THE DYNAMICAL EVOLUTION OF A STAR CLUSTER

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ABSTRACT. Using the second order tensor virial equations and the equation for the rate of change of the kinetic energy tensor, we follow the dynamical evolution of a star cluster with anisotropic velocity distribution. We show that the cluster executes finite amplitude oscillations both in size and eccentricity. However, unlike in the isotropic case, the amplitude now depends on the initial eccentricity.

1. THE BASIC EQUATIONS

$$\frac{1}{2}\frac{d^2 I_{ii}}{dt^2} = 2K_{ii} + W_{ii}, \tag{1}$$

$$\frac{d}{dt}K_{ii} = \frac{1}{2}\frac{W_{ii}}{I_{ii}}\frac{dI_{ii}}{dt}. \tag{2}$$

Here various symbols have their usual meaning, and summation convention is not used. It can be seen that

$$M_{ii} I_{ii} = const. \tag{3}$$

See Chandrasekhar & Elbert (1972) and Som Sunder & Kochhar (1985, 1986=Paper 1, Paper 2) for details. The total energy $E = K + W$ is conserved.

We consider homogeneous spheroidal systems with axes $a_1 = a_2$, and a_3. Then

$$I_{ii} = \frac{1}{5}Ma_i^2; \qquad W_{ii} = -\frac{3}{10}\frac{GM}{a_1^2 a_3}a_i^2 A_i. \tag{4}$$

Here A_i are the index symbols (Chandrasekhar & Elbert 1972). The gravitational energy of a spheroid is

$$W = -\frac{3}{5}\frac{GM^2}{a_1}S(y), \tag{5}$$

where $S(y)$ is a function of the eccentricity: $y = e^2$ for $y > 0$ (oblate spheroids). For prolate spheroids $S(y) = 1$. Equations (1) and (2) now reduce to

$$\frac{d^2 a_1}{dt^2} = \frac{2\left[Q + S(y_0)\right]}{a_1^3} - \frac{3}{2}\frac{A_1}{a_1^2} \tag{6}$$

T. de Zeeuw (ed.), Structure and Dynamics of Elliptical Galaxies, 525–526.

$$\frac{d^2 a_3}{dt^2} = \frac{2\,[Q + S(y_0)]}{a_3^2}(1 - y) - \frac{3}{2}\frac{A_1}{a_1^2} \tag{7}$$

where $Q = E/|W_0|$. Here subscript 0 refers to values at $t = 0$; a_1, a_3 are in the units of $a_0 (= a_1$ at time $t)$; t is in the units of $\sqrt{(a_0^3/GM)}$. Special cases of equations (6)–(7) have been considered by various authors (see Paper 2).

2. RESULTS AND DISCUSSION

We have numerically solved equations (6) and (7) subject to the conditions $da_1/dt = da_3 dt = 0$ at $t = 0$, and for various values of the initial eccentricity y_0, and Q. Note that equilibrium corresponds to $K = 0.5|W|$, or, for a sphere, to $Q = -0.5$.

Figure 1 shows the behavior of the a_1-axis and y for $Q = 0.75$. $Q = -0.75$ corresponds to $y_0 = 0$ to $K = 0.25|W|$ so that the kinetic energy is less than (or gravitational energy is more than) the equilibrium value. Consequently the a_1 axis contracts. This contraction is accompanied by an increase in the pressure, which causes the system to expand once again. These oscillations have a period $P \approx \pi/[2(-Q)^{\frac{3}{2}}] \approx 3.5$. The amplitude of the oscillations increases with decreasing y_0. The eccentricity also decreases initially. At about 0.5 P however there is a steep rise and a fall, after which the system slowly returns to $y \approx y_0$. The system however does not in general return to the exact initial conditions after a cycle, except for the spherical case. Other cases are discussed in Paper 2.

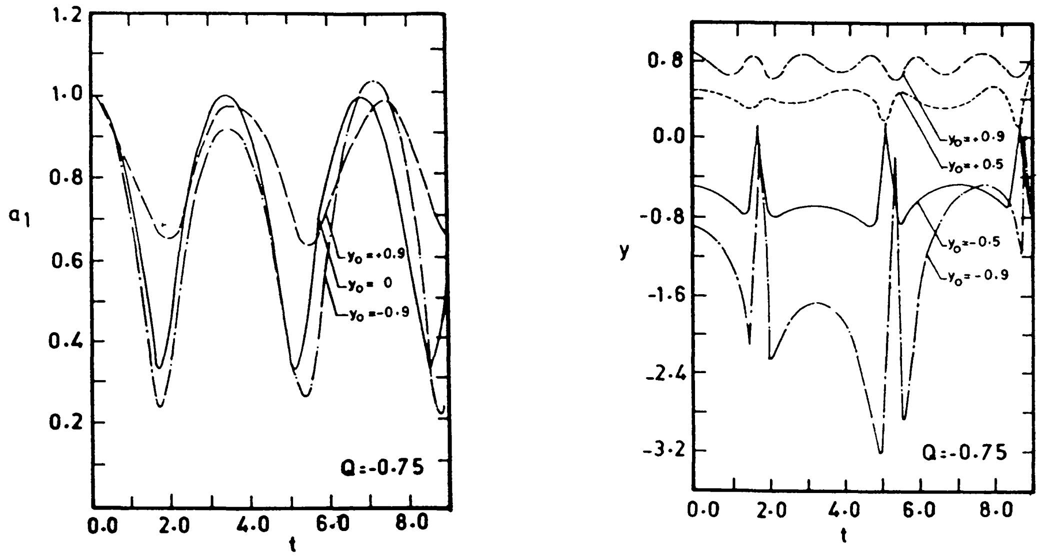

Figure 1. a) Behavior of the a_1-axis with time for $Q = -0.75$ and for various values of initial eccentricities y_0. b) Behavior of the eccentricity with time for $Q = -0.75$. The ordinate y measures the eccentricity; it is positive (negative) for oblate (prolate) spheroids.

REFERENCES

Chandrasekhar, S. and Elbert, D.D., 1972. *M.N.R.A.S.*, **155**, 435.
Som Sunder, G. and Kochhar, R.K., 1985. *M.N.R.A.S.*, **213**, 381 (Paper 1).
Som Sunder, G. and Kochhar, R.K., 1986. *M.N.R.A.S.*, **221**, 553 (Paper 2).