Inhibition of convective collapse of solar magnetic flux tubes by radiative diffusion

P. Venkatakrishnan

Indian Institute of Astrophysics, Bangalore 560034, India

Interaction of convection with a magnetic field leads to an intermittent distribution of magnetic flux[1]. Such a process operating on the solar surface can lead to 'equipartition' fields of 700 G (ref. 2). These fields are further prone to a convective instability and eventually collapse to kilogauss intensity[3-5]. I show here that radiative diffusion can inhibit this collapse to a varying degree, depending on the field strength and the thickness of the flux elements. As a consequence, one would expect the field strength of the photospheric magnetic flux elements to depend on their sizes. It is shown that at one end of such a distribution there would be kilogauss tubes with small dispersion in field strength and large dispersion in size. At the other extreme of the spectrum would be thin tubes of fairly constant size but with a wide range in field strength, from kilogauss intensities to the equipartition values of 700 G. High-resolution observations from space-borne telescopes should reveal the existence of the latter variety of tubes.

An approximate but effective method of examining the stability of magnetic equilibria resembling solar flux tubes is to use the slender-flux-tube equations[6]. The linearized equations for slender, radiating, optically thin flux tubes have been considered by Webb and Roberts[7] in the context of radiative damping of tube waves. One can use the same equations for optically thick tubes by re-defining the radiative relaxation time τ_R of ref. 7 as $\tau_R = 4\chi/r_0^2$ where χ is the radiative diffusivity and r_0 is the cross-sectional radius of the tube[8]. For mathematical simplicity and to focus attention on the physics of the problem, we assume isothermal stratification in the tube. The imposition of boundary conditions of vanishing velocity perturbation at the two ends of the tube on equation (12) of ref. 7 (after correcting a misprint in that equation by replacing Λ^2 with Λ_0) leads to the following dispersion relation:

$$\tilde{\sigma}^3 + a_2\tilde{\sigma}^2 + a_1\tilde{\sigma} + a_0 = 0 \quad (1)$$

where $a_0 = \varepsilon b_0/\delta$, $a_1 = -\{(1-\gamma)(1+\beta_0)/2 - \gamma b_0\}/\delta$, $a_2 = \varepsilon(1+\beta_0/2)/\delta$, $\delta = (1+\gamma\beta_0/2)$, $b_0 = 1/16 + \pi^2 n^2 \Lambda_0^2/d^2$, $\varepsilon = \tau_D/\tau_R$, $\tilde{\sigma} = \sigma\tau_D$ and $\tau_D = (\Lambda_0/g)^{1/2}$. Here, σ is the complex frequency of the perturbation, assumed to grow in time as $e^{\sigma t}$, β_0 is the ratio of gas pressure to magnetic pressure in the tube, n is the harmonic of the perturbation, d is the length of the tube, Λ_0 is the isothermal scale height of the atmosphere and g is the acceleration due to gravity. To simulate a superadiabatic temperature gradient (necessary for convective collapse) one must assume $\gamma < 1$. Although this is strictly unphysical, it may be used for purely illustrative purposes. Note that choosing a non-isothermal stratification leads to a quartic equation in $\tilde{\sigma}$ (ref. 8). (An algebraic error in an earlier analysis for isothermal stratification[9] resulted in a quartic rather than a cubic equation.)

Inspection of the discriminant of equation (1) shows that increase of ε (the ratio of the dynamical to the radiative timescale) leads to a decrease of the growth rate for a tube which is convectively unstable at $\varepsilon = 0$. This decrease continues until $\tilde{\sigma}$ acquires an imaginary part, when the discriminant vanishes. It can also be seen, by examining the terms of leading power in ε (for $\varepsilon > 1$) in the expression for the real part of $\tilde{\sigma}$, that the growth rate tends asymptotically to a constant value.

What are the implications of the above results for solar flux tubes? A straightforward inference is the inhibition by radiation of convective collapse of the tubes. Increase of ε implies decrease of r_0 for a given diffusivity χ. Thus, thinner tubes will collapse to a lesser extent than thicker ones. We shall now examine this phenomenon in a more quantitative manner, without forgetting the simplifications inherent in the assumption of isothermal stratification. We choose Λ_0 corresponding to 10^4 K (representing the region of largest convective instability due to hydrogen ionization), $\gamma = 0.7$ (to yield a very high superadiabaticity of ~0.4) and $d = 1{,}200$ km, assuring a stable tube at $\beta_0 \simeq 2.0$ for $\varepsilon = 0$. With this choice of parameters, Fig. 1 shows the real part of $\tilde{\sigma}$ as a function of ε for different values of β_0. We assign a value $\beta_0 = 7.0$ for the equipartition field of 700 G, so that $\beta_0 = 2.0$ corresponds to kilogauss intensity. Notice the curious cusps in the curves, which correspond to the transition from purely growing modes to overstable modes.

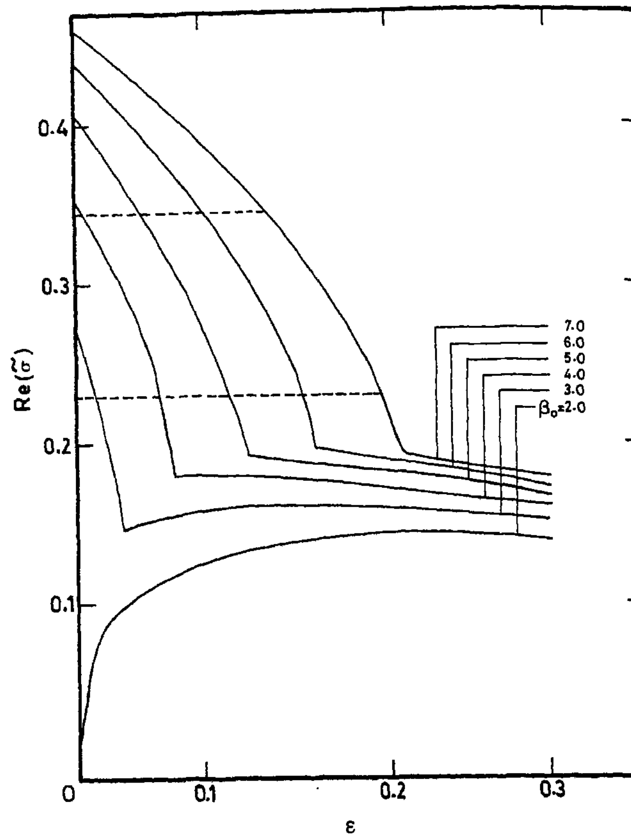

Fig. 1 Plot of Re ($\tilde{\sigma}$) against ε for different values of β_0. Dashed lines at Re($\tilde{\sigma}$) = 0.345 and 0.230 correspond to flux-tube lifetimes of 10 and 15 min, respectively.

In the Sun, the flux tubes are presumably jostled around, with frequent rearrangement of the field lines into different concentrations[10,11]. Thus, a given tube probably exists as a separate identity for a finite lifetime, dictated externally by its environment. Tubes with growth times larger than such externally dictated lifetimes can therefore be considered as 'stable'. In Fig. 1, two horizontal lines (dashed) are drawn at Re($\tilde{\sigma}$) = 0.345 and 0.230. These correspond to stable tubes with lifetimes of 10 and 15 min, respectively.

The 10-min timescale is representative of the normal granulation which rearranges the field lines[10], whereas the 15-min timescale represents the facular granules[12]. From the intersection of these horizontal lines with the curves for different values of β_0 (Fig. 1), one can construct a plot of ε versus β_0, showing the demarcation of stable and unstable regimes in ε–β_0 space. By choosing $\chi = 10^{10}$ cm^2 s^{-1} (consistent with the solar value at 10^4 K)[13] and assuming magnetic flux conservation as well as depth-independent β_0 for the tube, one can write the photospheric radius of the tube as $r_{ph} = (p_{ion}/p_{ph})^{1/4} \cdot (4\tau_D/\varepsilon)^{1/2}$ km, where p_{ion}/p_{ph} is the ratio of the pressure at 10^4 K to photo-

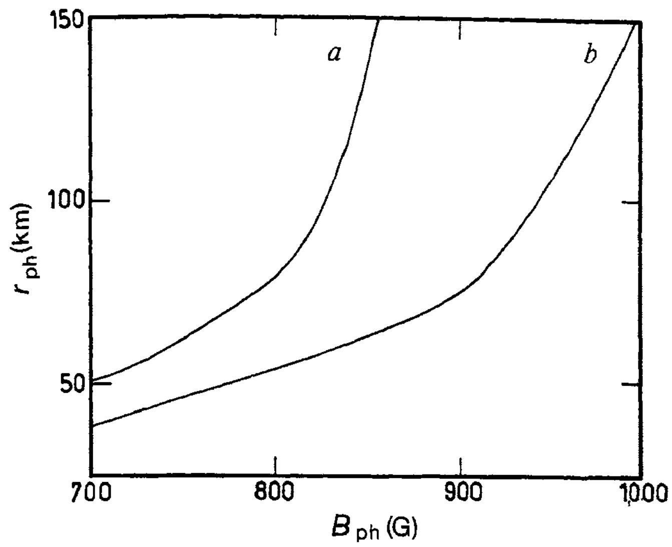

Fig. 2 Predicted photospheric cross-sectional radius of magnetic flux tubes as a function of their field strength, for stable tubes of lifetime 10 (*a*) and 15 (*b*) min.

spheric pressure and the dynamical timescale τ_D is expressed in seconds. Similarly, the surface magnetic field can be written as $B_{ph} = 700\sqrt{8/(1+\beta_0)}$ G, remembering our assignment of $\beta_0 = 7.0$ to a 700-G tube. Figure 2 shows the resulting plot of r_{ph} versus B_{ph} for the two values of lifetime.

We conclude that stable tubes on the Sun would show a typical distribution of field strength versus size (Fig. 2). At one end of the spectrum, there would be intense kilogauss tubes with small dispersion in strength but large dispersion in size. At the other extreme of the distribution, there would be a group of thin tubes of fairly constant size but with a continuous distribution of field strengths ranging from kilogauss levels down to equipartition values of 700 G. The tubes observed until now from ground-based telescopes[14] have perhaps belonged only to the stronger variety; observations from the new space-based telescopes should reveal the existence of the tubes at the thin end of the spectrum. A further prediction which can be made from Fig. 2 is that longer-lived tubes of a given size are magnetically more intense.

We emphasize the simple isothermal nature of the model which led to the above predictions: for a more precise prediction of the size distribution, the flux-tube model must be improved. A stability analysis[15] of such an improved model has shown a similar influence of the radiative diffusion on the convective collapse, although the instability was not examined in the ϵ–β_0 plane as we have done here. Finally, it must be remembered that mechanisms other than convective interaction might create strong tubes below the photosphere, which could emerge above the surface by buoyancy. In this case, the curves in Fig. 2 should be taken as the stable limit of the weakest field for a given size, or conversely the limit of the largest tube of a given strength. Thus, curves like these can be used to distinguish between tubes concentrated by convection and those formed otherwise.

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