

Fast electron generation in quasiperpendicular shocks and type II solar radiobursts

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Summary. The paper is devoted to the theory of type II solar radiobursts caused by collisionless shock waves propagating in the coronal plasma. The model developed is based on the theory of collisionless shock waves, which predicts that a small fraction of ions is reflected from the shock front. This ion beam is unstable and can drive low-frequency waves ($\omega_{\text{Hi}} < \omega < \omega_{\text{He}}$) which, as shown in this paper, are quickly absorbed by the magnetized electrons of the background plasma, leading to the formation of nonmaxwellian electron tails. When entering the cold background plasma, these hot electrons, in turn, drive high frequency Langmuir oscillations with $\omega \approx \omega_{pe}$ up to the high level $W_L/n_0 T_e \sim 10^{-5} - 10^{-4}$. The conversion of plasma waves into electromagnetic waves is caused by the induced scattering of plasma waves of ions ($\omega \approx \omega_{pe}$) or by merging of two Langmuir waves ($\omega \approx 2\omega_{pe}$).

The role of nonlinear processes is studied. The brightness temperature calculated from the theory, $T_b \sim 10^{11}$ K, appears to be in very good agreement with observations.

Key words: shocks – electron acceleration – solar radio-emission

1. Introduction

It is generally agreed that the solar radiobursts of spectral type II are generated by the passage outward through the solar corona of fast-mode MHD shock waves which originate in relatively intense solar flares and which, at any given height in the solar corona, excite radioemission of the appropriate plasma frequency (Wild and Smerd, 1972; McLean and Nelson, 1977; McLean, 1980).

Estimates of the velocities of the shocks, made from the frequency drift-rate of the type II bursts, in conjunction with appropriate models for the electron densities, generally give velocities of about thousands kilometers per second (Maxwell and Thompson, 1962; Maxwell and Dryer, 1981).

Nevertheless, despite the fact that the connection between solar radiobursts of the second type and shocks has become evident, the details of burst generation are not clear yet.

One of the possible means of burst generation by collisionless shock waves moving across the magnetic field was proposed by Pikelner and Ginzburg (1963) and developed by Zaitsev (1977); it is connected with the Buneman instability developing at the shock front. This instability is due to the relative motions of ions and electrons. However, the plasma waves excited have low frequen-

cies. The rising of the plasma wave frequency needs subsequent induced scattering on nonthermal electrons that leads to the broadening of the spectrum up to $2\omega_{pe}$ and to its isotropisation. So the transformation of plasma waves into electromagnetic ones gives use to the radioemission in a wide frequency band $\omega_{pe} \leq \omega \leq 2\omega_{pe}$. The harmonic structure of the emission spectrum can be obtained only in the case when the plasma wave spectrum is isotropic and has two maxima with respect to k : one in the region of plasma wave pumping and the second in the region of dissipation due to the radiation losses.

Lampe and Papadopoulos (1977) proposed that type II emission can be associated with acceleration of electrons by lower hybrid waves followed by nonlinear conversion. The lower hybrid waves were assumed to be generated by a current driven instability in the shock front.

However, the mechanisms proposed are not general. In particular, they do not describe adequately an analogy between type III and type II bursts, which is often observed (the example is the herringbone structure). The type III radiobursts, as it is well known, are produced by fast electron streams propagating along the magnetic field lines (Ginzburg and Zheleznyakov, 1958). From the analogy between the components of the second type bursts and the type III bursts one can suppose that type II bursts are also generated by streams of accelerated electrons (McLean and Nelson, 1977). Note that the intense streams of ions and electrons and waves driven by them are very often observed near the Earth's bow shock (Eastman et al., 1981; Anderson et al., 1981). It is natural to connect the radioemission of the coronal shock with the fluxes of the electrons as the most general and effective way of direct generation of plasma waves (Langmuir oscillations).

In this case the key questions are those of the effective electron acceleration and of the transformation of electron energy into electromagnetic radiation. As it is known, the radioemission of the type II bursts may be very strong [$T_b \sim 10^{11}$ K (Nelson and Robinson, 1975)].

In our opinion, the approach which makes it possible to answer these questions is that proposed by Papadopoulos (1981) and Vaisberg et al. (1983) for the Earth's bow shock. Its main features are the following:

The ions, accelerated during the reflection from the front of the quasiperpendicular shock wave, form the beam which advances almost perpendicular to the external magnetic field. It is widely known that such an ion beam can easily drive low-frequency oscillations ($\omega_{\text{Hi}} \ll \omega \ll \omega_{\text{He}} \ll \omega_{pe}$) with wavevectors almost normal to the magnetic field. The magnetized electrons may absorb these oscillations due to the resonance $\omega = k_z v_z$. As a result the non-maxwellian "tail" of the electron distribution function grows.

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Entering the background plasma, non-maxwellian electrons form the beam, which effectively drives the Langmuir oscillations at the local plasma frequency ω_{pe} . In this case the emission of electromagnetic wave with $\omega = \omega_{pe}$ is due to the merging of Langmuir and low-frequency waves, the emission of harmonics – due to the coalescence of two Langmuir waves.

In our paper this process is studied in more detail in connection with the acceleration of the electrons at the front of a shock wave traverse by the Solar corona and the generation of type II solar radioemission.

2. The acceleration of electrons of the front of the shock

1. The general theoretical picture of quasiperpendicular collisionless shock waves shows that in the plasma stream moving towards the shock there is a small fraction of ions with energies too small to overcome the potential barrier of the shock (Sagdeev, 1964; Biskamp, 1973). When the Mach number, i.e. the ratio of the velocity v_0 with which the plasma flows through the front of the shock to the Alfvén velocity $v_A = (H^2/4\pi n_i m_i)^{1/2}$ reaches some critical value, $M = v_0/v_A > M_{cr} \approx 2$, the potential barrier which decelerates the ions in the shock equals the energy of the ions in the incoming stream.

In this case the shock wave overturns because of the reflection of the ions; the flow of the ions becomes multistreaming and the structure of the shock becomes nonlaminar. Now there is no full theoretical description of the turbulent shock wave structure. Nevertheless, one can state that the number of the reflected ions for $M < 10 \div 12$ is relatively small and they form an ion beam in the foreshock region. In the frame of reference moving with the running stream the velocity of such a beam is $v_b \sim 2v_0$, its density is $n_b < (0.1 \div 0.3)n_0$, the density of the background plasma. These estimates are confirmed both by numerous observations *in situ* near the Earth's bow shock (Russel and Hoppe, 1983; Gosling, 1983) and computer simulations of collisionless shock waves (Leroy et al., 1981, 1982; Shodura, 1975).

The reflected ions are turned by the magnetic field, face the shock front again and move with the background plasma through the shock (there is no secondary reflection). The presence of the reflected ions in the foreshock region leads to the formation of the so called “forerunner” or “pedestal” of the shock. The thickness of this region d is of the order of v_0/ω_{Hi} . It is this region, in which there is a small preliminary deceleration of plasma flow, that the increase of the density and of the magnetic field and the acceleration of the electrons take place. We shall consider the mechanism of this acceleration in connexion with the waves driven by an ion beam.

Dealing with the mean parameters of plasma in solar corona, let us suppose the following inequalities are valid: $\omega_{He} \ll \omega_{pe}$, $\beta_{e,i} = 8\pi n_0(T_e, T_i)/H^2 \ll 1$. Here H is the strength of the magnetic field, n_0 is the background plasma density, T_e, T_i are the electron and ion temperatures, $\omega_{pe} = (4\pi ne^2/m_e)^{1/2}$ and $\omega_{He} = (eH/m_e c)$ are respectively the plasma frequency and cyclotron frequency. The low density ion beam ($n_b/n_0 \ll 1$) with small thermal spread ($\Delta v_b \ll v_b$) moves almost perpendicular to the magnetic field and hence can effectively interact with low-frequency plasma oscillations, which have wavevectors almost perpendicular to the magnetic field lines. If the electrons are magnetized and ions are not, $\omega_{Hi} \ll \omega \ll \omega_{He}$, one can easily write the following dispersion relation for non-potential waves

$$\omega^2 = \frac{\omega_{He}^2}{1 + \omega_{pe}^2/k^2 c^2} \left(\mu + \frac{\cos^2 \theta}{1 + \omega_{pe}^2/k^2 c^2} \right). \quad (1)$$

Here $\mu = m_e/m_i$ is the electron to ion mass ratio, $\cos^2 \theta \equiv k_z^2/k^2 \ll 1$, $\mathbf{H} = H\mathbf{e}_z$. For $\omega_{pe}^2/k^2 c^2 \ll 1$ (1) describes the low-hybrid oscillations, while for $\omega_{pe}^2/k^2 c^2 \gg 1$ it is relevant for fast magnetosonic and whistler waves.

Note that it is necessary to consider only waves with $k_\perp d \sim kv_b/\omega_{Hi} > 1$, because the ion beam exist only in a region of finite thickness $d \sim v_b/\omega_{Hi}$. So the ions are automatically unmagnetized.

The oscillations described by (1) are unstable in the presence of an ion beam (Mikhailovskii, 1974). They grow with a rate:

$$\begin{aligned} \gamma_k &= -\frac{\omega}{2} \cdot \frac{\omega_{He}^2}{\omega_{pe}^2} \cdot \frac{\text{Im } \varepsilon}{1 + \omega_{pe}^2/k^2 c^2} \\ &\approx \frac{\pi}{2} \cdot \frac{\mu \omega_{He}^2}{(k \cdot \Delta v_b)^2} \omega \cdot \frac{n_b/n_0}{1 + \omega_{pe}^2/k^2 c^2}. \end{aligned} \quad (2)$$

To calculate this rate we suppose that the ion beam distribution function is Maxwellian:

$$f_b(v) = n_b [(2\pi)^{1/2} \Delta v_b]^{-3} \exp[-(v - v_b)^2/2(\Delta v_b)^2].$$

Hence

$$\begin{aligned} \text{Im } \varepsilon &= -\pi \mu \frac{\omega_{pe}^2}{n_0 k^2} \cdot \int \delta(\omega - \mathbf{k} \cdot \mathbf{v}) \mathbf{k} \cdot \frac{\partial f}{\partial \mathbf{v}} \\ &= \pi \mu \frac{n_b}{n_0} \cdot \frac{\omega_{pe}^2}{k^2} \cdot \frac{(\mathbf{k} \cdot \mathbf{v}_b - \omega)}{(k \cdot \Delta v_b)^2} \\ &\quad \cdot \exp[-(\mathbf{k} \cdot \mathbf{v}_b - \omega)^2/2k^2(\Delta v_b)^2]. \end{aligned}$$

One can argue that $\cos \varphi \equiv k_x/k$ is to be determined in an optimal way: $kv_b \cos \varphi - \omega \approx \Delta v_b$, so $\text{Im } \varepsilon$ reaches its maximum value:

$$\text{Im } \varepsilon = \pi \frac{\omega_{pe}}{k^2 (v_b)^2} \cdot \frac{n_b}{n_0}.$$

Finally,

$$\gamma_b = \frac{\pi}{2} \omega \frac{\mu \omega_{He}^2}{k^2 (\Delta v_b)^2} \cdot \frac{n_b/n_0}{1 + \omega_{pe}^2/k^2 c^2}. \quad (2)$$

The growth of oscillations resonant with an ion beam is in principle, limited a) by convection of waves through the shock front; b) by the energy losses due to the resonance with the electrons, $\omega = k_z v_z$, and their acceleration and c) by the nonlinear effects. We shall suppose that the quasi-static amplitudes of the waves, are small and neglect their nonlinear interaction (see discussion below).

In such a case the excitation of waves by an ion beam upstream, the relaxation of the beam and the quasilinear acceleration of electrons are described by the following system of equations (Vaisberg et al., 1983):

$$(v_g - v_0) \frac{\partial}{\partial x} E_k^2 = 2E_k^2 (\gamma_b + \gamma_e + \gamma_i); \quad (3)$$

$$\begin{aligned} v \frac{\partial f_i}{\partial x} + \frac{e}{m_i c} [\mathbf{v} - \mathbf{v}_0 \times \mathbf{H}] \cdot \frac{\partial f_i}{\partial \mathbf{v}} \\ = \pi \left(\frac{e}{m_i} \right)^2 \int \frac{d^3 k}{(2\pi)^3} \mathbf{k} \cdot \frac{\partial}{\partial \mathbf{v}} \cdot \frac{E_k^2}{k^2} \delta(\omega - \mathbf{k} \cdot \mathbf{v}) \mathbf{k} \cdot \frac{\partial f_i}{\partial \mathbf{v}}; \end{aligned} \quad (4)$$

$$\begin{aligned} v_z \sin \theta_0 \frac{\partial f_e}{\partial x} = \pi \left(\frac{e}{m_e} \right)^2 \int \frac{d^3 k}{(2\pi)^3} \frac{\partial}{\partial v_z} \frac{k_z^2 E_k^2}{k^2 (1 + \omega_{pe}^2/k^2 c^2)^2} \\ \cdot \delta(\omega - k_z v_z) \frac{\partial f_e}{\partial v_z}. \end{aligned} \quad (5)$$

These equations are valid in the frame of reference moving with the shock wave. Here θ_0 is the angle between the magnetic field vector and the normal to the wave front, i.e. the angle between H and x axis, $v_g = \partial\omega/\partial k$. The LHS of (3) describes the convection of oscillations from the region of interaction, while the RHS of (3) deals with generation of low-frequency waves by an ion beam (γ_b), their damping due to the resonant interaction with the electrons (γ_e) and the ions of the back-ground plasma (γ_i).

To analyze (3)–(5) one can make some simplifications. First, one can suppose that the part of the beam energy transmitted to the waves is small and does not affect the relaxation of the ion beam described by Eq. (4). This assumption is confirmed by the observations near the Earth's bow shock and the computer simulations mentioned above and is in good agreement with the results obtained below.

Second, since the electrons are magnetized, their transversal energy does not change and a one-dimensional distribution function is enough to describe the behaviour of the electrons: $F(v_z) = \int \pi dv_\perp^2 f_e(v_z, v_\perp)$. With this simplification the damping rate of waves due to the Cherenkov resonance with the electrons is

$$\gamma_e = \frac{\pi}{2} \omega \frac{\omega_{\text{He}}^2}{k^2} \left(1 + \frac{\omega_{pe}^2}{k^2 c^2}\right)^{-3} \frac{\partial F}{\partial v_z} \Big|_{v_z = \omega/k_z} \quad (6)$$

and the velocity of resonant electrons is

$$v_z^2 = \left(\frac{\omega}{k_z}\right)^2 = \frac{v_A^2 \cdot \frac{\omega_{pe}^2}{k^2 c^2}}{\mu(1 + \omega_{pe}^2/k^2 c^2)^2} + \frac{v_A^2 \cdot \frac{\omega_{pe}^2}{k^2 c^2}}{\cos^2 \theta (1 + \omega_{pe}^2/k^2 c^2)}. \quad (7)$$

The third, and last, simplification is due to the fact that the term describing the wave convection is small in comparison with γ_b : $v_b = Mv_A > v_g \sim v_A$, and the LHS of (3) is

$$(v_0 - v_g) \frac{\partial E_k^2}{\partial x} \approx \frac{v_0}{d} E_k^2 \approx \alpha \omega_{\text{Hi}} E_k^2. \quad (8)$$

Here $\alpha \lesssim 1$ and we use the estimate $\partial/\partial x \sim d^{-1} \sim \omega_{\text{Hi}}/v_0$. Comparing (8) with $\gamma_b E_k^2$ one can conclude that the convection is important only for long wavelength oscillations for which $\gamma_b < \omega_{\text{Hi}}$ or (for simplicity it is assumed that $\cos^2 \theta < \mu$)

$$\mu^{1/2} \cdot \frac{\omega}{\omega_{\text{Hi}}} \cdot \frac{1}{1 + k^2 c^2/\omega_{pe}^2} \approx \frac{kc/\omega_{pe}}{(1 + k^2 c^2/\omega_{pe}^2)^2} \lesssim \frac{\mu^{1/2}}{\frac{n_b}{n_0} \cdot \left(\frac{v_A}{\Delta v_b}\right)^2}. \quad (9)$$

The important parameter $\frac{n_b}{n_0} \left(\frac{v_A}{\Delta v_b}\right)^2 \sim \frac{n_b}{n_0} \cdot \frac{1}{\beta}$, connected with the convection, determines, in particular, the threshold of the ion beam instability. Since the LHS of (9) is less than 3/8 for all k , for the instability to occur it is necessary to fulfill the condition $\frac{n_b}{n_0} \left(\frac{v_A}{\Delta v_b}\right)^2 > \frac{8}{3} \mu$. For $n_b/n_0 > 10^{-1} \beta$ this inequality is satisfied and one can take the convection into account only in the small part of phase volume occupied by the oscillations. The energy of these waves is small too. So the energy of the ion beam which is transferred to the waves, is mainly absorbed by the electrons. As a result, strong electron acceleration along the magnetic field lines will occur and $F(v_z)$ will have a non-maxwellian “tail”.

In the rest of the paper we shall describe this process in detail. The scheme of the solution is the following. Balancing $\gamma_b + \gamma_e \approx 0$ [Eq. (3)] one can find the fraction of electrons accelerated and

determine $F(v_z)$ such that the energy lost by an ion beam is absorbed by the electrons. Note that, when solving Eq. (3), we do not take into account the finite size of the system, while the wave convection is considered in a simplified way, see e.g. (8). Knowing $F(v_z)$, one can determine the wave spectrum and estimate if the relaxation of an ion beam is important.

In that part of phase space (k_\perp, k_z) where $E_k^2 \neq 0$ it follows from (3) that

$$-\frac{\partial F}{\partial v_z} \geq \frac{\mu}{v_A^2} \left(\frac{\omega_{pe}^2}{k^2 c^2}\right)^2 \left(1 + \frac{k^2 c^2}{\omega_{pe}^2}\right)^3 \cdot \left[\frac{n_b}{n_0} \left(\frac{v_A}{\Delta v_b}\right)^2 \frac{1}{1 + k^2 c^2/\omega_{pe}^2} - \alpha \frac{\omega_{\text{Hi}}}{\omega}\right]. \quad (10)$$

In the rest of phase space the LHS of (10) is negative, because the waves should damp. Taking into account (2) and (6), (10) can be written as

$$\gamma_b + \gamma_e - \alpha \omega_{\text{Hi}} \approx 0. \quad (11)$$

Note that resonance condition (7) and dispersion relation (1) allow us to find $\cos \theta$ and ω as functions of k, v_z , so that the RHS of Eq. (11) depends upon both independent variables. The exact equality takes place only for a single $k = k_{\text{max}}$, for which the RHS of (11) reaches its maximum. The waves with another k are damped, i.e. the spectrum of the oscillations is streamer-type and the energy is concentrated in a narrow band near the line $k = k_{\text{max}}$ on the phase plane (k_\perp, k_z).

Now our aim is to find the maximum of RHS of (11) for a given electron velocity v_z . For this purpose find $\omega = \omega(k, v_z)$, excluding $\cos \theta$:

$$\omega^2 = \mu \omega_{\text{He}}^2 \cdot \frac{1 + k^2 c^2/\omega_{pe}^2}{(1 + \omega_{pe}^2/k^2 c^2)(1 + k^2 c^2/\omega_{pe}^2) - (v_A^2/\mu v_z^2)} \quad (12)$$

and substitute ω into Eq. (11). To simplify the formula let us introduce

$$u = \mu^{1/2} \frac{v_z}{v_A}; \quad \xi = \frac{k^2 c^2}{\omega_{pe}^2}; \quad a = M^{-2} \frac{n_b}{n_0} \left(\frac{v_A}{\Delta v_b}\right)^2$$

and write down (11) in a form

$$-\frac{\partial F}{\partial u} = \frac{\mu^{1/2}}{v_A} \left(\frac{1 + \xi}{\xi}\right)^2 \left\{ a - \alpha \mu^{1/2} \left[\frac{(1 + \xi)^3}{\xi} - \frac{1 + \xi}{u^2} \right]^{1/2} \right\}. \quad (11')$$

We are interested in the region of superthermal velocities $v_z > 2v_T$, i.e. $u^2 > 4\mu v_T^2/v_A^2 = 4\beta_e$. Hence, if β_e is not too small, one can neglect the term with u^{-2} in (11'). Then the maximum of the RHS

of (11) is reached for $\xi = \xi_* = \left(\frac{5}{4} \frac{\sqrt{\mu}}{\alpha}\right)^2 < 1$ and it does not depend on u . It implies that the distribution function of the accelerated electrons is linear:

$$F(v_z) = \frac{\mu}{5\xi_*^2} \cdot \frac{n_b}{n_0} \cdot \frac{v_A^2}{(\Delta v_b)^2} \cdot \frac{v_h - v_z}{v_A^2}.$$

We do not dwell upon the dependence of ξ_* upon the parameters for the convenience and the compactness of formulae. When $\beta_e < 1/4 \sqrt{\mu}/\alpha$, ξ_* slightly depends upon u for small velocities. Although this dependence $\xi_*(u)$ is easy to find and determines the deviation of $F(v_z)$ from the linear law, this effect proved to be insignificant and we shall ignore it.

For small velocities $v_z = v_* \approx (2 + 3)v_T$ the distribution function of the accelerated electrons merges with the thermal electron

distribution function, which is supposed to be Maxwellian ($v_h \gg v_*$):

$$F(v_*) = \frac{\mu v_h}{5v_A^2} \cdot \frac{n_b}{n_0} \cdot \frac{1}{\xi_*^2} = \frac{\exp[-v_*^2/2v_{Te}^2]}{\sqrt{2\pi}v_{Te}}. \quad (13)$$

Before we find the maximum velocity v_h of the accelerated electrons, the analysis of a streamer-type spectrum is in order. As mentioned above, the waves are concentrated near the line

$k_\perp = k_\perp(k_z) = \frac{\omega_{pe}}{c} \sqrt{\xi_*} = k_* \approx \text{const}$ on a phase plane. Their frequency

$$\omega = \left[\frac{\mu \omega_{He}^2 \xi_*}{1 - (v_A^2 \xi_* / \mu v_z^2)} \right]^{1/2} \approx k_z v_A \approx \omega_{Hi} \left(\frac{\xi_*}{\mu} \right)^{1/2}$$

is practically constant for about the whole spectrum, where $v_z^2 \gg \frac{v_A^2}{\mu} \xi_*^2$. As it follows from the resonant condition, $\omega = kv_b \cos \varphi$, the streamer consists of two straight lines on (k_x, k_y) making angles $\pm \varphi_*$ with k_x axis. Here

$$\cos \varphi_* = \frac{k_x}{(k_x^2 + k_y^2)^{1/2}} \approx \frac{v_A}{v_b}.$$

To find the wave spectrum one can rewrite Eq. (5) in the form

$$-\frac{v_z \cos \theta_0}{d} F(v_z) = \pi \left(\frac{e}{m_i} \right)^2 \frac{\partial}{\partial v_z} \left[\frac{v_A^2}{v_z^2} \cdot \frac{1}{\xi_*} \cdot \int \frac{d^2 k_\perp}{(2\pi)^2} E_k^2 \frac{\partial}{\partial v_z} F(v_z) \right] \quad (14)$$

using the fact that the spectrum is of the streaming-type. For simplicity we shall consider only the region $v_z^2 > \frac{v_A^2}{\mu} \xi_*^2$ which is most important for the following. One can obtain from (7) that $\cos^2 \theta \approx \frac{v_A^2}{v_z^2} \xi_*$. Substituting (13) into (14) one find after integration

$$\int \frac{d^2 k_\perp}{(2\pi)^2} E_k^2 = \frac{\xi_* \sin \theta_0}{\pi d v_A^2} \left(\frac{m_e}{e} \right)^2 v_z^3 \int_{v_*}^{v_z} v_z (v_h - v_z) dv_z \approx 4\pi \mu^{3/2} \xi_*^{7/2} \sin \theta_0 \left(\frac{v_A}{c} \right)^2 \cdot \frac{m_i n_0 v_A^2}{2} \cdot \left(\frac{\omega_{pe}}{k_z c} \right)^4 \cdot \frac{1}{k_z} \cdot \left(1 - \frac{2}{3} \frac{v_A}{v_z} \cdot \frac{k_x}{k_z} \right). \quad (15)$$

The density of the wave energy is

$$W_1 = \int \frac{dk_z}{2\pi} \left(1 + \frac{c^2}{v_A^2} \right) \int \frac{d^2 k_\perp}{(2\pi)^2} \frac{E_k^2}{8\pi} = \frac{1}{8} (\mu \xi_*)^{3/2} \sin \theta_0 \cdot \frac{m_i \cdot n_0 \cdot v_h^2}{2}. \quad (16)$$

Knowing the wave intensity one can easily find the change of the velocity spread in the beam, $\delta(\Delta v_b)$. In agreement with (4),

$$\delta(\Delta v_b) \approx \sqrt{D d / v_b} \approx \sqrt{D / \omega_{Hi}}, \quad (17)$$

where the diffusion coefficient D is determined by the waves

$$D = \pi \left(\frac{e}{m_i} \right)^2 \int \frac{d^3 k}{(2\pi)^3} \cos^2 \varphi E_k^2 \delta(\omega - \mathbf{k} \cdot \mathbf{v}) = 4\pi^2 \left(\frac{e}{m_i} \right)^2 \cdot \frac{(v_A/v_b)^2}{[1 - (v_A/v_b)^2]^{1/2}} \cdot \frac{W_1}{k_x v_b}. \quad (18)$$

The assumption that the change of the growth rate of the ion beam small and that the energy of the beam is transmitted to the electrons is fulfilled if $\delta(\Delta v_b) < \Delta v_b$ or, as it follows from (16)–(18), if

$$\frac{\mu v_h^2}{v_A^2} \lesssim \frac{(\Delta v_b)^2 v_b^3}{v_A^5} \cdot \frac{\omega_{He}^2}{\omega_{pe}^2} \cdot \frac{8}{\xi_*^{1/2} \sin \theta_0}. \quad (19)$$

To find v_h it is necessary to balance the energy fluxes, i.e. the energy flux lost by the ion beam must be equal to the energy flux gained by the accelerated electrons:

$$\delta(n_b m_i v_b^3) = n_b m_i v_b^2 \delta(\Delta v_b) = \sin \theta_0 \int n_0 m_e v_z^3 F(v_z) dv_z. \quad (20)$$

From (12), (17), and (18) it follows that

$$\frac{\mu v_h^2}{v_A^2} = 10 \xi_*^{5/4} (\sin \theta_0)^{-1/4} \left(\frac{\omega_{pe}}{\omega_{He}} \right)^{1/2} \left(\frac{v_b}{v_A} \right)^{1/4} \cdot \frac{\Delta v_b}{v_A}. \quad (21)$$

Note that the maximum velocity of the accelerated electrons does not depend explicitly upon the density of the ion beam. However, n_b/n_0 determines ξ_* . For example, for shock waves with $M \approx M_{cr} = 2$, $\beta_i \sim 10^{-1}$, $\theta_0 \sim 10^{-1}$, $n_b/n_0 \sim 10^{-2}$, $\omega_{pe}^2/\omega_{He}^2 = 900$, $(\Delta v_b)^2/v_A^2 \sim \beta_i$ one can find from Eq. (9) that $\xi_*^{1/2} \approx \frac{1}{4} \div \frac{1}{3}$ and after substitution in (21) obtain

$$m_e v_h^2 \approx (1 \div 2) \frac{v_A^2}{\mu} m_e \approx \frac{T_e}{\beta_e} > T_e$$

so the electrons are accelerated up to superthermal velocities ($\beta_e \sim \beta_i \sim 10^{-1}$). In accordance with (16), the level of turbulence appears to be:

$$\frac{W_1}{n_0 T_e} \approx \frac{\sin \theta_0}{16} \mu^{1/2} \xi_*^{1/2} \frac{m_e v_n^2}{T_e} \approx 5 \cdot 10^{-4}$$

and the total density of the accelerated electrons:

$$n_{eh} = \int_{v_*}^{v_h} F dv_z = \frac{1}{10 \xi_*^2} \cdot \frac{n_b}{n_0} \left(\frac{v_A}{\Delta v_b} \right)^2 \frac{v_n^2}{v_A^2} \mu n_0 \approx 10^{-2} n_0.$$

2. The self-consistent theory of electron acceleration near the shock front, developed above, is based on the quasilinear approach. However, in the frame of a quasilinear theory for waves with $k_z < \omega/v_h \approx \sqrt{\mu} k_\perp$ there are no resonant electrons which can limit the growth of these waves. So the growth of the waves in this region of phase space is limited by nonlinear interaction. The full nonlinear analysis is beyond the scope of this paper, so below we restrict ourselves only to a qualitative analysis of nonlinear interactions.

For $k_z/k < \sqrt{\mu}$ the strongest nonlinear effect is the induced scattering of waves by electrons. The physics of this process is the following: the beat of two low-hybrid waves may have a phase speed close to that of the thermal electrons. So the beat will be quickly absorbed by the electrons, leading to the nonlinear “coupling” of primary waves and forcing the low-hybrid waves to move outwards in k -space. Thus, the waves driven by a beam will quickly leave the resonance region in k -space, limiting the density of the waves there.

For potential waves this process and the collapse of the low-hybrid waves, closely connected with it, were studied in numerous papers (Sturman, 1974; Sotnikov et al., 1978; Hasegawa and Chen, 1975). With nonpotentiality of waves taken into account, the nonlinear damping rate of induced scattering by electrons may be found in a similar way:

$$\begin{aligned} \gamma_{NL}(k) &= \sum_{k'} \frac{1}{1 + \omega_{pe}^2/k^2 c^2} - \frac{W(k')}{n_0 T_e} \cdot \frac{[\mathbf{k} \times \mathbf{k}']_z^2}{k^2 k'^2} \\ &\cdot (k_z - k'_z) \left(\frac{k_z}{\omega} - \frac{k'_z}{\omega'} \right) v_{Te}^2 \operatorname{Im} \varepsilon(k - k') \\ &\approx \frac{\pi}{2} \cdot \frac{\omega_{pe}^2}{1 + \omega_{pe}^2/k^2 c^2} \cdot \frac{\partial}{\partial k_z} \int \frac{d\pi k'_\perp}{2\pi} \frac{W(k'_\perp, k_z)}{n_0 T_e} \cdot \frac{\omega - \omega'}{v_{Te}^2} \end{aligned} \quad (22)$$

Here we use the fact that the spectral “repumping” of the waves occurs in a differential way: for a single scattering

$$\Delta k_z \approx (\omega - \omega')/v_{Te} < \frac{v_A}{v_{Te}} k_\perp \ll k_z.$$

As the result of induced scattering the energy of the waves is transmitted along the line $k_z = 0$ to the region of greater k_\perp , where it is absorbed by the thermal electrons. Part of the energy is lost during the spectral “repumping” because of angular scattering of waves into that region of phase space, where waves are effectively absorbed by the accelerated electrons.

Thus, the streamer-type spectrum $W(k) \sim \delta(k - k_*)$ for $k_z \sim v_A k_*/v_h$ turns and is continued in a form of a streamer along the k axis. In this branch of a streamer there is a balance between the induced scattering of waves and their generation by an ion beam: $\gamma_b \approx \gamma_{NL}$. Estimating the thickness of the streamer as $\Delta k_z \sim \omega/v_h$ and substituting in (22) $\omega(k) - \omega(k') \approx \sqrt{\mu} \omega_{He}$, one obtains

$$\begin{aligned} \gamma_{NL} &\approx \frac{\pi}{2} \cdot \frac{\omega_{pe}^2}{1 + \omega_{pe}^2/k^2 c^2} \cdot \frac{\sqrt{\mu} \omega_{He}}{(\Delta k_z v_{Te})^2} \cdot \int \frac{d\pi k'_\perp \Delta k_z}{(2\pi)^3} \\ \cdot \frac{W(k)}{n_0 T_e} &\approx \frac{1}{\sqrt{\mu}} \left(\frac{v_h}{v_{Te}} \right)^2 \cdot \left(\frac{\omega_{pe}}{\omega_{He}} \right)^2 \omega_{He} \frac{W_2}{n_0 T_e}. \end{aligned} \quad (22')$$

Now, balancing $\gamma_b \approx \gamma_{NL}$, one can find the wave energy density on the streamer, passing along k_\perp axis, assuming that the waves are concentrated near the maximum of the growth rate γ_b and $k_\perp c/\omega_{pe} \approx 1$. The result is

$$\frac{W_2}{n_0 T_e} \approx \mu \left(\frac{v_{Te}}{v_h} \right)^2 \frac{\omega_{He}^2}{\omega_{pe}^2} \cdot \frac{n_b}{n_0} \cdot \left(\frac{v_A}{\Delta v_b} \right)^2. \quad (23)$$

It should be noted that the presence of shortwave oscillations (i.e. the streamer passed along the k_\perp axis) may change the position of the main part of a streamer, $k \approx k_* \ll \omega_{pe}/c$ and thus may change ξ_* . To understand this fact one should keep in mind that the nonlinear interaction of waves belonging to the main streamer in k -space is unimportant, because the main nonlinear process for $k_z/k > \sqrt{\mu}$ is the induced scattering of ions and the corresponding phase velocity of the beat waves is $v_{ph} \sim (\omega - \omega')/(k - k') \sim v_A \gg v_{Te}$, so only an exponentially small fraction of the ions takes part in this process. On the contrary, the induced scattering caused by absorption of the beat waves by waves from the main part of the streamer, $k \sim k_*$, and the shortwave oscillations from the tail of a streamer, $kc > \omega_{pe}$, is very effective because in this process all the ions may play a role. The growth rate of this last process is (Sturman, 1974)

$$\gamma_{NL} \approx \frac{\omega_{pe}^2}{\omega_{He}^2} \frac{1}{\sqrt{\mu}} \frac{W_2}{n_0 T_e} \approx \sqrt{\mu} \omega_{He} \left(\frac{v_{Te}}{v_h} \right)^2 \frac{n_b}{n_0} \frac{v_A^2}{(\Delta v_b)^2}. \quad (24)$$

Thus, for $(n_b/n_0)(v_A/\Delta v_b)^2 > (v_h/v_{Te})^2 \sqrt{\mu} \gamma_{NL} > \omega_{Hi}$ and in this case the “cross-section” induced scattering on ions is more effective in stabilizing the waves with $k_\perp < k_*$ than the convection of

oscillations considered earlier [see Eq. (8)]. In this case the position of the streamer in k -space does not depend upon n_0 :

$$\frac{k_* c}{\omega_{pe}} \sim \left(\frac{v_{Te}}{v_h} \right)^2 \sim \beta_e.$$

Since it is difficult to determine $k_* c/\omega_{pe}$ taking into account, nonlinear effects, we estimate it as $\xi_* \sim 0.1 \div 0.3$.

Summarizing the results of this section, we arrive at the following conclusions.

The electrons near the front of the shock may be accelerated up to an energy of the order of $m_i v_A^2/2$, and the density of the accelerated particles may reach a value of $n_{eh} \sim (10^{-3} \div 10^{-2}) n_0$, depending upon the Mach number M and the angle θ_0 . In addition, the acceleration of electrons up to high energies may also take place, due to their resonant interactions with waves located in an arrow cone $k_z \approx 0$ in k -space. However, the number of such electrons is small.

3. The radiation caused by electrons moving from the shock front

Let us consider a magnetic field line moving with the plasma flow. At some time this field line will touch the front of the shock. The electrons accelerated by the shock are then injected into the background plasma. Since the electrons are magnetized, they move along the field line, and there is an analogy between this process and that considered by Ryutov and Sagdeev (1970), when the flow of the hot plasma enters the half-space occupied by the cold plasma. If $F(v_z)$ is the distribution function of hot electrons, one can assume that at a given point $z > 0$ in a moment $t > 0$ the distribution function of electrons $f_e(v_z, z, t)$ is equal to $F(v_z)$ for $v_z > z/t$ and is small for all the other velocities (we consider $t = 0$ as the moment of injection). However, this distribution function is unstable and can drive Langmuir waves, which, in turn, cause fast diffusion in v -space and form a plateau:

$$f(z, t, v_z) = \begin{cases} P(z, t), & v_z < u(z, t); \\ f_0(v_z), & v_z > u(z, t). \end{cases}$$

The quasihydrodynamic equations for p and u were obtained and solved by Ryutov and Sagdeev (1970). Knowing $p(z, t)$ and $u(z, t)$, one can find the energy density of plasma waves. This approach was successfully used in a number of papers for explaining some features of type III solar radiobursts (Zaitsev et al., 1972; Zaitsev et al., 1974). In these papers it was shown that at a given point on the field line the wave density grows when the first group of hot electrons arrives, reaches its maximum value and then decreases during the passage of slow electrons. Since we do not know the details of the injection process, we shall use estimates following from the works of Ryutov and Sagdeev (1970) and Zaitsev et al. (1972, 1974): the energy density of Langmuir waves is approximately equal to one tenth of the energy density of hot electrons:

$$W_L \sim \frac{1}{10} n_{eh} \cdot \frac{m_e v_h^2}{2}. \quad (29)$$

Substituting the density and the energy of the hot electrons, obtained in Sect. 2, in (29), we find $W_L > (10^{-5} \div 10^{-4}) \cdot n_0 T_e$.

Now let us consider the processes responsible for the radioemission of the shocks moving in the Solar corona.

The high level of turbulence ensures the high efficiency of the nonlinear transformation of Langmuir waves into electromagnetic ones at frequencies close to ω_{pe} or $2\omega_{pe}$. A number of processes causing the generation of electromagnetic waves by plasma

turbulence were studied in connection with solar type III radiobursts and kilometric radioemission of the Earth and Jupiter. Among them are: induced scattering of Langmuir waves by ions (Melrose, 1970, 1974; Tsytovich, 1966; Kaplan and Tsytovich, 1972), merging of two Langmuir waves (Tsytovich, 1966; Smith, 1977; Papadopoulos et al., 1974; Smith et al., 1979), coalescence of upper-hybrid waves with low-frequency electrostatic waves (Galeev and Krasnoselskikh, 1978), radiation due to the collapse of Langmuir waves (Galeev and Krasnoselskikh, 1976; Kruchina et al., 1980; Goldman et al., 1980).

Returning to the process considered in our paper, one can argue that the collapse of Langmuir waves does not play any role, since the spectrum of plasma waves driven by hot electrons is broad enough, $(k\lambda_D)^2 \sim v_{Te}^2/v_h^2 \sim \beta_e > W_L/n_0T_e$, and the condition of modulational instability (OTSI) is not valid. The ion-sound waves are absent because the plasma of the Solar corona is considered to be isothermal ($T_e = T_i$). In such a case the generation rate for electromagnetic waves with $\omega \approx \omega_{pe}$ is determined by the induced scattering on ions. If W_T is the energy density of electromagnetic waves and v_{ph} is the phase velocity of Langmuir waves this generation rate is (Tsytovich, 1966; Melrose, 1974):

$$\frac{d}{dt}W_T \approx \omega_{pe} \frac{W_L}{n_0T_e} \cdot \frac{v_{Te}^3}{c^2 v_{ph}} \cdot W_T. \quad (30)$$

To estimate the brightness temperature of the emission one should know the optical depth of the radiative region, which, in turn, is determined by the length at which Langmuir waves exist in the background plasma. This length may be estimated as $l \sim (1/2 \div 1/3)v_h t_{in}$, where t_{in} is the time of electron injection into the given field line, sliding along the surface of the shock wave.

It is easy to understand that the acceleration of electrons along the given field line continues only till the moment when the angle between the shock surface and this line exceeds some critical value $\theta > \theta_{cr} \approx 30^\circ$. Thus, if R is the curvature radius of the shock, the time of the injection may be estimated as $t_{in} \sim \frac{R}{2v_0} \cdot \sin \theta_{cr} < \frac{R}{4v_0}$, which gives $l \sim \frac{R}{10} \cdot \frac{v_h}{v_0}$. So, one can conclude that electrons, accelerated at the shock, excite intense Langmuir oscillations ($W_L/n_0T_e \sim 10^{-4} \div 10^{-5}$) in the wide foreshock region. The size of this region is comparable with the radius of curvature of the shock wave or is determined by the long-scale irregularities of the magnetic field in the solar corona (if their typical size is $l_i \ll R$). Using Eq. (30) and estimating $l \sim 10^{11}$ cm, as $1/10 \div 1/30$ of the shock wave front curvature, one can find that the plasma layer emitting the radiation is optically thick for $W_L/n_0T_e > 10 (c/\omega_{pe}R) \cdot (c^2 v_h/v_{Te}^2) \sim 10^{-5}$. In this case there is an equilibrium between the electromagnetic and Langmuir waves, and the brightness temperature of the radioemission is equal to the effective temperature of Langmuir waves:

$$T_b \sim T_{eff} \sim T_e (n_0 \lambda_D^3) \frac{W_L}{n_0 T_e}. \quad (31)$$

For $n_0 = 10^8$, $T_e = 10^6$ K one obtains $T_b \sim 10^{11}$ K.

We are not going to treat the mechanisms of radioemission of harmonics in details. It should only be pointed out that the optical depth in this case is large too, so the brightness temperature of the harmonic emission appears to be approximately equal to that of the fundamental emission. This conclusion is confirmed, at least for a part of type II bursts, by observations.

The value of the brightness temperature obtained for $W_L/n_0T_e = 10^{-3} \div 10^{-5}$, $T_b \sim 10^9 \div 10^{11}$ K is also in good agreement with

the observational data. For $W_L/n_0T_e < 10^{-5}$ the plasma becomes optically thin, the brightness temperature may stay at the same level $\sim 10^9$ K, but the difference between the rates of generation of fundamental and harmonic emission becomes significant, and their brightness temperatures differ; the fundamental emission should be brighter.

The self-consistent theory of radioemission by shocks moving in the Solar corona developed above allows us to explain in a natural way at least two more features of the type II radio bursts. One is the finite band width $\Delta\omega/\omega_{pe} \sim 10^{-1}$, the second is the relatively high degree of burst polarization at ω_{pe} (Suzuki et al., 1980). The former may be explained by the finite width of the Langmuir wave spectrum: $\Delta\omega \sim (3/2)\omega_{pe}(\Delta k\lambda_D)^2 \sim \omega_{pe} v_{Te}^2/v_h^2 \sim 0.1\omega_{pe}$. In addition, the density irregularities in the emitting volume may also play a significant role. To explain the polarization one should keep in mind that ordinary and extraordinary waves are generated in different ways.

The induced scattering of ions considered above, $l + i \rightarrow t$, is a very effective way to generate ordinary waves, since the frequency of this electromagnetic mode slightly differs from the frequency of Langmuir (upper-hybrid) waves, while the frequency of the extraordinary waves is shifted from that of upper-hybrid waves at a value of the order of ω_{He} . So the process of induced scattering affects mainly the ordinary waves.

The extraordinary waves may be, in principle, generated by the merging of Langmuir waves with the lower branch of electrostatic oscillations. However, as was pointed out by Galeev and Krasnoselskikh (1978), this process occurs only in the quadruple approximation and so it is very slow.

Summarizing these arguments, we conclude that the plasma wave turbulence near the shock front generates mainly ordinary electromagnetic waves with left-hand polarization. Of course, the degree of polarization is changes during the propagation of the wave in the coronal plasma (Zheleznyakov, Zlotnik, 1977). Moreover, the polarization of the second harmonic in the region of generation is connected with the coupling of two Langmuir waves and the difference between intensities of the left- and right-hand polarized waves is of the order of the difference between their refraction indexes, which is of the order of $2\omega_{He}/\omega_{pe} \sim 10\%$.

4. Conclusion

We propose a self-consistent model for radiowave generation by collisionless shocks moving in the plasma of the Solar corona. This model is based on basic principles of the plasma theory of shocks and needs no additional assumptions. The main steps of the generation process are the following:

1. The small fraction of background ions, reflected by the shock, forms the ion beam in the foreshock region.
2. The ion beam drives the low-frequency ($\omega_{Hi} < \omega < \omega_{He}$) waves due to the instability.
3. The low-frequency waves are quickly absorbed by the magnetized electrons. This leads to the formation of non-maxwellian "tails" on the electron distribution function.
4. The hot electrons enter the background plasma along the magnetic field lines and drive the Langmuir oscillations which may reach a rather high level ($W_L/n_0T_e \sim 10^{-5} \div 10^{-4}$).
5. The Langmuir waves are scattered by ions and transformed into ordinary electromagnetic waves of the same frequency $\omega \approx \omega_{pe}$. Since the optical depth of the emitting plasma layer is large, the brightness temperature of the radiation does not depend upon the fine structure of the spectra and appears to be of the order

of 10^{11} K. The radiation at harmonics is due to the merging of two Langmuir waves. For typical parameters of the coronal plasma the brightness temperature of harmonics may reach that observed at ω_{pe} .

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