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Influence of variable permeability on combined free and forced convection about inclined surfaces in porous media

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Abstract—The analysis is carried out for mixed convection about inclined surfaces in a saturated porous media incorporating the variation of permeability and thermal conductivity due to packing of particles. Similarity solutions are obtained, for two cases, namely (i) uniform wall temperature, (ii) linear variation of temperature with distance from the leading edge, for both aiding and opposing flows. It is found that the non-dimensional parameters Gr/Re^2 and σ^2/Re control the flow. The variation of permeability increases heat transfer rate for all values of σ^2/Re. Applications to convective flow in a liquid dominated geothermal reservoir are discussed. Criteria are given for flows which are purely forced, purely free and mixed.

1. INTRODUCTION

In recent years, the requirements of modern technology have stimulated interest in fluid flow studies which involve the interaction of several phenomena. One such study is related to the combined forced and free convection boundary layer flow about inclined surfaces. Sparrow *et al.* [1] treated the above problem by a similar solution approach and later it was extended by Gebhart [2] for arbitrary values of wedge angle and wall temperature distribution.

The convection problem in a porous medium has important applications in geothermal reservoirs and geothermal energy extraction. Cheng and Lau [3] and Cheng and Teckchandani [4] obtained numerical solutions for the convective flow in a porous medium bounded by two isothermal parallel plates in the presence of withdrawal of the fluid. Recently a number of papers [5–8] have appeared on the combined free and forced convection in a porous medium involving horizontal surfaces. Cheng [9] has investigated the free and forced convection about inclined surfaces in a porous medium using the Darcy equation. All the above-mentioned studies treat the permeability and the conductivity or thermal resistance of the medium as constants. However, porosity measurements by Schwartz and Smith [10], Tierney *et al.* [11], and Benenati and Brosilow [12] show that porosity is not constant but varies from the wall to the interior due to which permeability also varies. Chandrasekhara *et al.* [13–15] have incorporated the variable permeability to study the flow past and through a porous medium and have shown that the variation of porosity and permeability have greater influence on velocity distribution and on heat transfer.

The aim of the present investigation is to study the combined forced and free convection about inclined surfaces embedded in porous media, using the generalised momentum equation (after Chandrasekhara and Vortmeyer [16] and Choudhary *et al.* [17]) which accounts for the no-slip at the wall, incorporating the variation of permeability and thermal conductivity.

It is found that similar solutions exist when both wall temperature distribution of the plate and the free stream velocity vary according to the same power function of x, the coordinate in the free stream direction. The non-dimensional parameters which control the flow are Gr/Re^2 and σ^2/Re. Numerical solutions are obtained, for uniform wall temperature and for linear variation of temperature with distance from the leading edge, for both aiding and opposing flows. Variation of permeability and thermal resistance increases the heat transfer rate markedly.

2. ANALYSIS

The physical model consists of a wedge configuration shown in Fig. 1 (a) and (b) where x and y are the Cartesian coordinates along and perpendicular to the direction of free stream velocity. To seek similarity solutions, we impose that the free stream velocity and the wall temperature vary as

$$U_\infty = Ax^m \tag{1}$$

and

$$T_w = T_\infty \pm Bx^n. \tag{2}$$

NOMENCLATURE

A	constant defined in equation (1)
B	constant defined in equation (2)
b	ratio of thermal conductivity of the solid to the liquid, $b = k_s/k_f$
C	specific heat of the convective fluid
c_f	friction coefficient defined in equation (38)
C_1, C_2	constants defined in equations (15) and (17)
d	constant defined in equation (30)
d^*	constant defined in equation (29)
f	dimensionless stream function defined in equation (17)
Gr	local Grashof number, $Gr \equiv \frac{\lvert g_x\rvert \lvert T_w - T_\infty\rvert}{\nu^2}\beta x^3$
g_x	gravitational acceleration in the x direction
h	local heat transfer coefficient
$K(y)$	permeability of the porous medium
K_0	permeability of the porous medium at the edge of the boundary layer
$k(y)$	thermal conductivity of the saturated porous medium
m	constant defined in equation (1)
n	constant defined in equation (2)
Nu	local Nusselt number, $Nu \equiv hx/k_0$
p	pressure
Pr	Prandtl number, $Pr \equiv \nu/\alpha_0$
Q	total surface heat transfer rate defined in equation (44)
q	local heat transfer rate
Re	local Reynolds number, $Re \equiv U_\infty x/\nu$
T	temperature
U_∞	free stream velocity in x direction
U	velocity in x direction
V	velocity in y direction
w	constant defined in equation (15)
x	coordinate along the inclined impermeable surface
y	coordinate perpendicular to the inclined impermeable surface.

Greek symbols

$\alpha(y)$	equivalent thermal diffusivity
α_0	equivalent thermal diffusivity at the edge of the boundary layer
β	coefficient of thermal expansion
δ_T	thermal boundary layer thickness
$\varepsilon(y)$	porosity of the saturated porous medium
ε_0	porosity of the saturated porous medium at the edge of the boundary layer
η	dimensionless similarity variable defined in equation (15)
θ	dimensionless temperature defined in equation (16)
λ	constant defined in equation (17)
μ	viscosity of the convective fluid
ν	kinematic viscosity of the convective fluid
ρ	density of the convective fluid
σ	local porous parameter, $\sigma \equiv x/\sqrt{K_0}$
ψ	stream function.

Subscript

∞	condition at infinity
w	condition at the wall.

Flows are designated as aiding flows for which the buoyancy force has a positive component in the direction of the free stream velocity. Flows for which the buoyancy force has a component opposite to the free stream velocity are designated as opposing flows. For the physical model presented in Fig. 1, the aiding case will require $T_w - T_\infty > 0\,(B > 0)$ and the opposing case will require $T_w - T_\infty < 0\,(B < 0)$. For the mathematical analysis of the problem we assume that

(i) the convective fluid and the porous medium are everywhere in local thermodynamic equilibrium,
(ii) the temperature of the fluid is everywhere below the boiling point,
(iii) permeability and thermal resistance are functions of the vertical coordinate, y,
(iv) Boussinesq's approximation is valid.

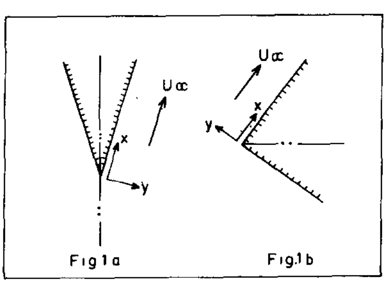

FIG. 1. (a) and (b). Physical model.

Under these assumptions, the boundary layer equations have the form

$$\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = 0 \tag{3}$$

$$0 = -\frac{dp}{dx} - \rho g_x + \mu \frac{\partial^2 U}{\partial y^2} - \frac{\mu U}{K(y)} \tag{4}$$

$$U\frac{\partial T}{\partial x} + V\frac{\partial T}{\partial y} = \alpha(y)\frac{\partial^2 T}{\partial y^2} + \frac{\partial \alpha(y)}{\partial y}\frac{\partial T}{\partial y} \tag{5}$$

and

$$\rho = \rho_\infty[1-\beta(T-T_\infty)] \quad (6)$$

where μ is the dynamical viscosity, $K(y)$ is the permeability of the medium and $\alpha(y) = K(y)/\rho c$ is the thermal diffusivity. The third and fourth terms of equation (4) represent the viscous resistance term and the Darcy resistance term, respectively. The boundary conditions for the problem are

$$\begin{aligned} U = V = 0;\; T = T_w \quad &\text{at} \quad y = 0 \\ U = U_\infty;\quad T = T_\infty \quad &\text{at} \quad y = \infty. \end{aligned} \quad (7)$$

The continuity equation (3) is satisfied by defining the stream function ψ such that

$$\begin{aligned} U &= \frac{\partial \psi}{\partial y} \\ V &= -\frac{\partial \psi}{\partial x}. \end{aligned} \quad (8)$$

The pressure gradient dp/dx appearing in equation (4) is evaluated from the free stream momentum equation neglecting dp/dy,

$$\frac{dp}{dx} + \rho_\infty g_x + \frac{\mu}{K_0} U_\infty = 0 \quad (9)$$

where K_0 is value of $K(y)$ at $y = \infty$. Incorporating equation (9) in (4), we have

$$0 = g_x\beta(T-T_\infty) + \frac{\nu}{K_0}U_\infty - \frac{\nu U}{K(y)} + \nu\frac{\partial^2 U}{\partial y^2}. \quad (10)$$

The momentum (10) and energy (5) equations expressed in terms of ψ become

$$0 = g_x\beta(T-T_\infty) + \frac{\nu}{K_0}U_\infty - \frac{\nu}{K(y)}\frac{\partial \psi}{\partial y} + \nu\frac{\partial^3 \psi}{\partial y^3} \quad (11)$$

$$\frac{\partial \psi}{\partial y}\frac{\partial T}{\partial x} - \frac{\partial \psi}{\partial x}\frac{\partial T}{\partial y} = \alpha(y)\frac{\partial^2 T}{\partial y^2} + \frac{\partial \alpha(y)}{\partial y}\frac{\partial T}{\partial y}. \quad (12)$$

The boundary conditions in terms of ψ are

$$\left.\begin{aligned} \frac{\partial \psi}{\partial y} = -\frac{\partial \psi}{\partial x} = 0 \\ T = T_w \end{aligned}\right\} \text{at } y = 0 \quad (13)$$

$$\frac{\partial \psi}{\partial y} \to U_\infty, \quad T \to T_\infty \quad \text{as} \quad y \to \infty. \quad (14)$$

To seek similarity solutions for equations (11) and (12) we define the following variables, after Sparrow *et al.* [1]

$$\eta = C_1 y x^w \quad (15)$$

$$\theta(\eta) = \frac{T-T_w}{T_w - T_\infty} \quad (16)$$

$$f(\eta) = \frac{\psi}{C_2 x^\lambda} \quad (17)$$

where C_1, C_2, λ and w are constants yet to be determined. Substitution of equations (15)–(17) in equations (11) and (12) leads to

$$0 = g_x\beta\theta(\eta)\beta x^n + \frac{\nu}{K_0}Ax^m - \frac{\nu}{K(\eta)} \times f'C_1C_2x^{\lambda+w} + \nu f'''C_2x^\lambda C_1^3 x^{3w} \quad (18)$$

$$f'n\theta x^{w-\lambda+1} - f\lambda\theta' x^{w-\lambda+1} = \alpha(\eta)\theta''\frac{C_1}{C_2} + \theta'\frac{d\alpha(\eta)}{d\eta}\frac{C_1}{C_2}. \quad (19)$$

Equation (19) will be independent of x provided

$$w - \lambda + 1 = 0, \quad i.e.\ \lambda = w+1. \quad (20)$$

Further we observe from equation (18) that the third and the fourth terms are matched when $\lambda = 1$ and $w = 0$. This, in turn, leads to the relation $n = 1$ and $m = 1$, if the equation (18) is to be independent of x. The above discussion shows that the free stream velocity and the surface temperatures cannot be varied arbitrarily when similar solutions are sought. Using the above relations for the exponents n, m and λ, equations (18) and (19) reduce to

$$0 = g_x\beta\theta(\eta)B + \frac{\nu}{K_0}A - \frac{\nu}{K(\eta)}f'C_1C_2 + \nu f''C_2C_1^3 \quad (21)$$

$$nf'\theta - \lambda f\theta' = \alpha(\eta)\theta'\frac{C_1}{C_2} + \theta'\frac{d\alpha(\eta)}{d\eta}\frac{C_1}{C_2} \quad (22)$$

where the primes represent the differentiation with respect to η. Following Sparrow *et al.* [1], we find that C_1, C_2, η, f, U and V satisfy the following relations:

$$C_1 = \tfrac{1}{2}(A/\nu)^{1/2}, \quad C_2 = (A\nu)^{1/2} \quad (23)$$

$$\eta = \tfrac{1}{2}(y/x)(U_\infty x/\nu)^{1/2} \quad (24)$$

$$f = \psi/(U_\infty \nu x)^{1/2} \quad (25)$$

$$U = \frac{U_\infty}{2}f' \quad (26)$$

$$V = \tfrac{1}{2}(\nu U_\infty/x)^{1/2}[\eta f'(1-m) + f(1+m)]. \quad (27)$$

We also note that for $\lambda = 1$, $n = 0$ and $m = 0$ the constants C_1, C_2 take the values

$$C_1 = \tfrac{1}{2}(A/\nu x)^{1/2}. \quad C_2 = (A\nu/x)^{1/2} \quad (28)$$

leading to the same relations for η, f, U and V as given in equations (23)–(27). Thus for the present problem, the similarity solutions exist when the exponents n and m have values (a) $\lambda = 1, n = 1$ and $m = 1$, (b) $\lambda = 1, n = 0$ and $m = 0$.

Following Chandrasekhara *et al.* [14] we allow for the variation of $K(\eta)$ and $\alpha(\eta)$ in the form

$$K(\eta) = K_0(1 + d^* e^{-\eta}) \quad (29)$$

$$\alpha(\eta) = \alpha_0[\varepsilon_0(1+d\,e^{-\eta}) + b\{1-\varepsilon_0(1+d\,e^{-\eta})\}] \quad (30)$$

where d^* and d are constants, α_0, K_0 and ε_0 are the values of the diffusivity, permeability and porosity, respectively at the edge of the boundary layer, b is the ratio of the thermal conductivity of the solid to the conductivity of the fluid. Using the above relations, the governing equations (21) and (22) finally take the form

$$f'''-4f'\frac{\sigma^2}{Re}\frac{1}{(1+d\,e^{-\eta})}\pm 8\frac{Gr}{Re^2}\theta+\frac{8\sigma^2}{Re}+8m=0 \quad (31)$$

$$2Pr[mf'\theta-f\theta'] = [\varepsilon_0(1+d\,e^{-\eta})+b\{1-\varepsilon_0(1+d\,e^{-\eta})\}]\theta'' + [b\varepsilon_0 d\,e^{-\eta}-\varepsilon_0 d\,e^{-\eta}]\theta' \quad (32)$$

where

$$Gr=\frac{|g_x|\,|T_w-T_\infty|}{\nu^2}\beta x^3,$$

local Grashof number; $Re = U_\infty x/\nu$, local Reynolds number; $\sigma^2 = x^2/K_0$, local porous parameter; $Pr = \nu/\alpha_0$, Prandtl number. The plus and minus signs in the third term of equation (31) correspond to aiding and opposing flows, respectively since the Grashof number as defined in equation (31) is always positive.

The boundary conditions (13) and (14), with the help of equation (26), become

$$f(0)=f'(0)=0;\quad \theta(0)=1$$
$$f'\to 2,\ \theta\to 0 \quad \text{as} \quad \eta\to\infty. \quad (33)$$

As we see from equations (31) and (32), the problem involves four parameters, i.e. Gr/Re^2, σ^2/Re, Pr and exponent m. The magnitude of Gr/Re^2 gives the relative importance of forced and free convection in determining the combined flow. For small values of this parameter, forced convection will dominate, while for large values, free convection becomes important. The ratio σ^2/Re tells us about the relative importance of viscous and Darcy resistance terms on the combined flow. It is interesting to note that the inclusion of the viscous resistance term in the governing equations leads to Gr/Re^2 as the controlling parameter of the combined flow while in Cheng's [9] Darcy flow, Gr/Re is the controlling parameter. Equation (31), in the absence of the viscous resistance term and for constant permeability, reduces to the similar form given by Cheng [9] for f' since Gr/σ^2 in the present case corresponds to Gr in Cheng's case and, further, for $\sigma^2/Re = 1$, $Pr = 7$ and $b = 2$, the values of Gr/Re in Cheng's [9] case and Gr/Re^2 in the present case correspond to each other.

3. RESULTS AND DISCUSSION

Equations (31) and (32) with boundary conditions (33) are integrated numerically by Runge–Kutta Predictor–Corrector scheme with systematic guessing of the derivatives $f''(0)$ and $\theta'(0)$ using the automatic initial value technique [14]. The above method is accurate and fast converging compared to the shooting technique.

The solutions are obtained, for different Prandtl numbers and for $m = 0$ and $m = 1$, i.e. for uniform wall temperature and for linear variation of temperature with distance from the leading edge, for both aiding and opposing flows. For the purpose of numerical integration we have assumed $d^* = 3$, $d = 1.5$ and $\varepsilon_0 = 0.4$ after Chandrasekhara *et al.* [14]. The temperature and velocity profiles are presented in Figs. 2 and 3 for a wide range of the parameter Gr/Re^2 and for particular values of σ^2/Re. The following discussion is confined to uniform wall temperature since a similar trend is observed for the case $m = 1$. It is seen from Figs. 2 and 2(a) that the velocity profiles satisfy the no-slip condition and are different from the velocity profiles obtained by Cheng. As Gr/Re^2 is increased, the velocity profiles become sharply peaked near the wall indicating the influence of free convection in modifying the flow field from pure forced convection flow to pure free convection flow. The temperature profiles are

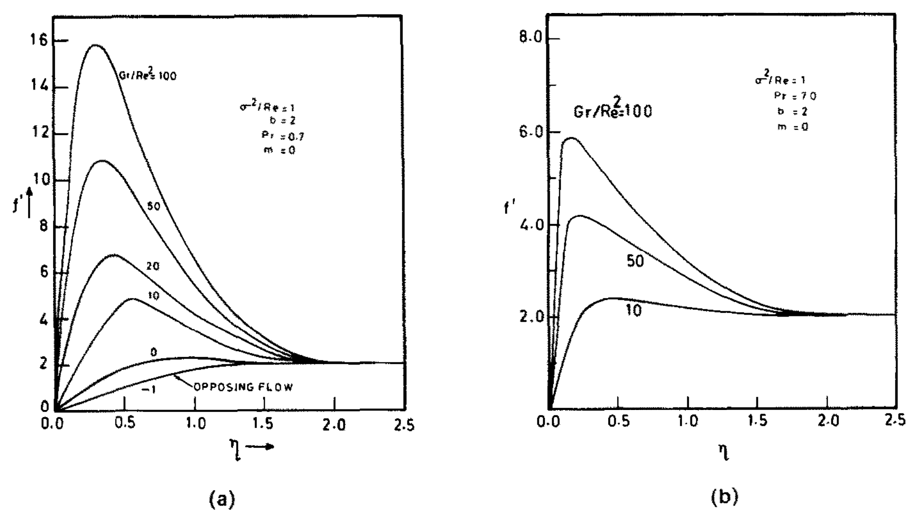

FIG. 2(a) and (b). Values of f' vs η.

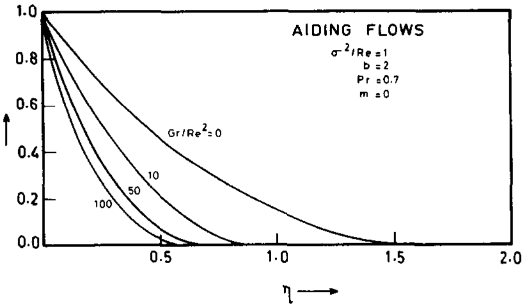

FIG. 3. Values of θ vs. η.

presented in Fig. 3. The profiles display a simple shape which is found in thermal boundary layers of the pure forced and pure free convection flows.

The results which are of greatest practical interest in geothermal application are heat transfer rate, shear stress and boundary layer thickness. We first consider the heat transfer rate. The expression for the local surface heat flux is given by

$$q = -k\left[\frac{\partial T}{\partial y}\right]_{y=0}$$

$$= k\frac{(T_w - T_\infty)}{2x}Re^{1/2}\theta'(0). \quad (34)$$

Using the definition of heat transfer coefficient and Nusselt number

$$h = \frac{q}{T_w - T_\infty}, \quad Nu = \frac{hx}{k}. \quad (35)$$

We obtain the expression for the dimensionless heat transfer in the form

$$\frac{Nu}{Re^{1/2}} = -\tfrac{1}{2}\theta'(0). \quad (36)$$

The wall's shear stress and friction coefficients are defined as

$$\tau_w = \mu\left[\frac{\partial U}{\partial y}\right]_{y=0}$$

$$C_f = \tau_w/[\rho U_\infty^2/2]. \quad (37)$$

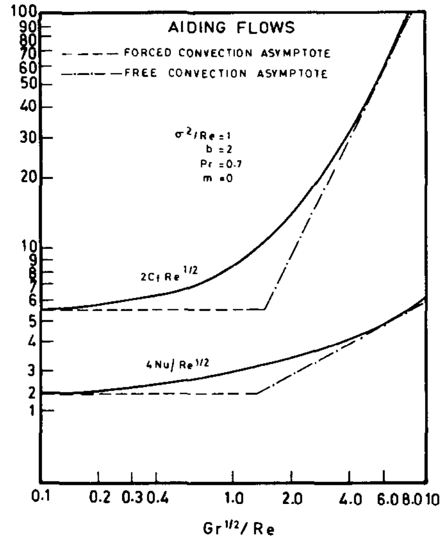

FIG. 4. Heat transfer and friction factor results, (aiding flows).

The dimensionless shear stress, from equation (37), is written as

$$c_f Re^{1/2} = \tfrac{1}{2}f''(0). \quad (38)$$

The numerical values of $f''(0)$ and $\theta'(0)$ corresponding to different values of Gr/Re^2 and σ^2/Re are given in Tables 1 and 2. We observe that $\theta'(0)$ has higher values for a given value of σ^2/Re compared to Cheng's [9] case of constant permeability. In Fig. 4 we have plotted both dimensionless heat transfer and shear stress as functions of $Gr^{1/2}/Re$ for aiding flows together with the asymptotes representing pure forced convection and pure free convection.

The expressions for these asymptotes are

$$\left.\begin{aligned}\frac{4Nu}{Re^{1/2}} &= 2.448\\ 2c_f Re^{1/2} &= 5.4\end{aligned}\right\}\text{ forced convection} \quad (39)$$

$$\left.\begin{aligned}\frac{4Nu}{Re^{1/2}} &= 1.98\,(Gr^{1/2}/Re)^{1/2}\\ 2c_f Re^{1/2} &= 4.335(Gr^{1/2}/Re)^{1/2}\end{aligned}\right\}\text{ free convection.} \quad (40)$$

Table 1. Values of $f''(0)$ and $[-\theta'(0)]$ for aiding and opposing flows, $b = 2$ $Pr = 0.7$

	$\sigma^2/Re = 1, m = 0$ (aiding flow)				$\sigma^2/Re = 100, m = 1$ (aiding flow)			$\sigma^2/Re = 1, m = 1$ (aiding flow)			$\sigma^2/Re = 1, m = 0$ (opposing flow)	
Gr/Re^2	$f''(0)$	$[-\theta'(0)]$	η_T	Gr/Re^2	$f''(0)$	$[-\theta'(0)]$	Gr/Re^2	$f''(0)$	$[-\theta'(0)]$	Gr/Re^2	$f''(0)$	$[-\theta'(0)]$
100	137.49	3.145	0.5	100	128.07	3.731	100	123.02	4.179	0	5.40	1.225
50	81.06	2.677	0.62	50	99.99	4.337	50	74.09	3.573	0.16	5.39	1.224
20	40.29	2.210	0.75	20	84.97	4.802	20	39.21	2.975	0.25	5.12	1.183
10	24.79	1.949	0.85	10	80.91	4.316	10	25.42	2.678	0.36	4.77	1.148
4	14.83	1.659	1.10	0	75.98	2.200	2	12.56	2.598			
1	8.19	1.512	1.15				0	7.45	1.173	0.50	3.92	0.825
0.64	7.34	1.500	1.16									
0.36	6.71	1.470	1.17									
0.25	6.47	1.441	1.18									
0	5.40	1.225	1.31									

Table 2. Values of $f''(0)$ and $[-\theta'(0)]$ for aiding flows, $b = 2$

Gr/Re^2	$\sigma^2/Re = 1, m = 0, Pr = 7.0$ $f''(0)$	$[-\theta'(0)]$	Gr/Re^2	$\sigma^2/Re = 1, m = 0, Pr = 3.5$ $f''(0)$	$[-\theta'(0)]$
100	82.64	5.647	100	96.00	4.732
50	49.466	4.825	50	57.58	4.037
20	25.82	4.00	20	29.78	3.334
10	16.43	3.565	10	18.674	2.960
4	9.83	3.255	4	10.899	2.685
0	5.701	2.650	0	5.862	2.186

The heat transfer prediction based on asymptotes would be in maximum error of around 25% which occurs near $Gr^{1/2}/Re = 1.35$.

We can also establish the criteria for pure mixed convection applying the 5% deviation rule suggested by Sparrow *et al.* [1], leading to the following subdivisions:

$$\begin{aligned} &0 < Gr/Re^2 < 0.18 && \text{forced convection} \\ &0.18 < Gr/Re^2 < 25 && \text{mixed flow} \\ &25 < Gr/Re^2 && \text{free convection.} \end{aligned} \qquad (41)$$

The results for the opposing flow situation are represented in Fig. 5 for isothermal boundary conditions which show that at small values of $Gr^{1/2}/Re$, the curve approaches the forced convection asymptotes. The friction factor is very much reduced as $Gr^{1/2}/Re$ is increased and finally leads to flow separation at still higher values of $Gr^{1/2}/Re$.

We now consider an expression for the thermal boundary layer thickness. If η_T is the value of η at the edge of the boundary layer, i.e. the value of $\theta(\eta)$ has a value 0.01, we have

$$\frac{\delta T}{x} = \frac{2\eta_T}{Re^{1/2}}. \qquad (42)$$

The values of η_T for $m = 0$ and for selected values of Gr/Re^2 are presented in Table 1. The variation of $Re^{1/2}$ $\delta T/x$ with respect to $Gr^{1/2}/Re$ is shown in Fig. 6 for aiding flows in which the expression for the free convection asymptote is given by

$$\frac{\delta T}{x} Re^{1/2} = \frac{10.0}{(Gr/Re^{1/2})^{1/2}}, \quad m = 0. \qquad (43)$$

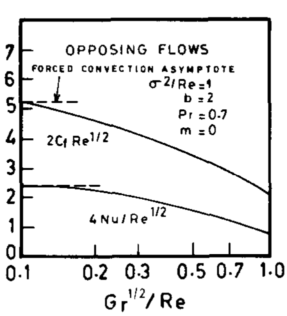

FIG. 5. Heat transfer and friction factor results, (opposing flows).

The above discussions are confined to the case $Pr = 0.7$. A similar kind of behaviour is also observed for other values of Pr. The values of $f''(0)$ and $\theta'(0)$ for $Pr = 7.0$ and 3.5 are shown in Table 2.

In geothermal applications, the quantity of interest is the total surface heat transfer rate, Q, which for a flat plate with length L and width S can be computed from the relation

$$Q = S \int_0^L q(x)\,dx \qquad (44a)$$

$$Q = \frac{SkB}{3m+1}[A/v]^{1/2}[-\theta'(0)]L^{(3m+1)/2} \qquad (44b)$$

In order to have an idea of order of magnitude of various physical quantities involved in geothermal application, we consider a heated isothermal impermeable surface ($m = 0$) 1 km × 1 km embedded in an aquifer where a pressure gradient exists. If the temperature of the impermeable surface and the aquifer are at 215 and 15°C, respectively, then we find that the value of Q is 130 MW for the following values of the physical quantities: $\beta = 1.8 \times 10^{-4}$°C, $\rho_f = 10^6$ g cm^{-3}, $K = 10^{-12}$ m^2, $\mu = 0.27$ g s^{-1} m^{-1}, $k = 0.58$ cal s^{-1} °C^{-1} m^{-1} and $U_\infty = 1$ cm h^{-1}. Whereas, in the case of Cheng [9] it is 45 MW.

4. CONCLUSIONS

The problem of mixed convection about inclined surfaces in a saturated porus media incorporating the variation of permeability and thermal conductivity is analysed by a similarity solution approach. The analysis is based on the boundary layer approxim-

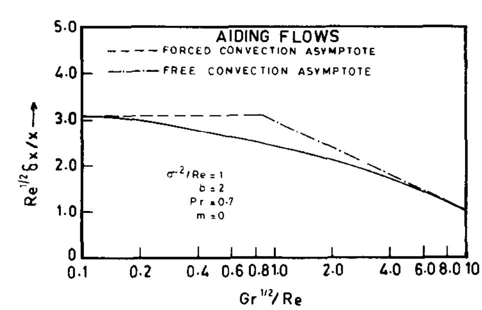

FIG. 6. Dimensionless boundary layer thickness, vs. $Gr^{1/2}/Re$.

ations and neglecting the component of buoyancy force normal to the inclined surface. The problem admits similarity solutions for $m = 0$ and $m = 1$. The variation of permeability and, in turn, the conductivity of the medium brings about an increase in the heat transfer rate which is large for large values of σ^2/Re. The velocity profiles exhibit skewing near the boundaries due to the Brinkman resistance term and are different from those obtained by Cheng [9] for pure Darcy flow.

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INFLUENCE DE LA PERMEABILITE VARIABLE SUR LA CONVECTION MIXTE AUTOUR DES SURFACES INCLINEES DANS DES MILIEUX POREUX

Résumé—L'analyse concerne la convection mixte autour des surfaces inclinées dans des milieux poreux en incorporant la variation de perméabilité et de conductivité thermique due à l'entassement des particules. Des solutions similaires sont obtenues pour deux cas, à savoir (i) température uniforme de paroi, (ii) variation linéaire de température avec la distance au bord d'attaque, à la fois pour des écoulements favorables et opposés. On trouve que les paramètres adimensionnels Gr/Re^2 et σ^2/Re contrôlent l'écoulement. La variation de perméabilité augmente le transfert thermique pour toutes les valeurs de σ^2/Re. Des applications à l'écoulement convectif dans une réservoir géothermique saturé de liquide sont discutées. Des critères sont donnés pour des écoulements qui sont purement forcés, purement naturels et mixtes.

EINFLUSS DER VERÄNDERLICHEN PERMEABILITÄT AUF DIE MISCHKONVEKTION ÜBER GENEIGTEN OBERFLÄCHEN IN PORÖSEN MEDIEN

Zusammenfassung—Die Untersuchung wird für Mischkonvektion an geneigten Oberflächen in einem gesättigten porösen Medium durchgeführt, wobei die Permeabilität und die Wärmeleitfähigkeit in Abhängigkeit der Packungsart variiert wurden. Für zwei Fälle wurden Ähnlichkeitslösungen ermittelt: (i) einheitliche Wandtemperatur, (ii) lineare Änderung der Temperatur mit der Entfernung von der Anströmkante, und zwar für aufwärts- und abwärts-gerichtete Strömungen. Es wurde herausgefunden, daß die dimensionslosen Parameter Gr/Re^2 und σ^2/Re die Strömung kennzeichnen. Eine Änderung der Permeabilität bewirkt eine Zunahme des Wärmetransportvermögens für alle Werte von σ^2/Re. Anwendungen für eine konvektive Strömung in einem geothermischen Sammelbecken mit überwiegendem Flüssigkeitsanteil werden diskutiert. Kriterien werden für Strömungen aufgestellt, welche vollständig erzwungen, vollständig natürlich und gemischt sind.

ВЛИЯНИЕ ИЗМЕНЕНИЯ ПРОНИЦАЕМОСТИ НА СМЕШАННУЮ СВОБОДНУЮ И ВЫНУЖДЕННУЮ КОНВЕКЦИЮ У НАКЛОННЫХ ПОВЕРХНОСТЕЙ, ПОМЕЩЕННЫХ В ПОРИСТЫЕ СРЕДЫ

Аннотация—Проведен анализ смешанной конвекции у наклонных помещенных в пористую среду поверхностей при изменении проницаемости и теплопроводности среды из-за различной плотности упаковки частиц. Получены автомодельные решения для двух случаев: (i) одномерной температуры стенки и (ii) линейного изменения температуры с расстоянием от передней кромки как при спутных, так и противоточных течениях. Найдено, что параметрами, определяющими течение, являются безразмерные критерии Gr/Re^2 и σ^2/Re. При изменении проницаемости среды отмечается рост интенсивности теплопереноса при всех значениях параметра σ^2/Re. Результаты анализа использованы для описания конвективного течения в геотермальном резервуаре, в основном заполненном жидкостью. Приведены критерии для описания чисто вынужденных, чисто свободных и смешанных течений.