# TORSION AND THE COSMOLOGICAL CONSTANT PROBLEM

VENZO DE SABBATA and C. SIVARAM

*World Laboratory, Lausanne, Switzerland; Dipartimento di Fisica, Università di Ferrara, Italy, and Indian Institute of Astrophysics, Bangalore, India*

(Received 27 July, 1989)

**Abstract.** It is shown that the recently suggested energy-dependent torsion coupling constant can make the spin contributions of matter sources large enough to cancel the cosmological constant term at all stages in the early universe from the Planck epoch.

The so-called problem of the cosmological constant has been receiving considerable attention in recent years. A good current review of the situation concerning the problem is due to Weinberg (1989). Basically the problem is to explain why the effective cosmological constant is so small if not zero at the present epoch vastly less than the values one would expect from elementary particle physics theories. Anything that contributes to the energy density of the vacuum acts just like a cosmological constant the effective vacuum energy density being of the form

$$\rho_v = \Lambda_{\text{eff}} c^4/8\pi G \, .$$

There would have been large changes in the vacuum energy in the early universe as a result of phase transitions due to the breaking of some symmetry group as the expanding universe cooled. If the symmetry breaking takes place at some energy $M$, then the induced vacuum energy density is $\sim M^4$. For instance at the Planck epoch, $t \approx 10^{-43}$ s, $M \approx 10^{19}$ GeV, one would expect a large vacuum energy density term $\approx 10^{114}$ ergs cm$^{-3}$, corresponding to an effective cosmological constant $\Lambda_{\text{Pl}} \simeq 10^{66}$ cm$^{-2}$. This could arise as a result of scale invariance breaking and quantum gravitational contributions to the vacuum energy (Sivaram, 1986a, b, c, 1985) in the very early universe at the Planck epoch. Again the GUTS phase transition at energies $\approx 10^{15}$ GeV, would similarly induce another large $\Lambda$ of $\Lambda_{\text{GUTS}} \approx 10^{50}$ cm$^{-2}$. There would also be other large contributions from other symmetry breaking phase transitions at the electroweak scale for instance at somewhat later epochs in the early universe.

The question is what has happened to all these large contributions to the $\Lambda$-term. Why is the present value of $\Lambda^{50}$ vanishingly small?

Weinberg summarizes five different approaches undertaken in recent years to understand this question. He consider supersymmetry (exact global supersymmetry would indeed make the vacuum energy and, hence, $\Lambda$ vanish). But we know that supersymmetry must be broken quite strongly and this would give a large contribution to $\Lambda$ which would not vanish. There is no symmetry principle known (like gauge invariance in electromagnetism implying zero-photon mass) which would make $\Lambda$ vanish exactly and Weinberg states that it is very hard to see how any property of

*Astrophysics and Space Science* **165**: 51–55, 1990.

supergravity or superstring theory could make the effective cosmological constant sufficiently small. He also resorts to the anthropic principle to explain why $\Lambda$ is so small but this is a rather weak argument (see also de Sabbata, 1983, 1984). Again most attempts have involved some sort of adjustment mechanism (e.g., Dolgov, 1982; Wilzcek, 1985) which requires some extra scalar field, which evolves and acts as a counterterm to cancel the cosmological term. However, it turns out that in all such attempts the scalar field must have some very special ad hoc properties and involves a lot of 'fine-tuning' at all stages, apart from there being no evidence of such extra fields. Other attempts have dealt with changing the structure of Einstein's equations but this also leads to several consistency problems. Recently there has been a lot of excitement about a new mechanism suggested by Coleman (1988) which follows up an earlier work of Hawking which described how in quantum cosmology there could arise a distribution of values for the effective cosmological constant with an enormous peak at $\Lambda_{\rm eff} = 0$. Coleman considers the effect of topological fixtures known as wormholes, consisting of two asymptotically flat spaces joined together at a 3-surface, and shows that the probability distribution or expectation values has an infinite peak at $\Lambda_{\rm eff} \to 0$. However, several objections have been raised, including the reality of wormhole existence, the use of Euclidean quantum cosmology (it is essential that the path integral be given by a stationary point of the Euclideanized action) which may have nothing to do with the real world. Moreover, if the path integral has a phase, that might eliminate the peak in the probability distribution at zero cosmological constant. In short there are too many controversies with Coleman's very speculative proposal.

One promising possibility which has not been considered so far in understanding the cosmological constant problem is the use of torsion in a framework such as the Einstein–Cartan (E–C) theory which is natural in considering the gravitational contributions of particles with spin which is indeed a universal property of elementary particles. In fact at sufficiently early epochs the energy content of the Universe can indeed be spin dominated and the temporal evolution of the spin-density tensor is important in describing the cosmological dynamics (Trautman, 1973; de Sabbata 1988a, b).

In the Einstein–Cartan theory, the Lagrangian is the usual scalar curvature (de Sabbata, 1985)

$$L_{\rm E-C} = (-g)^{1/2} R(\Gamma)\,, \tag{1}$$

where $\Gamma$ is non-symmetric affine connection

$$\Gamma_{\alpha\beta}{}^{\mu} = \begin{Bmatrix} \mu \\ \alpha\beta \end{Bmatrix} - K_{\alpha\beta}{}^{\mu} \tag{2}$$

and $K_{\alpha\beta}{}^{\mu}$ is the contorsion tensor which is related to the torsion tensor $Q_{\alpha\beta}{}^{\mu} \equiv \Gamma_{[\alpha\beta]}{}^{\mu}$ by

$$K_{\alpha\beta}{}^{\mu} = -Q_{\alpha\beta}{}^{\mu} - Q^{\mu}{}_{\alpha\beta} + Q_{\beta}{}^{\mu}{}_{\alpha}\,. \tag{3}$$

The dynamical spin-density tensor can be written:

$$\tau^{\alpha\beta\mu} = 1/(-g)^{1/2}(\delta L_m/\delta K_{\mu\beta\alpha}), \tag{4}$$

where $\tau^{\alpha\beta\mu}$ is connected with torsion tensor by $T^{\alpha\beta\mu} = \chi\tau^{\alpha\beta\mu}$ being $T^{\alpha\beta\mu}$ the modified torsion tensor:

$$T_{\alpha\beta}{}^{\mu} = Q_{\alpha\beta}{}^{\mu} + \delta^{\mu}_{\alpha} Q_{\beta\nu}{}^{\nu} - \delta^{\mu}_{\beta} Q_{\alpha\nu}{}^{\nu}. \tag{5}$$

The torsion algebra is related to the matter spin density and one can substitute spin for torsion everywhere. The variation with respect to $g_{\alpha\beta}$ and $K_{\alpha\beta\mu}$ gives the field equations:

$$G^{\alpha\beta}(\{\,\}) = \chi(T^{\alpha\beta} + \tau^{\alpha\beta}), \tag{6}$$

where $T^{\alpha\beta}$ is the usual energy-momentum tensor and $\tau^{\alpha\beta}$ can be considered as representing the contribution of an effective spin-spin interaction (Hehl *et al.*, 1976), i.e., product terms:

$$\tau^{\alpha\beta} = \chi[-4\tau^{\alpha\mu}{}_{..[\nu}\tau^{\beta\nu}{}_{..\mu]} - 2\tau^{\alpha\mu\nu}\tau^{\beta}{}_{.\mu\nu} + \tau^{\mu\nu\alpha}\tau_{\mu\nu}{}^{\beta} + \\ + (\tfrac{1}{2})g^{\alpha\beta}(4\tau_{\mu.[\rho}{}^{\nu}\tau^{\mu\rho}{}_{..\nu]} + \tau^{\mu\nu\rho}\tau_{\mu\nu\rho})]. \tag{7}$$

This is equivalent to an effective cosmological term (Sivaram, 1974). $L_m$ describes spinning fluid coupled minimally to the metric and torsion of Riemann–Cartan manifold.

The dynamical spin tensor can be written

$$\tau^{\alpha\beta\mu} = (\tfrac{1}{2})S^{\alpha\beta}u^{\mu}) \tag{8}$$

where $S^{\alpha\beta}$ is spin density and $u^{\mu}$ the 4-velocity of the fluid and $S_{\alpha\beta}u^{\beta} = 0$ holds.

The dynamical energy momentum tensor can be decomposed into the usual fluid part $T_F^{\alpha\beta}$ and an intrinsic spin part $T_S^{\alpha\beta}$. $S_{\alpha\beta}$ is associated with the quantum mechanical spins of elementary particles and effective sources of the gravitational field described by space-time averaging of $T^{\alpha\beta} + \tau^{\alpha\beta}$. Even if the spins are randomly oriented, the average of the spin-squared terms is not zero in general (Hehl *et al.*, 1976). For unpolarized spinning fluid we have

$$\langle S_{\alpha\beta}\rangle = 0 \quad \text{and} \quad \sigma^2 = (\tfrac{1}{2})\langle S_{\alpha\beta}S^{\alpha\beta}\rangle, \tag{9}$$

$$\langle \tau^{\alpha\beta}\rangle = (\tfrac{1}{2})\chi\sigma^2 u^{\alpha}u^{\beta} + (\tfrac{1}{4})\chi\sigma^2 g^{\alpha\beta}. \tag{10}$$

For the fluid and spin parts of the energy-momentum tensor we have

$$\langle T_F^{\alpha\beta}\rangle = (\rho + p)u^{\alpha}u^{\beta} - pg^{\alpha\beta}, \tag{11}$$

$$\langle T_S^{\alpha\beta}\rangle = -\chi\sigma^2 u^{\alpha}u^{\beta}. \tag{12}$$

The simplest E–C generalization of standard Big Bang cosmology is obtained by considering the Universe filled with unpolarized spinning fluid and solving modified Einstein equations $G^{\alpha\beta}(\{\,\}) = \chi\theta^{\alpha\beta}$, where

$$\theta^{\alpha\beta} = \langle T^{\alpha\beta}\rangle + \langle\tau^{\alpha\beta}\rangle = (\rho + p - (\tfrac{1}{2})\chi\sigma^2)u^{\alpha}u^{\beta} - (\rho - (\tfrac{1}{4})\chi\sigma^2)g^{\alpha\beta}, \tag{13}$$

where $\rho$, $p$, and $\sigma$ depend only on time. In the co-moving frame $u^\mu = (0, 0, 0, 1)$, we get the following modified field equations of the Robertson–Walker universe, which in general for $k \neq 0$ and $\Lambda \neq 0$ is of the form:

$$\dot{R}^2/R^2 = (8\pi G/3)\,[\rho - (\tfrac{2}{3})\pi G\sigma^2/c^4] + \Lambda c^2/3 - kc^2/R^2\,. \tag{14}$$

We immediately note that the torsion term in Equation (14) (the second term within brackets) is of *opposite* sign to that of the cosmological constant term. This raises the possibility that a sufficiently large spin-torsion term in the early universe might cancel a correspondingly large cosmological constant. We shall see that this is indeed the case. For instance consider the Universe at the Planck epoch when as we noted earlier the $\Lambda$ term was $\approx 10^{66}$ cm$^{-2}$ implying $\Lambda_{\mathrm{Pl}} c^2 \approx 10^{87}$ in Equation (14). At $t_{\mathrm{Pl}} \simeq 10^{-43}$ s, the Universe had a density of $c^5/G^2h \approx \rho_{\mathrm{Pl}} \approx 10^{93}$ g cm$^{-3}$, and as the particle masses were $\approx 10^{19}$ GeV $\approx 10^{-5}$ g, the particle number density was $n_{\mathrm{Pl}} \approx 10^{98}$ cm$^{-3}$, so that $\sigma$, the spin density, was $\sigma_{\mathrm{Pl}} \approx 10^{98} \times 10^{-27}$ (i.e., $n_{\mathrm{Pl}} h) \approx 10^{71}$. This gives for the term $-(8\pi G/3)\,(\frac{2}{3})\pi G\sigma_{\mathrm{Pl}}^2/c^4$ (i.e., the torsion term in Equation (14)) the value of $\approx -10^{87}$, which is exactly equal and of opposite sign to that of the cosmological term $\Lambda_{\mathrm{Pl}} c^2 \sim +10^{87}$ so that the two terms would have cancelled each other in the early universe at the Planck epoch. We can see that they would continue to cancel at later epochs in the early universe.

The $\Lambda$ term would evolve with temperature $T$ as $\Lambda \sim T^2$ (Sivaram *et al.*, 1976). $\sigma$ being the spin density (i.e., proportional to number density of spins) would scale as $T^3$ ($n \sim T^3$ for relativistic particles). So $\sigma^2$ would scale as $T^6$. Now in an earlier paper (de Sabbata, 1989) it was argued that the spin-torsion coupling in the early universe was energy-dependent and scaled as $G \sim T^{-2}$, i.e., $G \sim T^{-2} \sim t$. So $G^2 \sim T^{-4}$, so that the $G^2\sigma^2$ term would scale as $T^2$, the same dependence on $T$ as the $\Lambda$-term so that if they cancel each other at the Planck epoch, they would also cancel at later epochs in the early universe.

Thus, in short, we have a more natural mechanism for understanding of a vanishing $\Lambda$ term by the simple incorporation of spin effects (a universal property of particles!) in general relativity.

## References

Coleman, S.: 1988, *Nucl. Phys.* **B307**, 867.
de Sabbata, V. and Gasperini, M.: 1985, *Introduction to Gravitation*, World Sci., Singapore.
de Sabbata, V.: 1983, *J. Tensor Soc. India* **1**, 39.
de Sabbata, V.: 1984, *Giornale di Astronomia*, p. 275.
de Sabbata, V.: 1988, *Gravitational Measurements, Fundamental Metrology and Constants*, Kluwer Academic Publ., The Netherlands, p. 115.
de Sabbata, V. and Sivaram, C.: 1988, *Nuovo Cimento* **102B**, 107.
de Sabbata, V. and Sivaram, C.: 1989, *Astrophys. Space Sci.* **158**, 347.
Dolgov, A. D.: 1982, in G. Gibbons *et al.* (eds.), *The Very Early Universe*, Cambridge Univ. Press, Cambridge, p. 449.
Hehl, F. W., van der Heyde, P., Kerlick, G. D., and Nester, G. M.: 1976, *Rev. Mod. Phys.* **48**, 393.
Sivaram, C.: 1985, *Bull. Astron. Soc.* **13**, 339.
Sivaram, C.: 1986a, *Astrophys. Space Sci.* **125**, 189.

Sivaram, C.: 1986b, *Astrophys. Space Sci.* **127**, 133.
Sivaram, C.: 1986c, *Int. J. Theor. Phys.* **25**, 825.
Sivaram, C., Sinha, K. P., and Lord, E. A.: 1974, *Nature* **249**, 64.
Sivaram, C., Sinha, K. P., and Sudarshan, E. C. G..: 1976, *Found. Phys.* **6**, 717.
Trautman, A.: 1973, *Nature* **242**, 7.
Weinberg, S.: 1989, *Rev. Mod. Phys.* **61**, 1.
Wilczeck, F.: 1985, in A. Zichichi (ed.), *Proc. of Erice 1983 Conf.*, Plenum, New York, p. 208.