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Relativistic star clusters with large central redshifts

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Summary. Previous calculations of the velocity distribution function for a star cluster scenario of a class of core-envelope type static, stable, massive equilibrium configurations, capable of giving high internal gravitational redshifts, have been extended to show that positive distribution functions can be obtained for these objects by suitably altering the envelope equation of state, without affecting the overall structure significantly. Limits on the central redshift obtainable from such configurations, subject to conditions of pulsational stability and positivity of the distribution function, have been obtained. Finally, the relevance of the new composite configurations as models for high redshift quasi-stellar objects (QSOs) is discussed.

1 Introduction

Supermassive objects with large gravitational redshifts, in addition to their intrinsic theoretical importance, are of great interest in the astrophysical scenario as possible models for objects such as quasars (quasi-stellar objects or QSOs). In a series of papers previously (Das & Narlikar 1975 (Paper I); Das 1975, 1976 (Paper II), 1979) spherically symmetric, static, bound, core envelope-type massive configurations in stable equilibrium had been considered as models for the QSOs. It was shown that these models could provide an adequate interpretation of the observed large emission and absorption redshifts as well as various other physical properties of the QSOs without making undue demands on the equations of state.

In Paper II, while considering a star cluster scenario for these models, on the lines of the original cluster model suggested by Hoyle & Fowler (1967), an attempt was made to calculate the velocity distribution function for these models using a method due to Fackerell (1968). It was then noted that the distribution function became necessarily negative over a part of the phase space for all the models. But since this negativity occurred only near the surface it was conjectured that with a suitable modification of the equation of state near the surface the distribution function could perhaps be made positive throughout which is a necessary criterion for any physical object. The present paper provides an explicit detailed verification of this conjecture and also discusses the relevance of the new configurations as models for QSOs.

We start with a brief sketch of the original model in Section 2. In Section 3, we shall outline the

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1984MNRAS.210..753D

earlier calculations of the distribution function done in Paper II. The proposed modifications of the equations of state, derivations of the new equations and the results of numerical computations will be given in Section 4. Finally we shall discuss the significance of these models in Section 5.

2 The original model

We consider a static, spherically symmetric configuration of mass M and radius R (in Schwarzschild coordinates) in hydrostatic equilibrium. Unless otherwise mentioned geometrized units (speed of light $c=1$, gravitational constant $G=1$) will be used throughout.

Outside the body ($r>R$) we have the well-known exterior Schwarzschild solution, viz.

$$ds^2=\left(1-\frac{2M}{r}\right)dt^2-\frac{dr^2}{(1-2M/r)}-r^2(d\theta^2+\sin^2\theta\, d\phi^2). \tag{1}$$

Inside the body ($r\leq R$) we write the line element in the form

$$ds^2=\exp(\nu)\,dt^2-\exp(\lambda)\,dr^2-r^2(d\theta^2+\sin^2\theta\, d\phi^2) \tag{2}$$

where

$$\exp(-\lambda)=\left(1-\frac{2m(r)}{r}\right), \tag{3}$$

and $m(r)$ is the mass within a radius r given by

$$m(r)=\int_0^r 4\pi r^2\varrho(r)\,dr. \tag{4}$$

The energy momentum tensor T^i_k is

$$T^i_k=\mathrm{diag}\,(-p,-p,-p,\varrho) \tag{5}$$

where pressure p, mass-energy density ϱ, ν and λ are functions of r only.

Following Bondi (1964) we introduce two dimensionless variables u and v defined by

$$u(r)=\frac{m(r)}{r},\quad v(r)=4\pi r^2 p(r). \tag{6}$$

The field equations can then be cast in a particularly convenient form in terms of u and v. Given an equation of state $p=p(\varrho)$ these can be solved to obtain v as a function of u and represent the given sphere by a $v(u)$ curve in the (u, v) plane.

It is seen that – *cf.* Bondi (1964) and Paper I for greater details – the hyperbola

$$H\equiv 2v-(u^2+6uv+v^2)=0 \tag{7}$$

divides the interior into two parts, $H>0$ and $H<0$, such that different conditions apply if the redshifts are to be maximized. Hence, as in earlier papers, we assume different equations of state in $H>0$ and $H<0$ with the transition taking place on $H=0$. We designate $H\geq 0$ as the core, $H<0$ as the envelope and the values of various physical quantities at the centre, at the core–envelope interface and at the surface of the sphere are denoted by the subscripts c, b and s respectively. The class of models put forward in Paper I, Das (1975), Paper II and Das (1979) has the following core–envelope combinations.

core: isothermal $p=k\varrho \qquad (k<1)$.

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1984MNRAS.210..753D

Envelope: limiting adiabatically stable

$$\frac{dp}{dp}=\frac{1}{n} \qquad (1\leqslant n<k^{-1}). \tag{8}$$

Numerical computations are necessary to determine the march of various dimensionless physical variables such as ν, u, v, p/p_c, ϱ/ϱ_c and $a=\{\sqrt{(4\pi/3)\varrho_c}\}\times r$ and to calculate the gravitational redshifts z_c, z_b and z_s of light leaving respectively from the centre, the core–envelope boundary and the surface and received by a remote observer, for a specified pair of equations of state that is for a given pair of values of n and k. The dimensionless quantities can be scaled once the value of the total mass M or the radius R or the central density ϱ_c is specified.

3 Distribution function (previous calculations)

In this section we shall summarize the results of the earlier calculations of the distribution function (for details see Paper II).

For a star cluster scenario of our model we assume that the constituent stars are either neutron stars or white dwarfs of identical proper masses (for an assumed total cluster mass $M\sim 5\times 10^{10}\, M_\odot$). Then our model is a collisionless, spherically symmetric system with isotropic pressure and non-vanishing pressure gradient at the surface with all the 'particles' (i.e. stars) having the same proper mass. For such a system the distribution function $F(x)$ depends only on the energy of a particle and for the models considered is given by (Paper II equations (B5) and (B6))

Core:

$$F(x)=\frac{(1-nk)}{(n+1)}\tilde{\varrho}_b\left\{\left(\frac{1+k}{1-nk}\right)^{\{-(k+1)\}/\{k(n+1)\}}\times\frac{k(n+1)}{(1-nk)}\,\frac{(1-3k)(1-k)(1+k)}{2k^3}\right.$$
$$\times\sqrt{x}\int_x^{x_b}\frac{dy}{\sqrt{y^{(1+3k)/k}\cdot(y-x)}}+\frac{(n-3)(n-1)(n+1)}{2}$$
$$\left.\times\sqrt{x}\int_{x_b}^{x_s}\frac{dy}{\sqrt{y^{(n+3)}\cdot(y-x)}}-(n+1)\sqrt{\frac{x}{(1-x)^3}}+(n-5)(n+1)\sqrt{\frac{x}{(1-x)}}\right\},$$
$$x_c\leqslant x\leqslant x_b,\qquad x_s=1 \tag{9}$$

envelope:

$$F(x)=\frac{(1-nk)}{(n+1)}\tilde{\varrho}_b\left\{\frac{(n-3)(n-1)(n+1)}{2}\sqrt{x}\int_x^{x_s}\frac{dy}{\sqrt{y^{(n+3)}\cdot(y-x)}}\right.$$
$$\left.-(n+1)\sqrt{\frac{x}{(1-x)^3}}+(n-5)(n+1)\sqrt{\frac{x}{1-x}}\right\}\qquad x_b\leqslant x<x_s \tag{10}$$

where $x=[\exp(\nu)/\exp(\nu_s)]$ is the square of the energy of the particle measured in units of its energy at the surface and $\tilde{\varrho}_b=\varrho_b/\varrho_c$.

It is seen that

1. For $k\geqslant ⅓$ and hence for $n<3$, since $n<k^{-1}$, $F(x)$ is negative throughout.

2. Even for $k<⅓$, $F(x)$ will become negative as $x\to 1$ that is near the surface, due to the negative term $-(n+1)\sqrt{x/(1-x)^3}$ in (10) which will dominate as $x\to 1$. This in fact is true for any configuration with non-vanishing density or equivalently non-vanishing pressure gradient at the surface.

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1984MNRAS.210..753D

Hence we cannot have star cluster scenario right up to the surface. However, explicit numerical computations show that for $k \lesssim 1/6$ and $n \gtrsim 5$ $F(x)$ becomes negative only in the outermost parts of the envelope. Hence, it was conjectured in Paper II that the negativity could possibly be avoided by letting $\varrho \to 0$ in a suitable manner in the outer parts of the envelope. We proceed to do this in the next section.

4 Distribution function with a polytropic patch-up

4.1 Derivation of the formulae

In our attempt to remove the negativity of $F(x)$ we replace the adiabatically stable equation of state with a polytropic equation of state in the outer parts of the envelope. This particular choice is motivated by the facts that relativistic polytropes have been extensively studied so that their properties are well known (see e.g. Tooper 1964) and they are often used as models for supermassive objects.

We thus have from a point $d(r_b<r_d<R)$ outwards in the envelope

$$p=K\cdot\varrho^{(1+1/m)}. \tag{11}$$

where $m \geqslant 1$ is the polytropic index. The constant K can be determined by demanding the continuity of p and ϱ at the point d.

Making use of the relevant equations given by Fackerell (1968) for the distribution function for a polytropic sphere (Fackerell equations (92)–(97)) we obtain, after a somewhat lengthy calculation the following expressions.

Core:

$$\begin{aligned} F(x)= & \frac{(1-nk)}{(n+1)}\tilde{\varrho}_b \left\{ \left(\frac{1+k}{1-nk}\right)^{\{-(k+1)\}/\{k(n+1)\}} \times \frac{k(n+1)}{(1-nk)}\,\frac{(1-3k)(1-k)(1+k)}{2k^3} \right. \\ & \times\sqrt{x}\int_x^{x_b}\frac{dy}{\sqrt{y^{(1+3k)/k}\cdot(y-x)}} \\ & \left. +\frac{(n-3)(n-1)(n+1)}{2}\sqrt{x}\int_{x_b}^{x_d}\frac{dy}{\sqrt{y^{(n+3)}\cdot(y-x)}}\right\} + \left(\frac{k\tilde{p}_d}{\tilde{\varrho}_d}\right)^{-m}\frac{\tilde{\varrho}_d}{2} \\ & \times\sqrt{x}\int_{x_d}^{x_s}\frac{(1-y^{\Delta})^{(m-2)}\{3(m+1)^2-(10\,m^2+17\,m+6)y^{\Delta}+(8\,m^2+10\,m+3)y^{2\Delta}\}}{(m+1)^2\sqrt{y^3\cdot(y-x)}}dy \\ & +\frac{(1-nk)}{(n+1)}\tilde{\varrho}_b\left\{-(n+1)\sqrt{\frac{x}{(1-x)^3}}+(n-5)(n+1)\sqrt{\frac{x}{1-x}}\right\} \\ & (x_c \leqslant x \leqslant x_b \leqslant x_d < x_s). \end{aligned} \tag{12}$$

Envelope (unpatched part)

$$\begin{aligned} F(x)= & \frac{(1-nk)}{(n+1)}\tilde{\varrho}_b\left[\frac{(n-3)(n-1)(n+1)}{2}\sqrt{x}\int_x^{x_d}\frac{dy}{\sqrt{y^{(n+3)}\cdot(y-x)}}\right]+\left(\frac{k\tilde{p}_d}{\tilde{\varrho}_d}\right)^{-m} \\ & \times\frac{\tilde{\varrho}_d}{2}\sqrt{x}\int_{x_d}^{x_s}\frac{(1-y^{\Delta})^{(m-2)}\{3(m+1)^2-(10\,m^2+17\,m+6)y^{\Delta}+(8\,m^2+10\,m+3)y^{2\Delta}\}}{(m+1)^2\sqrt{y^3\cdot(y-x)}}\,dy \\ & +\frac{(1-nk)}{(n+1)}\tilde{\varrho}_b\left\{-(n+1)\sqrt{\frac{x}{(1-x)^3}}+(n-5)(n+1)\sqrt{\frac{x}{1-x}}\right\} \quad (x_b \leqslant x \leqslant x_d < x_s). \end{aligned} \tag{13}$$

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1984MNRAS.210..753D

Envelope (patched polytrope part)

$$F(x)=\left(\frac{k\tilde{p}_d}{\tilde{\varrho}_d}\right)^{-m}\tilde{\varrho}_d\left\{\sqrt{\frac{x}{1-x}}\frac{m}{(m+1)}\delta_{m1}\right\}+\left(\frac{k\tilde{p}_d}{\tilde{\varrho}_d}\right)^{-m}$$

$$\times\frac{\tilde{\varrho}_d}{2}\sqrt{x}\int_x^{x_s}\frac{(1-y^{\Delta})^{(m-2)}\{3(m+1)^2-(10\,m^2+17\,m+6)y^{\Delta}+(8\,m^2+10\,m+3)y^{2\Delta}\}}{(m+1)^2\sqrt{y^3\cdot(y-x)}}dy$$

$$(x_d\leq x\leq x_s) \tag{14}$$

where $\Delta=1/\{2(m+1)\}$, δ_{m1} is the Kronecker delta symbol, the suffix d denotes the values of variables at the point d and $\tilde{p}_d=p_d/p_c$, $\tilde{\varrho}_d=\varrho_d/\varrho_c$, etc.

For the usual complete polytropic spheres there are two free parameters K and m to be chosen in (11). Here the constant K is to be determined by the condition of continuity of p and ϱ at the point d. We may adopt one of the following two approaches to determine m.

(a) Fix the value of m by imposing some suitable constraints on the composite model.

(b) Choose an arbitrary value of m and solve the equations of relativistic stellar structure again.

A constraint that the redshifts of the composite model be the same as those of the original model was imposed and calculations of $F(x)$ from equations (12)–(14) were done if the derived value of m was equal to or greater than unity (Das 1976a). Numerical computations for the range of the models $k^{-1}=4(1)10$ and $n=3(1)(k^{-1}-1)$ and $(k^{-1}-1)$ (0·1) $(k^{-1}$–0·1) showed that in general a $m\geq 1$ polytropic patch-up was not possible. In the very few cases where $m\geq 1$ polytropic patch-up was possible $F(x)$ became negative over an intermediate range of x in the envelope. Of course this does not mean that such a composite model is impossible since the calculations do not exhaust all possible core–envelope combinations for $k<1/3$. But it seems unlikely. In fact, in principle it is not necessary to restrict ourselves to $k<1/3$ for the composite models. But obtaining a positive distribution function with $k\geq 1/3$ $(n<3)$ is even more unlikely as can be seen from an inspection of equations (12)–(14).

The method (b), however, yields several models with positive distribution functions. The relevant calculations are as follows:

Following Tooper (1964) we introduce a variable θ for the polytropic part such that

$$\varrho=\varrho_d\,\theta^m,\quad\text{and}\quad p=\sigma\,\varrho_d\,\theta^{(m+1)},$$

where

$$\sigma=\frac{p_d}{\varrho_d}\quad\text{and}\quad\theta_d=1. \tag{15}$$

The metric coefficient exp (ν) for the polytropic part is then given by

$$\exp(\nu)=\exp(\nu_d)\left(\frac{1+\sigma}{1+\sigma\theta}\right)^{2(m+1)} \tag{16}$$

(Tooper equation (2.18)).

In analogy with equations (2.22)–(2.26) of Tooper we introduce two new dimensionless variables

$$\eta=A\cdot r$$

and

$$\omega=\frac{A^3}{4\pi\,\varrho_d}m(r),$$

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1984MNRAS.210..753D

where

$$A=\sqrt{\frac{4\pi\,\varrho_{\mathrm{d}}}{(m+1)\,\sigma}} \tag{17}$$

In our notation we have

$$\omega=\left\{\sqrt{\frac{3\tilde{\varrho}_{\mathrm{d}}}{(m+1)^{3}\,\sigma^{3}}}\right\}au=C_1\,au\ \text{(say)},$$

and

$$\eta=\left\{\sqrt{\frac{3\,\tilde{\varrho}_{\mathrm{d}}}{(m+1)\,\sigma}}\right\}a=C_2\,a\,\text{(say)}. \tag{18}$$

The relativistic stellar structure equations can then be written in terms of θ, η and ω as

$$\frac{d\omega}{d\eta}=\eta^2\,\theta^{\mathrm{m}},$$

$$\frac{d\theta}{d\eta}=-\frac{(\omega+\sigma\,\eta^3\,\theta^{(m+1)})(1+\sigma\theta)}{\eta^2\,\{1-2\,\sigma\,(m+1)\,(\omega/\eta)\}}. \tag{19}$$

The metric coefficient exp (ν) is given by equation (16) for the polytropic part of the envelope and by relevant equations of Paper I for the adiabatically stable part of the envelope and the core. In particular we have

$$\exp\,(\nu_{\mathrm{s}})=(1-2\,u_{\mathrm{s}})=\left(1-2\frac{C_2}{C_1}\frac{\omega_{\mathrm{s}}}{\eta_{\mathrm{s}}}\right),$$

$$\exp\,(\nu_{\mathrm{d}})=\frac{\exp\,(\nu_{\mathrm{s}})}{(1+\sigma)^{2(m+1)}}=\frac{\exp\,(\nu_{\mathrm{s}})}{\{1+k(\tilde{p}_{\mathrm{d}}/\tilde{\varrho}_{\mathrm{d}})\}^{2(m+1)}},$$

$$\exp\,(\nu_{\mathrm{b}})=\exp\,(\nu_{\mathrm{d}})\left\{\frac{(1+k)\cdot\tilde{p}_{\mathrm{b}}}{k(n+1)\,\tilde{p}_{\mathrm{d}}+(1-nk)\,\tilde{p}_{\mathrm{b}}}\right\}^{-2/(n+1)},$$

and

$$\exp\,(\nu_{\mathrm{c}})=\exp\,(\nu_{\mathrm{b}})\{(\tilde{p}_{\mathrm{b}})^{2k/(k+1)}\}. \tag{20}$$

The redshifts are given by the usual expressions

$$(1+z)=\exp\,(-\nu/2), \tag{21}$$

since $x=\exp\,(\nu)/\exp\,(\nu_{\mathrm{s}})$ $F(x)$ can be computed using (12)–(14).

4.2 NUMERICAL COMPUTATIONS

The following procedure was adopted. For a given original model (n, k) and polytropic index m, a series of points a_{i} in the envelope were chosen ($a_{\mathrm{b}}\leqslant a_{\mathrm{i}}<a_{\mathrm{s}}$). The polytropic patch-up was done from each point a_{i} in the envelope. The pair of ordinary first-order linear differential equations given by equation (19) were numerically integrated using Runge–Kutta method with the

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1984MNRAS.210..753D

following boundary conditions.

$\theta=\theta_d=1$, $\omega=\omega_d=C_1 a_d u_d$ at $\eta=\eta_d=C_2 a_d$, $\theta=\theta_s=0$ at $\eta=\eta_s$ (surface of the object).

The metric coefficient exp (ν) was evaluated as indicated above and $F(x)$ was computed numerically using equations (12)–(14). The new values of physical parameters such as redshifts z_c, z_b and z_s and radius a_s were also calculated. The range $k^{-1}=4(1)10$ and $n=3(1)$ $(k^{-1}-1)$ $(k^{-1}-1)$ (0.1) $(k^{-1}-0.1)$ with polytropic index $m=2$, 2.5, and 3 was investigated. As remarked upon earlier, cores with $k \geqslant 1/3$ and envelopes with $n<3$ were not considered since the possibility of getting a positive $F(x)$ is smaller for such cases.

4.3 DISCUSSION

The main conclusions of the numerical computations may be summarized as follows.

1. For a majority of models with $k^{-1}>5$, $n \geqslant 5$ the distribution function $F(x)$ remains positive throughout for the entire range of integration $x_c \leqslant x<1$.
2. The radius of a composite model is much larger compared with the radius of the original model.
3. The values of surface redshift z_s are much smaller and those of the central redshift z_c and core–envelope boundary redshift z_b are marginally larger compared with their values for the original model.
4. Differences in the old and new values of physical parameters diminish as the polytropic layer is made thinner.

These general trends can be observed from the values of the redshifts and radii given in Table 1 for the composite model $k^{-1}=10$, $n=9$, $m=3$ with polytropic patch-ups of different thicknesses as well as for the original model $k^{-1}=10$, $n=9$.

It is interesting to compare the properties of the composite models with those of the complete polytropic spheres discussed by Tooper (1964) and Fackerell (1968). For a given polytropic index m and 'relatively parameter' σ, which we here define as the ratio of the pressure and the energy density at the centre of the object, from relevant equations of Tooper (1964) we have

$$(a_s)_{\mathrm{Tooper}}=\left\{\sqrt{\frac{(m+1)\sigma}{3}}\right\}\xi_1$$

and

$$(u_s)_{\mathrm{Tooper}}=\sigma\,(m+1)\;\frac{v\,(\xi_1)}{\xi_1}, \tag{22}$$

Table 1. Variation of redshifts z_c, z_b and z_s and the radius a_s with the $m=3$ polytropic layer thickness parameter x_d (larger x_d indicates thinner layer) for the model with core $p=\varrho/10$ and envelope $dp/d\varrho=1/9$. The original values of the same variables are given in parentheses.

x_d	0.51	0.57	0.67	0.80	0.89
z_s	0.04	0.05	0.06	0.11	0.15
z_b	0.57	0.57	0.55	0.53	0.52
z_c	0.94	0.94	0.90	0.88	0.88
a_s	10.47	9.86	6.33	3.56	2.78

$(z_s=0.20,\ z_b=0.52,\ z_c=0.88,\ a_s=2.20)$

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1984MNRAS.210..753D

where the values of dimensionless radius ξ_1 and dimensionless mass function $v(\xi_1)$ have been tabulated by Tooper. [Note that $v(\xi_1)$ is not the $v(r)$ defined in equation (6).]

The Tooper model corresponding to our $k=1/10$ core has $\sigma=0{\cdot}1$. From table 1 and table 2 of Tooper this has, for $m=3$, $\exp(v_c)=0.4075$, $\xi_1=6.826$ and $v(\xi_1)=1.078$. Substitution in equation (22) yields

$$z_c=0.5665, \quad a_s=2.4925 \quad \text{and} \quad z_s=0.06986.$$

Comparing with the corresponding values for the composite model $k^{-1}=10$, $n=9$, $m=3$ with the thinnest polytropic layer $x_d=0.89$ from Table 1 it is seen that the redshifts obtained from the latter are significantly higher whereas the radius is marginally larger. These trends are observed for the entire range of models considered.

For a given m, σ can take any value from 0 to $m/(m+1)$ at which the speed of sound at the centre equals that of light (Tooper 1964). But in order that the distribution function be positive throughout, the condition $\sigma<\sigma_m<m/(m+1)$ must be satisfied (Fackerell 1968). For example for $m=3$, $\sigma_m=0.166989$ ($\approx 1/6$) (Fackerell 1968, Table 2). Our calculations show that for $m=3$ positive distribution function can be obtained even for cores with $1/6<k<1/5$. Thus the composite models can admit marginally stiffer equations of state for a given polytropic index.

5 Composite configurations as models for quasars

In the earlier papers in this series it has been shown that the original core–envelope configurations were fairly successful as central gravitational redshift models for high redshift quasars. In this section, we discuss the relevance of the composite configurations developed above as models for quasars.

As seen above, the requirement of positivity of the distribution function puts an upper limit on the stiffness of core-equation of state, viz. the core must have $p<\varrho/5$, i.e. $k<1/5$. However, stability considerations of the original models against small radial perturbations show that – *cf.* Paper II for details – for each k there exists a $n=n_{st}(k)\leqslant k^{-1}$ and a corresponding central redshift value $z_c=\{z_c(k)\}_{st}$ such that all models with a core k and envelopes with $n\geqslant n_{st}(k)$ and $z_c\geqslant\{z_c(k)\}_{st}$ are unstable against radial pulsations. Thus the stability requirement puts upper limits on the values of n and the central redshift z_c for a given k. While a similar rigorous analysis of the pulsational stability of the composite configurations is still under progress, preliminary results show that the above conclusions remain unchanged if the polytropic layer is kept sufficiently thin so that the overall structures are not disturbed significantly.

Taking $k=1/6$ as the maximum permissible stiffness of the core we have from Table 1 of Paper II $n_{st}(1/6)=5.1$ and $\{z_c(1/6)\}_{st}=1.50$. Thus the maximum admissible central redshift is 1.50.

If we assume that the total redshift z of a QSO has a cosmological component z_{cos} and a non-cosmological, i.e. gravitational, component $z_g(=z_c)$ we have

$$(1+z)=(1+z_{cos})\cdot(1+z_g) \tag{23}$$

Since galaxy redshifts up to $z\approx1$ are known (the largest known galactic redshift to date is $z=1.172$ for the radio source 3C 427.1) and the cosmological nature of galaxy redshifts is fairly well established it is reasonable to put $z_{cos}\leqslant1$. From equation (23) we then obtain, for $z_g\leqslant1.5$, $z\leqslant4$. Thus the maximum total redshift of a stable relativistic cluster is around 4.

It is interesting to note that this limit provides a natural explanation for the observed decrease in space density of quasars with $z\gtrsim3.5$. It is well-known that quasars are plentiful for redshifts as large as 3.2 but there is a sharp decrease in the space density at $3.7<z<4.7$ (Osmer 1982). In fact it was nearly a decade after the discovery of OQ172 with $z=3.53$ (Wampler *et al.* 1973) that a quasar with a higher redshift PKS 2000–330 with $z=3.78$, was discovered (Peterson *et al.* 1982). If the

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1984MNRAS.210..753D

observed paucity is real, on the basis of cosmological interpretation of quasar redshifts, it would imply that the quasars were formed at epochs when the cosmic scale factor was about one-fifth of its present value and became suddenly visible in an interval of a few hundred million years. However, there are difficulties associated with this interpretation (see e.g. Osmer 1982). In the light of the models discussed in the present paper we put forward the hypothesis that the limit on the internal gravitational redshifts imposed by the equations of state manifests itself as the observed upper bound for quasar redshifts.

6 Conclusions

The main results of the present paper may be summarized as follows:

1. The negativity of the Boltzmann distribution function of the stars in a star cluster scenario for the original core–envelope type configurations can be removed by altering the equation of state near the surface to a polytropic one, without disturbing the overall structures significantly.
2. The requirement of positivity of the distribution function puts an upper limit on the stiffness of the core equation of state, viz. $p<\varrho/5$.
3. The maximum central redshift that can be obtained from these configurations, compatible with the dual requirements of stability against radial pulsations and positivity of the distribution function, is around 1.5. Consequently the maximum total redshift is around 4, provided the cosmological component is not more than 1.

It is not our aim to claim that the models discussed here are necessarily correct. What the present work demonstrates is that it is possible to construct physically plausible configurations with large central gravitational redshifts which can account for many properties of the high redshift quasars. Contrary to the general belief we feel that the cosmological nature of quasar redshifts has not yet been established beyond doubt. Non-cosmological redshift models, such as the gravitational redshift models discussed here, hence merit serious consideration and further investigation.

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