivity and assume that χ is constant. Then the final energy equation is

$$\left(\frac{\partial}{\partial t}+\frac{1}{\tau}\right)\delta p-\left(\frac{\gamma p_0}{\rho_0}\right)\left(\frac{\partial}{\partial t}+\frac{1}{\gamma\tau}\right)\delta\rho+\delta v\left(\frac{\mathrm{d}p_0}{\mathrm{d}z}-\frac{\gamma p_0}{\rho_0}\frac{\mathrm{d}}{\mathrm{d}z}\rho_0\right)=0, \tag{20}$$

where

$$\tau=r_0^2/4\chi.$$

We impose the following boundary conditions,

$$\delta v = 0 \quad \text{at } z = 0 \text{ and } z = d. \tag{21}$$

Furthermore we choose perturbations of the form

$$q(z, t) = \hat{q}(z) \exp(\sigma t).$$

Substituting this form of the perturbation in Equations (16), (17) and (20), and eliminating δp, $\delta \rho$ and δB, we obtain

$$\frac{d^2}{dz^2}(\rho_0 \delta v) + a_1 \frac{d}{dz}(\rho_0 \delta v) + a_0(\rho_0 \delta v) = 0, \tag{22}$$

where

$$a_1 = \frac{1}{\Lambda}\left(\frac{\Gamma - 1}{\Gamma} + \frac{1}{2} - \frac{\delta_0 \sigma}{\sigma + 1/\gamma\tau}\right)$$

$$a_0 = \frac{1}{\Lambda^2}\left[\left(\frac{1}{2} - \frac{\delta_0 \sigma}{\sigma + 1/\gamma\tau}\right)\left(\frac{\sigma + 1/\tau}{\gamma\sigma + 1/\tau}\right) - \frac{\sigma}{\gamma g}\left(\frac{\gamma\beta}{2} + \frac{\sigma + 1/\tau}{\sigma + 1/\gamma\tau}\right) \times \left(\sigma\Lambda - \frac{g\delta_0}{\sigma + 1/\gamma\tau}\right)\right]$$

where

$$\delta_0 = \left(\frac{\Gamma - \gamma}{\Gamma\gamma}\right) \quad \text{and} \quad \Lambda = \mathscr{R}T_0/g.$$

Let us assume isothermal stratification, in which case the coefficients in Equation (22) would become independent of z, admitting solutions of the form $A_1 \exp(k_1 z) + A_2 \exp(k_2 z)$. Substituting this form into the boundary conditions yields the dispersion relation

$$b_4\sigma^4 + b_3\sigma^3 + b_2\sigma^2 + b_1\sigma + b_0 = 0, \tag{23}$$

where

$$b_4 = \beta_0 + 2(1 + \delta_0); \quad b_3 = 2\varepsilon(1 + \delta_0)(\beta_0 + \delta_0 + 2);$$

$$b_2 = -\left[1 + \delta_0(1 + \beta_0) - \left\{\frac{2\pi^2 n^2}{d^2} + \frac{7}{4} + \varepsilon^2(1 + \delta_0)^2(2 + \beta_0)\right\}\right];$$

$$b_1 = -\varepsilon(1 + \delta_0)\left[\delta_0(1 + \beta_0) + \frac{7}{4} - \frac{2\pi^2 n^2}{d^2}\right];$$

$$b_0 = -\varepsilon^2(1 + \delta_0)^2\left(\frac{1}{16} + \frac{2\pi^2 n^2}{d^2}\right);$$

where n is the order of the harmonic, $\varepsilon = (\Lambda/g\tau^2)^{1/2}$, β_0 is the ratio of gas to magnetic pressure, $\delta_0 = (\Gamma - \gamma)/\Gamma\gamma$ is the superadiabaticity and d is the length of the tube. We see that Equation (23) is of fourth degree in σ. It must be mentioned that Webb & Roberts (1980a) obtained a third degree polynomial for an unstratified tube and fourth degree polynomial for a stratified tube (Webb & Roberts 1980b). They have not commented on

Table 1. Frequencies and growth rates of slender radiating flux tube.

$\beta_0 = 6.0 \qquad \delta_0 = 0.3$

d	ε	Re(σ)	Im(σ)
1	0.05	0.0017	1.4454
	0.10	0.0018	1.4545
	0.15	−0.0009	1.4685
	0.20	−0.0069	1.4862
	0.25	−0.0162	1.5063
	0.30	−0.0287	1.5281
	0.35	−0.0438	1.5508
	0.40	−0.0614	1.5741
3.65	0.05	−0.0592	0.1120
	0.10	−0.0945	0.1970
	0.15	−0.1268	0.2509
	0.20	−0.1584	0.2926
	0.25	−0.1899	0.3264
	0.30	−0.2214	0.3543
	0.35	−0.2527	0.3777
	0.40	−0.2841	0.3973

the extra mode. An inspection of Equations (16) through (20) shows that the fourth mode arises out of stratification and nonadiabaticity. Following the nomenclature of Defouw (1970) we shall call this mode as the thermal-convective mode. In the limit $\varepsilon \ll 1$ the roots of Equation (23) separate clearly into thermal and dynamical modes. It can also be shown that in such an eventuality overstability is possible only if the equilibrium state is adiabatically stable and that for adiabatically unstable states the thermal effects only modify the growth rates.

For finite values of ε, such a demarcation of the modes is not obvious. Table 1 shows the complex roots determined numerically in the case of finite ε for two values of d corresponding to an adiabatically stable and unstable state respectively. Overstability is seen only for the adiabatically stable state. The growth rate increases with increasing ε and then decreases for large value of ε. The frequencies of the overstability are not sensitive to the value of ε. However, the frequencies in the nonadiabatic case must decrease as the equilibrium approaches a neutrally stable state in the adiabatic limit and thus the period of oscillation must be dependent on the magnetic field of the tube.

However, this limited study does not include the case of unequal zero-order temperatures inside and outside the tube. It is quite possible that unequal temperatures might lead to overstability even when the equilibrium is adiabatically unstable, by the excitation of new modes which are suppressed in the case of equal temperatures.

4. Initial and boundary conditions for the nonlinear calculations

Equations (1), (2) and (15) form a system of hyperbolic partial differential equations provided we treat the derivatives of temperature as source terms. Thus the problem is an initial value problem requiring specification of the initial conditions. We chose the initial conditions, in the case of constant conductivity K, as a polytropic stratification with

$$T_e = T_e(z=0) - \left(\frac{\Gamma-1}{\Gamma}\right)z,$$

$$T = T_e,$$

$$p \propto T^{\Gamma/(\Gamma-1)},$$

and

$$B = (8\pi p/\beta_0)^{1/2}.$$

Practical computational considerations require the imposing of boundary conditions at finite values of z. We chose the following boundary conditions:

$$p = p(t=0) \quad \text{at} \quad z=0$$

and

$$p = p(t-\Delta t) - \frac{\Delta t}{\Delta z} v(t-\Delta t, z=d)\{p(t-\Delta t, z=d) - p(t-\Delta t, z=d-\Delta z)\} \quad \text{at} \quad z=d.$$

The lower ($z = 0$) boundary condition implies that the radius of the tube does not change in time while the upper boundary condition implies that there is no lateral leak of matter at the top. This has been explained in Paper I. Sometimes more boundary conditions become necessary whenever some characteristic emanating from the boundary fails to communicate with the interior. The additional boundary conditions used in such cases were

$$\rho(t, 0) = \rho(t-\Delta t, 0)$$

and

$$\rho(t, d) = 2\rho(t, d-\Delta z) - \rho(t, d-2\Delta z).$$

It must be remarked here that the density boundary condition at $z = d$ is different from that used in Paper I where it was

$$\rho(t, d) = \rho(t-\Delta t, d).$$

The introduction of the new density boundary condition saved the calculations from the numerical breakdown of zero pressure encountered in Paper I.

The calculations were performed in terms of dimensionless units which are described in Paper I and which essentially use the base pressure as the unit for gas and magnetic pressure, the base density and temperature as the units for the corresponding variables, the sound speed at the base as velocity unit, the pressure scale-height at the base as unit of length and the free-fall time over this length as the unit of time.

5. Results

The convective instability will be a maximum in a region few hundred kilometres below the photosphere where the superadiabaticity is large. In this region the radiative diffusivity ranges from $2.33 \times 10^{10}\,\mathrm{cm^2\,s^{-1}}$ at a temperature of 1.003×10^4 K to $3.504 \times 10^{12}\,\mathrm{cm^2\,s^{-1}}$ at the photosphere (Spruit 1977). We have, however, first considered a constant radiative conductivity representing a 'mean' value, in order to understand the physical effects produced by heat diffusion. We shall also briefly describe results of a single calculation involving a variable diffusivity at the end of this section.

5.1 *Effect of Lateral Heat Exchange*

The results were all obtained in dimensionless units with the reference units that were defined in Paper I. In those dimensionless units, the radiative conductivity was 0.001 which is representative of a layer $\simeq 200$ km below the photosphere. Since the term representing lateral heat exchange in Equation (15) depends inversely on the area of cross-section of the tube, we shall first look at the effect of tube radius on the development of the instability.

Fig. 1 shows the time variation of longitudinal velocity at $z = 0.48$ for $\beta_0 = 6.0$, for 4 values of r_0, the tube radius. We see an oscillatory behaviour for the velocity. The oscillations have nearly the same period of $\simeq 12$ units for all values of r_0. The amplitude seems to be large for $r_0 = 0.5$. For smaller values of r_0 we find that after some time the gas pressure inside the tube exceeds the external gas pressure making the magnetic field vanish in the slender flux tube approximation. Thus, the calculations could not be extended for these cases. Fig. 2 shows the behaviour of β, the ratio of gas pressure to magnetic pressure inside the tube, at $z = 0.48$ as a function of time for different values of r_0. The behaviour of β is essentially similar to that of velocity except

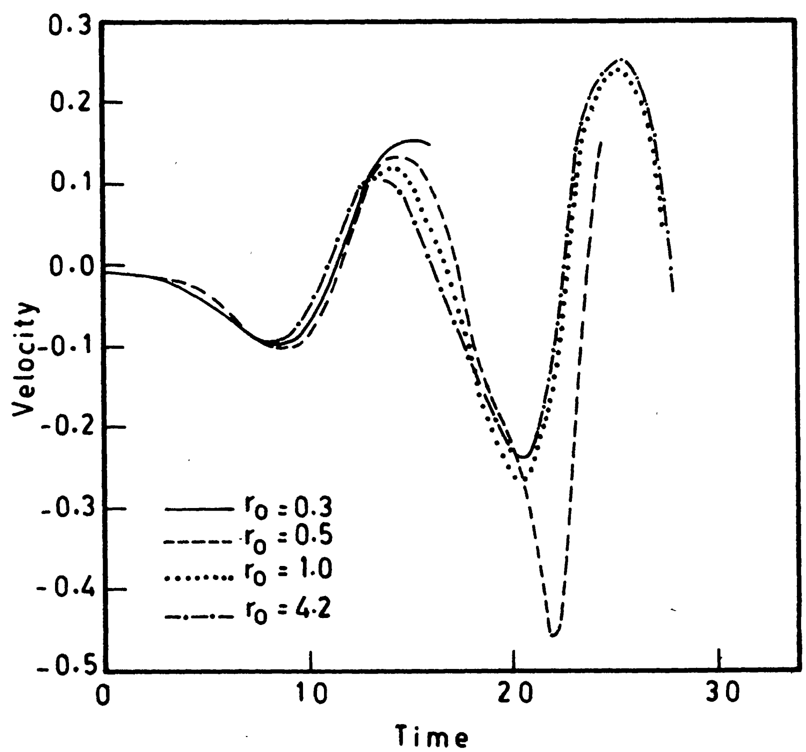

Figure 1. Time dependence of velocity at $z = 0.48$ in an 'open' tube, for $\beta_0 = 6.0$ and different values of radius r_0, laterally exchanging heat with its surroundings.

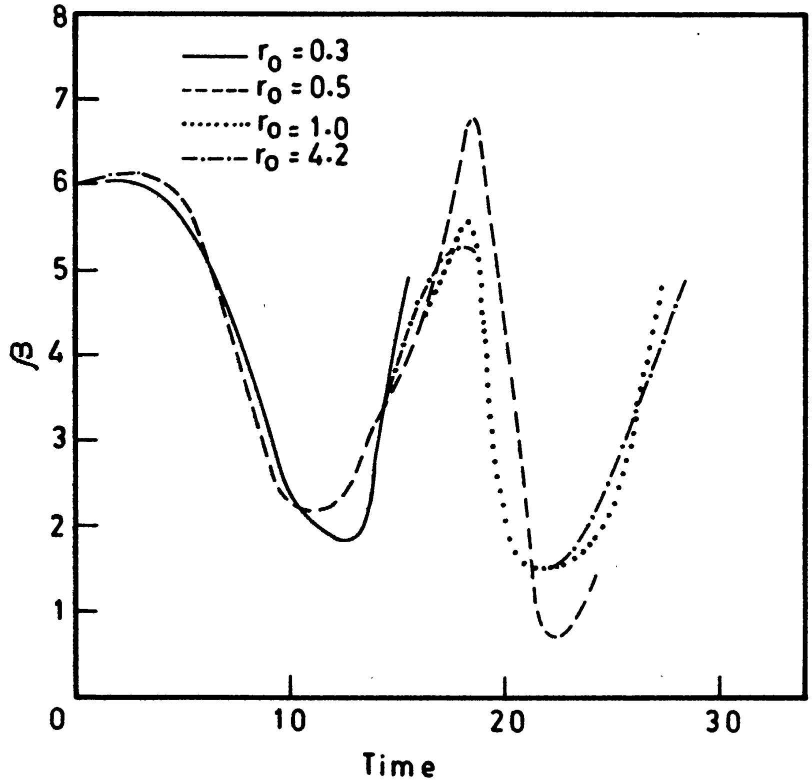

Figure 2. Variation of β with time in the tube considered for Fig. 1.

for the phase. We notice transient intensification of the field to values of β as low as 1.0.

The stabilizing influence of the initial magnetic field can be seen in Fig. 3. Here the velocity at $z = 0.48$ in a tube of radius $r_0 = 0.5$ is plotted as a function of time for different values of β_0. There is no discernible change in the period of oscillations but the amplitudes are considerably affected as can be seen from the low values for $\beta_0 = 4.0$ compared to those for larger β_0.

5.2 *Effect of Longitudinal Heat Transport*

Let us now consider the effect of longitudinal heat transfer with constant heat conductivity on convective instability. We, therefore, retain only the first two terms on the right-hand side of Equation (15). In a rigorous sense, the character of the system of differential equations changes here from hyperbolic to parabolic due to the appearance of the second derivative of temperature. We shall, however, continue to regard the system as hyperbolic and treat the derivatives of temperature as 'source' terms. These source terms were calculated on the previous time-line using a standard IBM subroutine for numerical differentiation. Such a procedure does not cause serious problems as long as the thermal conductivity is small. Figs 4 and 5 show the temporal behaviour of velocity and plasma-β in a tube with $\beta_0 = 6.0$ and 4.0 respectively, with $r_0 = 0.5$. One notices three facts, *viz.* the presence of overstability, the smaller period of oscillation and the greatly diminished amplitude of oscillation as compared to the case with lateral heat exchange alone. Compared to this, the differences between the case of lateral heat exchange alone and the adiabatic case are rather small. This indicates that

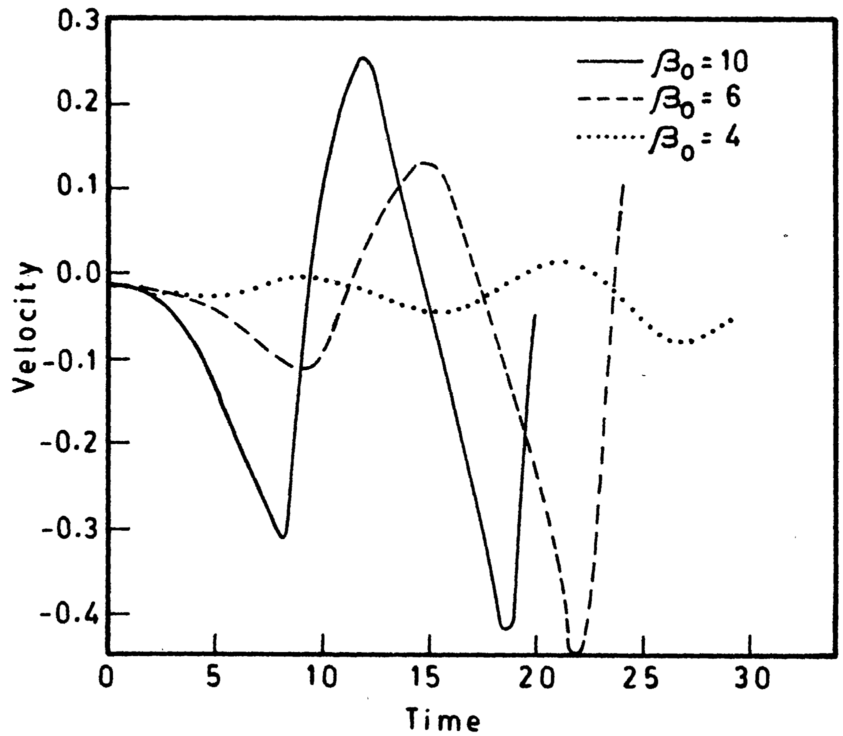

Figure 3. Dependence on β_0 of the time development of velocity in a tube of radius $r_0 = 0.5$.

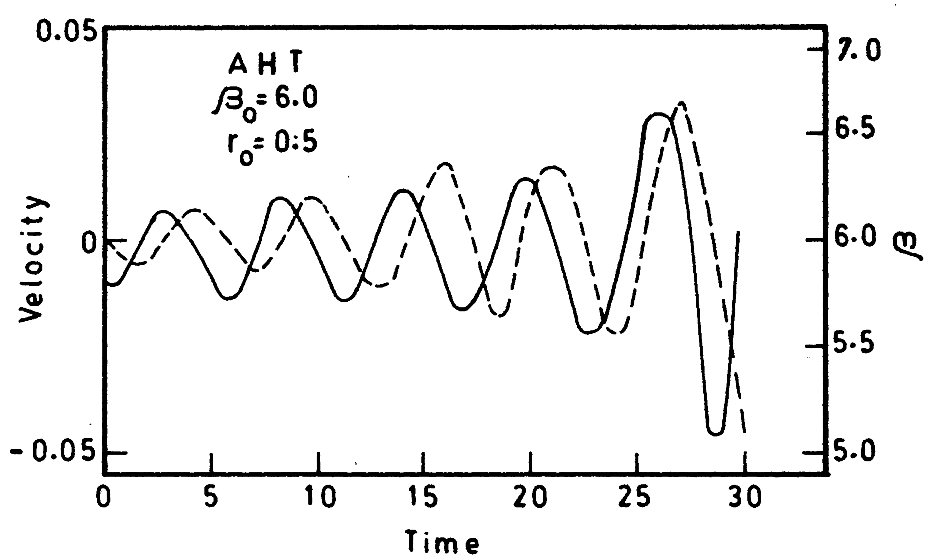

Figure 4. Time dependence of velocity (dashed curve) and β (solid curve) at $z = 0.48$ in an open tube with heat transport and with $\beta_0 = 6.0$.

longitudinal heat transport has a greater effect on the convective instability of a flux tube with $r_0 = 0.5$. However, one cannot predict the relative importance of longitudinal heat transport on thinner tubes. In the context of solar photospheric magnetic fields, it is also interesting to see that intensification of such tubes by convective instability with heat transport would most probably be transient and would be accompanied by only

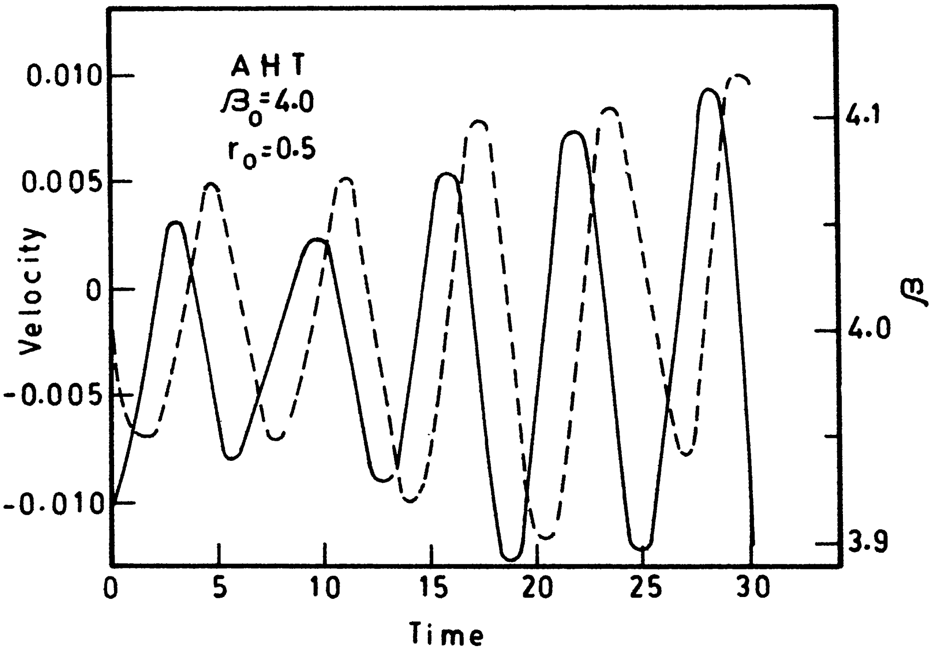

Figure 5. Time dependence of velocity (dashed curve) and β (solid curve) at $z = 0.48$ in an open tube with heat transport and with $\beta_0 = 4.0$.

small and oscillatory flows. Here the calculations were not continued beyond $t \simeq 30$ and, therefore, we do not know the saturation amplitudes of the overstability. However, $t \simeq 30$ corresponds to $\simeq 1000$ s for tubes with base temperature $T_b = 10^4$ K. Therefore, other processes such as granulation might interfere with the development of the overstability within this time.

5.3 *Effect of a Variable Heat Conductivity*

Below the solar photosphere the heat conductivity is not constant but varies by a few orders of magnitude as mentioned earlier. In order to study such a situation we need to first calculate a static equilibrium model for the environment of the flux tube with such a variable conductivity. For this we first calculated the opacity by obtaining a least-squares fit for the relation

$$\kappa = \kappa_0 (p/p_0)^{\mu} (T/T_0)^{\nu}, \tag{24}$$

using Spruit's (1977) values for κ, p and T. Here, $p_0 = 3.126 \times 10^5$ dyn cm^{-2} and $T_0 = 1.003 \times 10^4$ K corresponding to a depth of 1.779×10^2 km in the model. We obtained

$$\nu = 12 \quad \text{and} \quad \mu = -0.65.$$

From Equations (6), (7) and (24) one can write K as a function of p and T. Further, we simultaneously solve the static energy equation

$$K \frac{dT}{dz} = K_0 \left(\frac{dT}{dz} \right)_0 \tag{25}$$

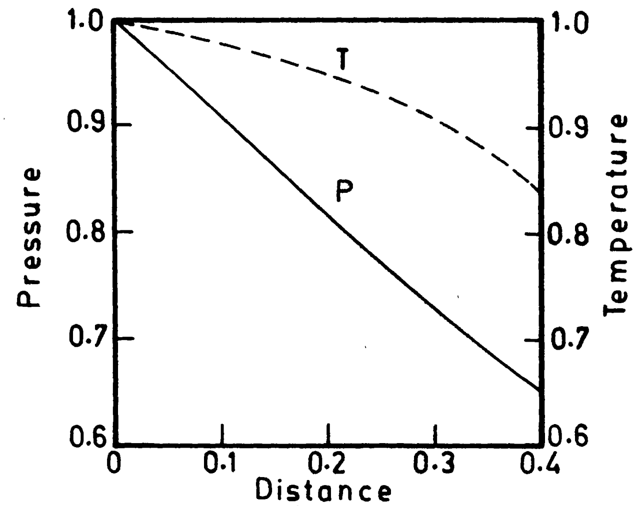

Figure 6. Initial hydrostatic state for a tube's environment (p_0, T_0) where heat transport is solely by radiation with opacity varying as $\kappa = \kappa_0 (T/T_0)^{12} (p/p_0)^{-0.65}$.

and the equation of hydrostatic pressure balance,

$$\frac{\mathrm{d}p}{\mathrm{d}z} = -\frac{pg}{\mathscr{R}T}, \tag{26}$$

using an Adam's predictor-corrector algorithm to obtain the equilibrium state which is given in Fig. 6. One should notice that the thickness of the layer between temperatures 10^4 K and 0.6×10^4 K in Fig. 6 is smaller than the corresponding thickness in Spruit's model because we have entirely ignored the convective transport of heat.

We can now calculate the initial state of the tube from the equilibrium state of its environment using the condition of horizontal pressure balance and by assuming $T_e = T_i$ (see Roberts & Webb 1979). In this particular case we chose $\beta_0 = 6.0$. This initial state was perturbed with a small initial velocity perturbation and the evolution of the flow was studied using the complete energy Equation (15). The development of the flow at two spatial points is shown in Fig. 7. There is once again oscillatory behaviour with a larger frequency and smaller amplitude as compared to the cases of constant conductivity. This calculation does not have direct relevance to the solar convection zone because of the neglect of convective transport in the environment of the tube. However, the general trend of oscillatory behaviour is seen even for this case. Another interesting feature of this calculation is the systematic flattening of the spatial profile of temperature gradient with time as seen from Fig. 8. However, one does not know from a single calculation whether the general tendency of heat transport is to smoothen out the variations in the temperature gradient.

6. Discussion

All these results, though not exhaustive, indicate that the general behaviour of convectively unstable flux tubes, in the presence of heat transport, is oscillatory. The change of period of oscillation on introducing longitudinal heat transport could be due to the excitation of a new mode or a new harmonic. The further decrease in period for

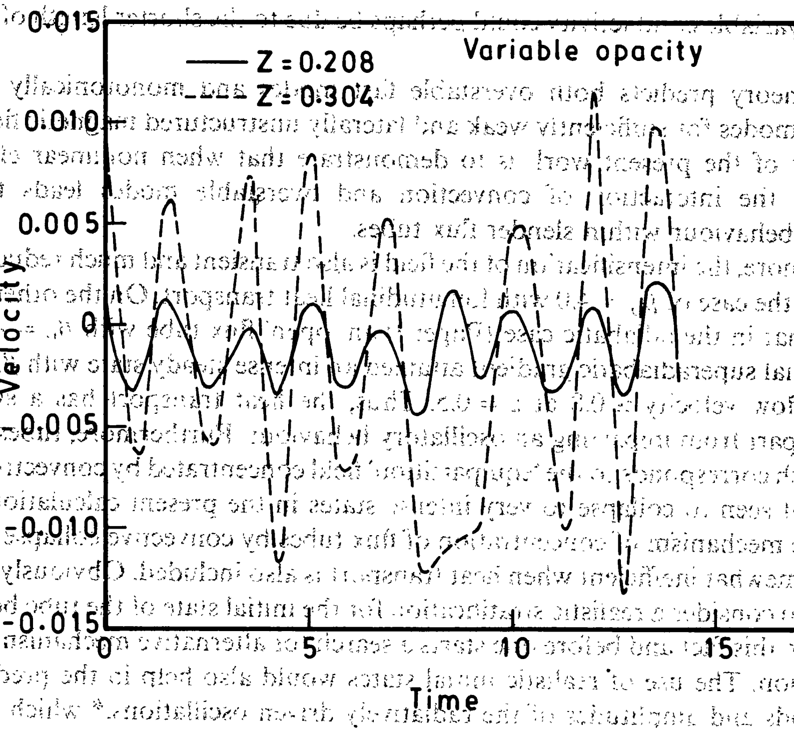

Figure 7. Variation of velocity in a tube embedded in an atmosphere given by Fig. 6 and with $\beta_0 = 6.0$.

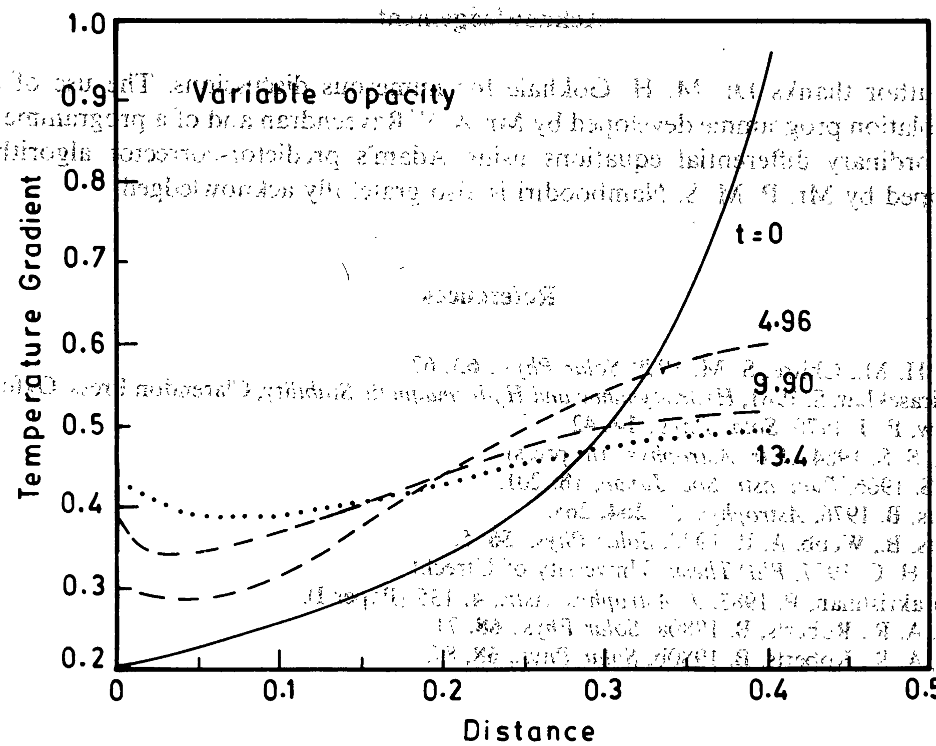

Figure 8. Evolution of the spatial profile of temperature gradient within the tube considered for Fig. 7.

the case of variable conductivity could perhaps be due to the shorter length of the tube considered.

Linear theory predicts both overstable fast modes and monotonically growing convective modes for sufficiently weak and laterally unstructured magnetic fields. The main result of the present work is to demonstrate that when nonlinear effects are considered, the interaction of convection and overstable modes leads to a net oscillatory behaviour within slender flux tubes.

Furthermore, the intensification of the field is also transient and much reduced as for example in the case of $\beta_0 = 4.0$ with longitudinal heat transport. On the other hand, it was seen that in the adiabatic case (Paper I) an 'open' flux tube with $\beta_0 = 4.0$ and a smaller initial superadiabatic gradient attained an intense steady state with $\beta_{\text{final}} \simeq 1.0$ and downflow velocity $\simeq 0.8$ at $z = 0.5$. Thus, the heat transport has a stabilizing influence apart from imparting an oscillatory behaviour. Furthermore, tubes with $\beta_0 = 6.0$, which corresponds to the 'equipartition' field concentrated by convective eddies, are also not seen to collapse to very intense states in the present calculation.

Thus the mechanism of concentration of flux tubes by convective collapse seems to become somewhat inefficient when heat transport is also included. Obviously it will be necessary to consider a realistic stratification for the initial state of the tube before one can confirm this fact and before one starts a search for alternative mechanisms of tube concentration. The use of realistic initial states would also help in the prediction of exact periods and amplitudes of the radiatively driven oscillations,* which will have important observational implications.

Acknowledgement

The author thanks Dr. M. H. Gokhale for numerous discussions. The use of an interpolation programme developed by Mr. A. V. Raveendran and of a programme to solve ordinary differential equations using Adam's predictor-corrector algorithm developed by Mr. P. M. S. Namboodiri is also gratefully acknowledged.

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* Recent results for solar flux tubes laterally exchanging heat with their surroundings (Hasan 1984) do show overstability similar to the polytropic tubes, but vertical transport of heat is yet to be considered there.