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Polarization line radiative transfer in the atmospheres of magnetic white dwarfs

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Some physical mechanisms which can affect the Zeeman line profiles of magnetic white dwarfs are studied. The pure absorption polarization transfer equation is solved including these mechanisms. The broadening of lines in magnetic white dwarfs is briefly discussed.

Es werden physikalische Mechanismen, die die Zeemann-Linienprofile Weißer Zwerge beeinflussen können, untersucht. Die Strahlungstransport-gleichung für Polarisation bei wahrer Absorption wird unter Einbeziehung dieser Mechanismen gelöst. Die Linienverbreitung in magnetischen Weißen Zwergen wird kurz diskutiert.

Key words: white dwarfs — magnetic stars — Zeeman effect — polarization

1. Introduction

The theoretical analysis of the polarization observations of magnetic white dwarfs needs careful consideration of all the relevant physical processes which operate in a dense magnetized plasma. In Section 2 we discuss the plasma polarization shift (PPS) of spectral lines — a process which is important in high density plasmas. It appears that this line shifting mechanism has not been used before in the computation of the lines of hydrogenic ions in the white dwarf atmospheres. In Section 3 we describe the differences in the line profiles produced by atmospheres in radiative or convective equilibrium. In Section 4 we have pointed out the importance of the quadratic Stark effect in neutral Helium lines through exact calculation. We hope that the qualitative features presented here will be of some use in the actual modelling of the line profiles of magnetic white dwarfs by quantitative means (disk integrated studies). Recently we have studied some physical mechanisms relevant to magnetic atmospheres of Ap stars and white dwarfs in NAGENDRA and PERAIAH (1986). The method of solving the vector transfer equation is described in NAGENDRA and PERAIAH (1985a).

2. The effect of plasma polarization shift (PPS) of spectral lines on the Zeeman lines in white dwarfs

PPS is a line shifting mechanism which affects the lines formed in dense plasmas ($N_e \gtrsim 10^{17}$ cm^{-3}). The examples are the atmospheres of white dwarfs and the polar cap emitting regions of accreting white dwarfs and neutron stars. The nearly linear dependence of this shift on B even for strong fields ($B > 10^7$ G), and on the electron densities N_e makes it relevant in strongly magnetized dense plasmas in general. It is an effect originating in the partial screening of nuclear charge by an excess of negative space charge, mainly caused by the perturbing free electrons moving in the Coulomb field of the emitting ion. It is important only when the radiating atom is ionized, the simplest example being He II. Here the interactions of the emitting ion with the plasma environment are responsible for an average negative space charge which partly lies "within the bound electron orbits", and therefore partially shielding nuclear charge, thus altering the energy level structure of the emitter. The static level shift arises due to initial correlations and is frequency independent. Also, the shift is negligible compared to Stark effects in the case of neutral atoms. The shift in the He-like ions is again smaller than the H-like ions. The explicit dependence of PPS on electron density (N_e) and the temperature (T) leads to stronger radiative transfer effects particularly in deep layers of the stellar atmosphere. See JAEGLE et al. (1985) for some recent experimental and theoretical work regarding PPS.

The quantum mechanical treatment of this problem is due to GRIEM (1974) and VOLONTÉ (1975). According to GRIEM's theory, the average perturber charge density due to plasma at the $(Z-1)e$ charged ion (radiating particle) is given by

$$\varrho(V) = -(eN_e)\,\frac{2\pi(Z-1)\,\dfrac{e^2}{\hbar V}}{1-\exp\left[-2\pi(Z-1)\,\dfrac{e^2}{\hbar V}\right]}\,, \tag{1}$$

assuming isotropic perturber distribution. V is the velocity of the perturbing electron. The shielding charge $e\,\Delta Z_n$ within the level n (principal quantum number) is given by

$$e\,\Delta Z_n = 2\pi \int_0^{r_n} \mathrm{d}r \int_0^{\pi} \mathrm{d}\theta \int_0^{\infty} \mathrm{d}V\, r^2 \sin\theta f(V)\,\varrho(V)\,, \tag{2}$$

where $r_n = (n^2/Z)a_0$ is the bound orbit radius, a_0 being the Bohr radius. $f(V)$ is the Maxwellian velocity distribution. The shielding by the bound electrons is negligible compared to that of the electrons in free states, because the contribution to negative space charge in the GRIEM's model comes mainly from free electrons in the plasma. However, in VOLONTE's (1975) model the perturbing electrons move in the Coulomb field of the emitting ion considered as a point charge. In principle, these electrons can be either in bound or in free states. In the quantum mechanical treatments of both authors, the radial integral shown above appears and r_n is used as a suitable cutoff radius in performing the integral. In more recent calculations radially dependent charge density distributions in the ionic volume are also used. The shift of the level $n(n > 1)$ is obtained in the hydrogenic approximation as

$$\Delta E_n \approx \Delta\left(-Z^2 \frac{E_\mathrm{H}}{n^2}\right) \approx -2Z\, \Delta Z_n \frac{E_\mathrm{H}}{n^2}\,, \tag{3}$$

from which the relative wavelength shift of the level follows

$$\frac{\Delta\lambda_n}{\lambda_n} \approx -\frac{32}{3}\pi^{3/2}\left(\frac{Z-1}{Z^4}\right)\left(\frac{E_\mathrm{H}}{kT}\right)^{1/2} a_0^3 n^2(n^2+1)\, N_\mathrm{e}\,, \tag{4}$$

where

$$a_0 = \frac{h^2}{4\pi^2 m_\mathrm{e} e^2}\,, \qquad E_\mathrm{H} = \frac{e^2}{2a_0}\,. \tag{5}$$

$\bar{\lambda}_n = \lambda_n + \Delta\lambda_n$ is the shifted position of the level. In GRIEM's theory the perturbing charge density is estimated "at the nucleus", where it takes its maximum value and is assumed to remain constant at that value throughout the ion volume. Such a treatment is justified for low n values and high ($N_\mathrm{e} \gtrsim 10^{17}\ \mathrm{cm}^{-3}$) density plasma environment. The ground state of an ion is not shifted in GRIEM's theory. Let ν and ν' be the frequency of a point in the unshifted and shifted profiles, respectively; the line shift being produced by PPS. Then the relative shift can be expressed in reduced frequency units as

$$v' = v + \frac{c}{V_\mathrm{T}}\left[\frac{\nu}{\nu_0} x_n - (x_{n'} - x_n)\frac{\nu_{n'}}{\nu_0}\right], \tag{6}$$

where, as usual,

$$v = \frac{\nu - \nu_0}{\Delta\nu_\mathrm{D}}\,, \qquad v' = \frac{\nu' - \nu_0}{\Delta\nu_\mathrm{D}}\,; \qquad \Delta\nu_\mathrm{D} = \frac{\nu_0}{c}\sqrt{\frac{2kT}{M}} = \frac{\nu_0}{c} V_\mathrm{T}\,, \tag{7}$$

and

$$x_n = \frac{32}{3}\pi^{3/2}\left(\frac{Z-1}{Z^4}\right)\left(\frac{E_\mathrm{H}}{kT}\right)^{1/2} a_0^3 n^2(n^2+1)\, N_\mathrm{e}\,. \tag{8}$$

$x_{n'}$ can be obtained by a replacement $n \to n'$. n' and n are the principal quantum numbers of the lower and upper energy levels. E_H is the binding energy of the ground state of hydrogen atom. We shall now estimate the effect of PPS on the spectra of He II ion in a strong magnetic field. The energy levels of this ion including linear and quadratic Zeeman effect in a strong magnetic field ($B > 10^7$ G) have been computed by SURMELIAN and O'CONNELL (1973) using the perturbation theory. We have taken those eigenvalues and included the PPS using the equations given above. It is well known that the binding energy of the ground state of hydrogen atom increases monotonically for field strengths $B > 10^8$—10^9 G (RAJAGOPAL et al. 1972, COHEN et al. 1970). The wavelength shift for any line in general can be computed using

$$\Delta\lambda_{nn'} \simeq -x_n\lambda_{nn'} + \frac{(x_{n'} - x_n)}{\lambda_{n'}}\lambda_{nn'}^2\,, \tag{9}$$

where x_n and $x_{n'}$ are as given before. The displaced wavelength is given by $\bar{\lambda}_{nn'} = \lambda_{nn'} + \Delta\lambda_{nn'}$. λ_n is the wavelength corresponding to the excitation energy of the n^{th} level. We have assumed in deriving the equation given above that $E_n > E_{n'}$. But in very strong magnetic fields ($B \gtrsim 10^9$ G) the intern mixing of highly excited states occurs, thus diminishing the impact of PPS and even leading to a shift of opposite sign compared to the corresponding transition in a lower field strength. The ground state ionization energy is calculated using

$$E_\mathrm{H} = \frac{\hbar^2}{2m_\mathrm{e}a_0^2} + \hbar\omega_\mathrm{L} - m_\mathrm{e}\omega_\mathrm{L}^2 a_0^2\,; \qquad \omega_\mathrm{L} = \frac{eB}{2m_\mathrm{e}c}\,. \tag{10}$$

It should be noted that the mean radius of the atom decreases for field strengths $B > 5 \cdot 10^8$ G. This has to be taken into account when extending the basic formulation of PPS theory in the presence of a strong magnetic field. Since we are not aware of whether such a strong field calculation exists, we have used the non-magnetic shift formula itself as a first approximation. However we have consistently included the magnetic field effect on the energy levels by employing the magnetic field modified eigenvalues for a given field strength. When PPS is at all important, the linear and/or quadratic Stark effects are comparitively much weaker and vice versa. The asymmetry of the resonance and subordinate lines of He II for example are caused mainly by PPS in a dense plasma. Hence we discuss further the impact of PPS on the line shift of He II lines in a strongly magnetized plasma.

In a strong magnetic field, a large number of Zeeman sub-components are produced (KEMIC 1974) in any line. We have

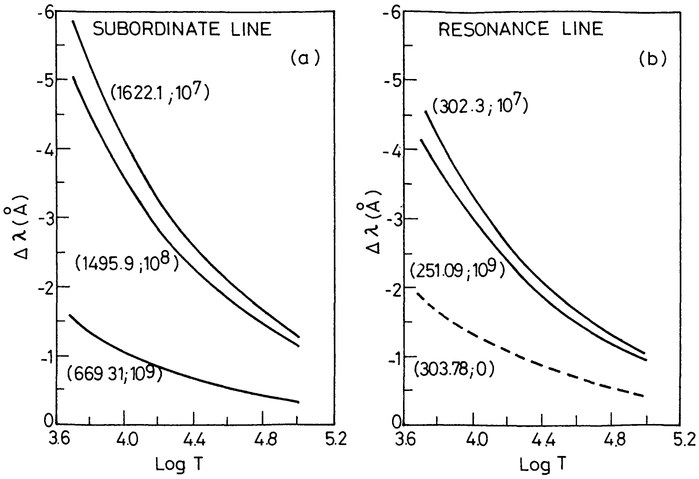

Fig. 1. (a) The temperature dependence of PPS wavelength displacements of a subordinate line Zeeman sub-component of He II, in strong magnetic fields. The first of the numbers in the parenthesis near the curves represents the steady state wavelength of the transition, corresponding to the field strength given as the second number. The electron density is $N_e = 10^{18}$ cm^{-3}. (b) Same as (a), but for a strong Zeeman component of the resonance line. For this case $N_e = 10^{20}$ cm^{-3}.

selected one such subcomponent, $3d_2$ — $2p_1$ which is a very strong transition. In Fig. 1a we have shown the wavelength displacement $\Delta\lambda$ produced by PPS and measured with respect to the steady state wavelength position of this transition at a given field strength. The displacements are quite large in a low temperature plasma, because the thermal velocity of the electrons is smaller, leading to a larger value of time averaged perturber charge density at the radiating ion. The displacements for a transition $2p_1$ — $1s_0$ are shown in Fig. 1b. The PPS displacements curve for $\lambda = 298.26$ Å, the wavelength position corresponding to $B = 10^8$ G is not shown, since it is unresolvable in the adopted scale from the curve corresponding to $\lambda = 302.3$ Å. The displacements are very small, because (i) they vary as the fourth power of the principal quantum numbers of the levels involved and (ii) they are directly proportional to the wavelength of the transition, both of which are smaller presently. The displacement of this resonance transition in a non-magnetic plasma is also shown (dashed line). In general, the displacements depend on the difference in shifts between the upper and lower levels and are directly proportional to the electron density N_e. While using the PPS formulae given above, in a strongly magnetized plasma it is safer to verify that the classical straight line path approximation is satisfied. This approximation means that $r_G = \frac{m_e c}{eB}\sqrt{\frac{2kT}{M}} \gtrapprox r_n = \frac{n^2 a_0}{Z}$, namely the gyroradius is larger than the radii of the Bohr orbits of the levels involved in the transition. It is not a stringent criterion, because the time averaged negative polarization charge overlapping the bound electron radius is more important than the details of the electron paths.

Now we shall compute a practically important line (He II 4685.7 Å), which is used sometimes in the spectral line analysis as a gravity indicator. In view of this application, we have employed a realistic model atmosphere of a DB white dwarf with $T_{\text{eff}} = 25000$ K and $\log g = 9$, given by WICKRAMASINGHE (1972). The normal Zeeman pattern of this line is computed for a field strength $B = 2$ MG. The transfer equation and the absorption matrix elements required for the calculation including the line plus continuum magneto-optical effects and the continuum polarization are given in NAGENDRA and PERAIAH (1984, 1985b). We use the same notation here and mention only the changes to be made to include the PPS in the computation of a realistic line profile of He II 4685.7 Å. Here unlike the problem in the next section all the physical parameters are depth dependent. The line absorption coefficients for example are now calculated using

$$\eta_i^{\text{L}} = \eta_0 H(a, v - v_i)\,, \qquad i = p, l, r\,, \tag{11}$$

where

$$v = \frac{\Delta\lambda}{\Delta\lambda_{\text{D}}} \quad \text{and} \quad v_i = \frac{\Delta M\,\Delta\lambda_B + \Delta\lambda_{nn'}}{\Delta\lambda_{\text{D}}}\,, \tag{12}$$

and $\Delta M = 0, \pm 1$ for $i = p, l, r$, respectively. The wavelength shift due to PPS is given by the equation (9). The Zeeman shift is $\Delta\lambda_B = eB\lambda_0^2/4\pi m_e c^2$. The parameter η_0 is calculated using

$$\eta_0 = \frac{\pi e^2}{m_e c}\frac{Nf}{\varrho k^{\text{C}}}\frac{1}{\Delta\gamma_{\text{D}}}\,. \tag{13}$$

The damping constant is $a = \Gamma/4\pi\,\Delta\gamma_{\text{D}}$. Γ being the total damping width of the line. ϱ is the mass density, f is the oscillator strength of the line and N the number density of the lower level of the transition. The results of calculation are shown in Fig. 2. The full lines correspond to the case of normal Zeeman effect in the line and without including PPS. The Zeeman components are well separated since the field strength is large, but they are narrow since the damping constant a is quite small. The Stokes profiles are already asymmetric because of the continuum polarization and the continuum magneto-optical effect, the latter being quite strong in deepening the line profile and depolarizing it. The p and q profiles have some difference in shape at the π and σ positions as compared to the Stokes profiles formed in constant opacity atmosphere (see e.g. Fig. 4). The position angle φ in particular is highly scrambled and less useful as a diagnostic. All these changes occur mainly because of the depth dependence here, of even the Doppler width and the damping constant, which are usually kept constant. The dashed lines show the impact of PPS on the Zeeman intensity and polarization profiles. The profiles clearly show a blueshift, but the shift is not

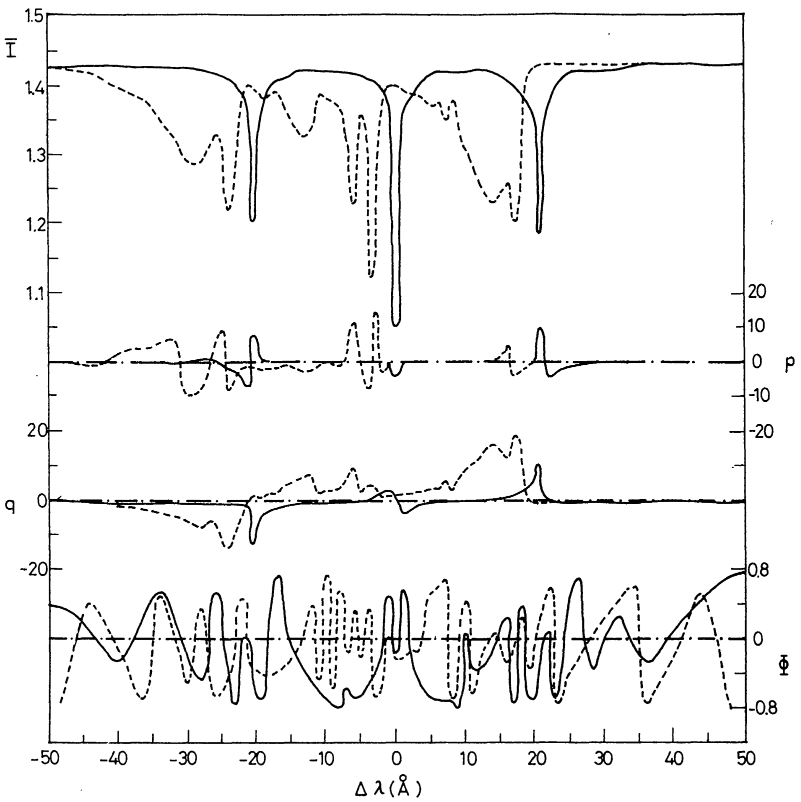

Fig. 2. He II 4685.7 Å line profile formed in a magnetic field are shown, assuming the normal Zeeman effect. These realistic Zeeman line profiles are computed adopting the model atmosphere of a helium-rich DB white dwarf ($T_{eff} = 25{,}000$ K, log g = 9.0) given by WICKRAMASINGHE (1972). The model atmosphere is truncated in the electron density range ($N_e \simeq 10^{17}$ to 10^{18} cm^{-3}) to take care of the validity of the simple model of PPS used in our computations. This however, provides the optical depth range sufficient to integrate the transfer equation accurately. The dashed curves correspond to the case when the PPS mechanism is included in the line profile calculation. The geometry is $\mu = 1$, $\psi = \pi/4$ and $\chi = \pi/4$. The polarization parameters are defined now onwards as $p = (\mathrm{sgn}\ U)\sqrt{Q^2 + U^2}/I$; $q = V/I$; $\varphi = 0.5 \tan^{-1}(U/Q)$.

uniform. This is because the PPS strongly varies with depth. The shift in the deepest layers of the atmosphere is nearly 20 times larger than the shift in the outermost layers. In the deepest layers the PPS in fact approaches the Zeeman splitting itself for the field strength $B = 2$ MG which we have used. This leads to an extreme overlapping of the π and σ components in these layers. Such an overlapping, coupled with Doppler width variation, is responsible for the appearance of additional structure in the shifted components and sharp variations in the p, q and φ profiles. The line strength and its polarization are also enhanced in general. Thus, an interpretation based on half width or Zeeman shift measurement becomes difficult if the PPS is not included in the calculation. Though the effect of PPS may have been overestimated here because of its approximate treatment, the qualitative features may not change in a more exact calculation. The impact of any such line shift mechanism depends also on the degree of Zeeman splitting. The large asymmetries in the polarized line profiles lead to an increase in the surface averaged continuum polarization near these lines, affecting consequently the field strength measurements using the continuum polarization. The increased line blanketing due to PPS in such high gravity white dwarf atmospheres slightly changes the original atmospheric structure, which indirectly influences the spectral lines.

In the atmospheres of white dwarfs the electric field strengths are largely due to high electron densities. Still, they cannot directly affect the He II lines, because — PPS being a correlation effect — it is the time averaged polarization charge density and not the steady state electric field which is responsible for the level shifts. Details of the electron paths are also not important for the same reason. In this way the PPS is rather insensitive to external electric and magnetic fields. The PPS is much larger than Doppler and Stark (quadratic) widths, but is very small compared to the Zeeman shifts of the π and σ components in a strong magnetic field, so as to act only as a perturbation. Hence in an approximate calculation, we can algebraically add up the PPS shifts to the respective Zeeman shifts of the π and σ components. However a consistent way of treating this problem is to calculate the atomic structure in the regime of strong magnetic and electric fields taken together — which is a strong field analogue of the combined Stark-Zeeman effect of hydrogen lines in many ways. The first approximation treatment of PPS we have used is quite sufficient to show the magnitude of the impact of PPS on line formation. In fact, these simple formulae are used even in the interpretation of the laboratory experiments on PPS (see GRIEM 1974).

Massive white dwarfs ($M \sim 1$—$2M_\odot$) have high gravity atmospheres. He II lines, in particular He II 4686 Å line, belong to the strongest lines in the optical spectra of such helium rich (He/H $\sim$ 5—10) white dwarfs. For example the equivalent widths of 10 Å and 4.5 Å have been observed in HD 149499 B and HZ 21, respectively. This line has been used as a gravity indicator. It is important to note that the disk integrated intensity and polarization profiles do not actually look like that shown in Fig. 2.

They would be quite smooth but highly broadened. Our interest here had been to show just the impact of PPS mechanism. A surface integration taking a realistic field distribution is needed basically in order to compare with observations. It is found by model line computations (WASEMAEL and VAN HORN 1979) that the equivalent width of this line increases strongly with the increase of effective gravity. The width is weakly dependent on T_{eff} for a given value of gravity, except of course at low temperatures ($T_{eff} < 40{,}000$ K).

3. Zeeman line formation in differing atmospheric structures and moving atmospheres

The line shape and depth of an absorption line largely depend on temperature structure and the source function gradient in the atmosphere. In Fig. 3 we have compared the intensity and polarization profiles formed in model atmospheres ($T_{eff} = 9000$ K, $\log g = 8$) of DA white dwarfs, which are in radiative (dashed line) and convective (solid line) equilibrium. We have used a hypothetical Zeeman triplet taking representative parameters typical of a weak line formed in a low field (10^5—10^6 G) magnetic white dwarf. The relevant equations are described in NAGENDRA and PERAIAH (1985b, Section 3; and 1984). For simplicity, the transfer matrix A is taken as depth independent; hence the constancy of Doppler width over the depth does not introduce significant errors. The temperature distribution is the key factor. It is clearly seen that the lines formed in a convective model are deeper and also wider than those formed in a radiative model. In the former case, slight distortions are produced in the Stokes profiles near the centres of the π and σ components. In the same figure we have shown the effect of altering the source function gradient in the outermost layers of the convective model (dotted line). Such changes in source function gradients may be caused, for example, by an accretion of matter by the white dwarf. Since the medium is in LTE we have done this by simply reducing the temperature smoothly by 5% (at $\tau = 10^{-2}$) to 10% (at $\tau = 0$), respectively. We see that a deeper π component is produced in this case and the line becomes narrow and develops triplet structure. Marked changes in linear and circular polarization also occur at the centres of Zeeman components. It is well known that a similar effect occurs when one increases the line blanketing in the model atmospheres. The position angle φ almost remains unaffected to such changes mainly because of constancy of A with depth.

In Fig. 4 we show the line shifting produced by macroscopic steady state mass motions in the atmosphere. The lines in a moving atmosphere can be computed by using

$$v(\tau) = v - \cos \alpha(\tau)\, V_m(\tau)\,, \tag{14}$$

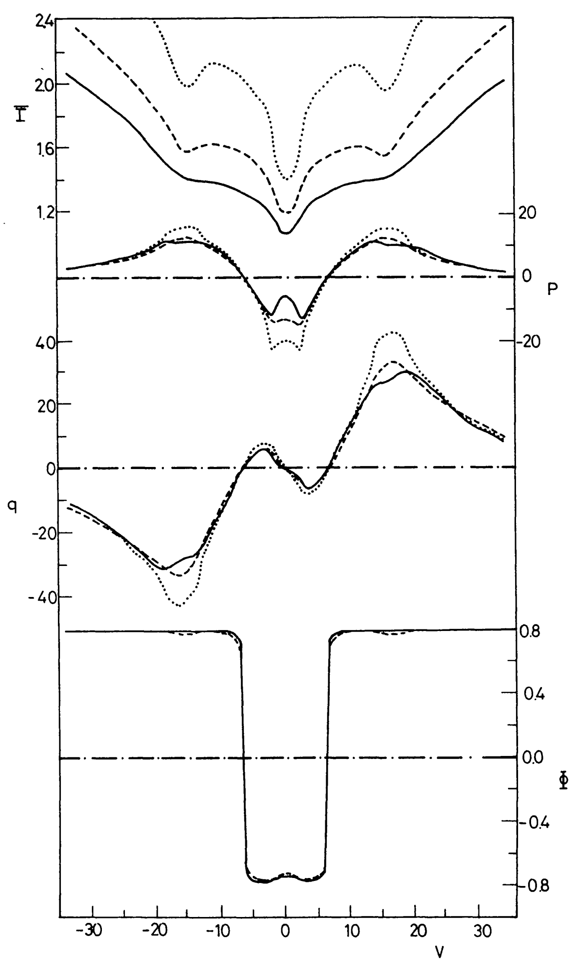

Fig. 3. The variation of $\bar{I}, p, q$ and φ for a Zeeman triplet with following parameters: $\eta_0 = 10^4$, $a = 0.1$, $\lambda_0 = 5000$ Å, $\mu = 1$, $\psi = \pi/4$, $\chi = \pi/4$, $v_{p,l,r} = 0, \pm 16$. The continuum is dichroic and magneto-optic: $\eta^C_{p,l,r} = 1, 0.94, 1.1$ and $\varrho^C_R = -10 \cos \psi$, $\varrho^C_W = -0.25 \sin^2 \psi$. Model atmospheres of DA type white dwarfs $T_{eff} = 9000$ K, $\log g = 8$, taken from WEHRSE (1976) are employed. Dashed and solid lines represent the profiles formed in radiative and convective models with the same parameters given above. Dotted lines correspond to the profiles formed in the convective model with a changed source function gradient as mentioned in the text.

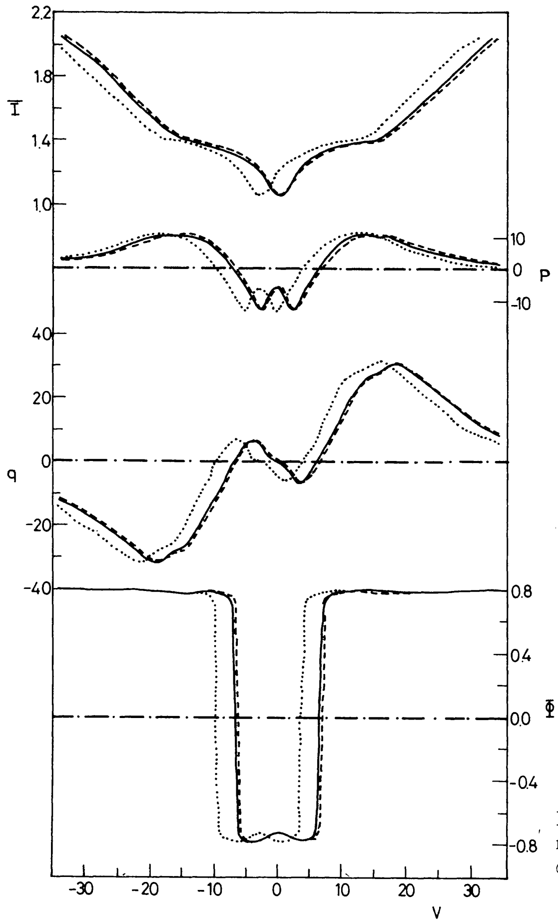

Fig. 4. Zeeman profiles and polarizations formed in a convective moving atmosphere with all the other parameters same as Fig. 3. $\alpha(\tau) = 0$ and depth independent velocities are employed.

where

$$V_m(\tau) = \frac{U_m(\tau)}{V_T(\tau)}\,; \qquad V_T(\tau) = \sqrt{\frac{2kT(\tau)}{M}}\,, \tag{15}$$

instead of v in the calculations. $V_m(\tau)$ is the dimensionless velocity parameter. $\alpha(\tau)$ is the angle between the velocity vector $U_m(\tau)$ and the line of sight. Here we have used τ-independent velocities and Doppler widths. The dashed lines indicate the Stokes profiles formed in a radially inward directed constant macroscopic velocity of 0.4 mtu ($\sim$5 km/sec); 1 mtu (mean thermal unity) $= V_T(\tau)$. The dotted lines are computed for an outwardly expanding medium with a velocity of 3 mtu. The Stokes profiles are blue shifted in this case. The solid lines represent the Stokes profiles formed in the static atmosphere. It should be noticed that the gravitational redshift of spectral lines, which can be computed using

$$v(\tau) = v + \frac{c}{V_T}\left\{\left(1 - \frac{2GM_*}{R_* c^2}\right)^{-1/2} - 1\right\}, \tag{16}$$

for a white dwarf of mass M_* and radius R_*, actually gives such a large (3—4 mtu) "red" shift of the Stokes profiles (Preston 1970). The gravitational redshift being isotropic over the stellar surface (unlike Zeeman splitting, which varies by a factor of 2 from equator to the pole) leads to a "residual continuum polarization" contributed by the Zeeman split redshifted lines, when integrated over the visible disk of the star. Also, a depth dependent velocity produces the Stokes profiles, which look wiggled, with deeper cores and increased half widths. We feel that such processes, which produce line distortions and line shifts in extremely overlapping Zeeman lines formed in strong magnetic fields, may contribute significantly to the continuum polarization and non-thermal like continuum energy distribution observed in these objects.

Near the centres of the Zeeman π and σ components the Doppler effects dominate and lead to the distorted profiles. Though there is no justification for assuming that the white dwarf atmospheres are either slowly expanding or contracting, it is nevertheless important to note that the macroscopic ordered motions do exist even in these rather high gravity atmospheres. The high resolution ($\lambda/\Delta\lambda = 10^4$) IUE observations of white dwarfs have shown the shortward shifted lines, which have been interpreted by Bruhweiler and Kondo (1983) as due to expanding halos or winds associated with these stars. These authors propose that the radiative levitation of certain ions having large absorption cross sections can explain the blue shift of the ionic resonance lines in hot white dwarfs. In the non-equilibrium situation the radiative acceleration may become stronger than gravity and these elements might leave the star via a "selective" wind. There are tentative suggestions of a slowly expanding halo consisting of ionised heavy elements by many authors. It is also well known that the magnetic field assists the radiative where $(\Delta\lambda_{LZ})_{p,l,r} = (0, \pm 1) |\Delta\lambda_{LZ}|$ represents the linear Zeeman shifts; $(\Delta\lambda_{QZ})_p = |\Delta\lambda_{QZ}|_0$ and $(\Delta\lambda_{QZ})_l = (\Delta\lambda_{QZ})_r = |\Delta\lambda_{QZ}|_{\pm 1}$ represent the quadratic Zeeman shifts all of which are blue shifts; $(\Delta\lambda_{QS})_{p,l,r} = |\Delta\lambda_{QS}|$ represents the quadratic Stark shifts, which are red shifts for the $2^3P - n^3S$ array.

Fig. 5 shows the relative importance of these three mechanisms. The LZ and QZ purely depend on the field strength B and the QS only one the perturber density N_p. From the figure it is clear that in the low field, high density media the quadratic Stark effect is an important line shift mechanism. Such conditions normally prevail in the deep layers of high gravity cool DB white dwarfs which have low magnetic fields. Just as QZ effect, QS effect also causes line asymmetries. The general impact expected on the line profiles is the deepening and broadening of the unresolved Zeeman triplet. In this way both QZ and QS act in a way similar to the rotational broadening of the spectral lines, though QZ has much more dramatic effects in the strong magnetic fields. QS is a second order redshifting mechanism, but has some significance in connection with the gravitational redshift measurements on the spectral lines of white dwarfs. The QS broadening can be easily included in the line transfer computations as already indicated in the previous sections. QS is normally the important broadening mechanism for the (l-degeneracy removed) sharp individual components in the spectra of magnetic white dwarfs. The procedure used here is simple and provides better estimate of this effect, than representing the QS effect via the Γ_4 constant in the estimation of total damping constant Γ and using the latter in the actual (Voigt) profile function computation.

4.2. The Lorentz effect

The total electric field acting on radiators (neutrals and ions) in a magnetoplasma can be written as $(F_0 + F_L)$, where F_0 is the usual Holtsmark electric microfield of the plasma and F_L is the so-called Lorentz electric field. $F_L = (\boldsymbol{V} \times \boldsymbol{B})/c$, where $\boldsymbol{V}$ is the velocity of the radiator moving in an external magnetic field $\boldsymbol{B}$. The Lorentz effect is simply the linear Stark effect due to F_L. The competition of this effect with the ordinary quasi-static Stark broadening requires $F_L \gtrsim F_0$, i.e. $(4\pi N_p/3) \lesssim (VB/ec)^{3/2}$ In the atmospheres of magnetić Ap stars and even white dwarfs this is not easily attained — the demands being high temperature low density plasma immersed in a strong magnetic field. Thus the Lorentz effect will be extremely weak when the Stark broadening is at all important. However, in the outermost layers of hot magnetic white dwarfs the Lorentz effect dominates the magnetic line broadening, whereas the Stark effects remain comparitively weak.

Galushkin (1970) has studied this effect in detail. We have employed a simple asymptotic wing formula given in that paper for just showing the actual impact of this effect on the spectral lines of magnetic white dwarfs. The computation of the general profile function valid in both the core and the wings of the line is slightly more difficult. We consider the radiation propagating

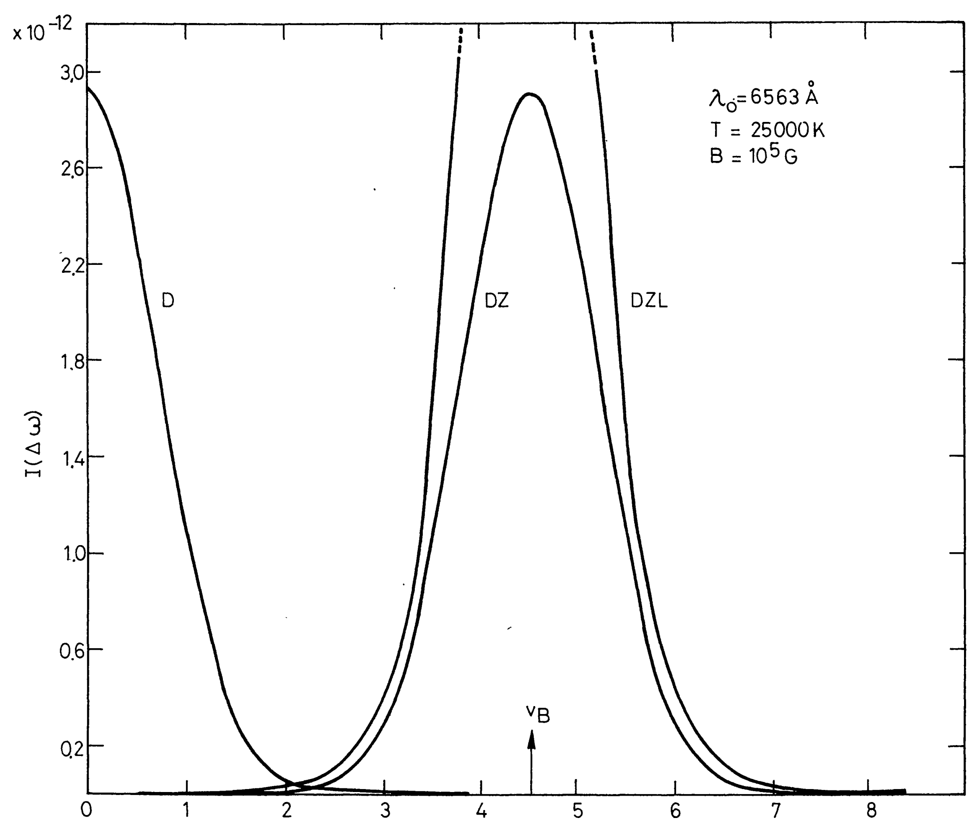

Fig. 6. The Lorentz effect on a σ-component of the H_α Zeeman triplet. The Landé g-factor is taken as unity. The profile function $I(\Delta\omega)$ is plotted against the reduced frequency $v = \Delta\omega/\Delta\omega_D$ for the indicated values of T and B. D = pure Doppler; DZ = Doppler-Zeeman and DZL = Doppler-Zeeman-Lorentz profile functions.

transversely to the magnetic field. The Lorentz effect will broaden only the component of the radiation that is polarized normal to H. The radiation polarized parallel to H will not be broadened. The asymptotic wing formula for transverse polarization is given by

$$I(\Delta\omega) = \frac{1}{2\sqrt{\pi}\,\Delta\omega_D} \left\{ \frac{\exp\left[-\left(\frac{\Delta\omega'}{\Delta\omega_D}\right)^2\right]}{\left(\frac{\Delta\omega'}{\Delta\omega_D}\right)^2} + \frac{\exp\left[-\left(\frac{\Delta\omega'}{\Delta\omega_D - \Delta\omega_E}\right)^2\right]}{\sqrt{1 - \frac{\Delta\omega_E}{\Delta\omega_D}}} + \frac{\exp\left[-\left(\frac{\Delta\omega'}{\Delta\omega_D + \Delta\omega_E}\right)^2\right]}{\sqrt{1 + \frac{\Delta\omega_E}{\Delta\omega_D}}} \right\}, \tag{20}$$

where $\Delta\omega' = \Delta\omega - \Delta\omega_B = \omega - \omega_0 - \Delta\omega_B$ and

$$\Delta\omega_D = \frac{\omega_0}{c}\sqrt{\frac{2kT}{M}} = \frac{\omega_0}{c} V_T\,; \qquad \Delta\omega_E = \frac{3ea_0}{\hbar(Z_i + 1)} \cdot \frac{VH}{c}\,. \tag{21}$$

$\Delta\omega_E$ represents the Lorentz (or electrodynamic) frequency shift. Z_i is the charge of the radiating ion ($Z_i = 0$ for hydrogen). We have adopted $V \equiv V_T$, the mean thermal velocity of the radiators. The linear Stark shift in comparison is given by

$$\Delta\omega_{LS} = \frac{3}{2}\frac{ea_0 n(n-1)}{\hbar(Z_i + 1)} F_0\,; \qquad F_0 = 2.603 e N_p^{2/3}\,. \tag{22}$$

N_p being the perturber number density. n is the principal quantum number of the upper level of the transition. This formula correctly represents the more general formula in the range $\Delta\omega \gtrsim \Delta\omega'_{1/2}$; $\Delta\omega_{1/2} = (\Delta\omega_D + \Delta\omega_E)$. Also, this formula is valid only for $\Delta\omega_E < \Delta\omega_D$. It can be seen clearly that the Lorentz effect basically produces the Doppler type profile function, except for the larger value of half width. Because of this nature, the usual Stark effect once again dominates in the far wings of the hydrogen lines.

In Fig. 6 we have shown the broadening due to Lorentz effect of one of the σ components in the transverse Zeeman case. The σ component is substantially widened (see DZL), but it still peaks at $\Delta\omega = \Delta\omega_B$. Since we have used an asymptotic formula, no significance is to be placed on the behaviour of the DZL profile closer (< 1 Doppler width) to the peak. The general formula however, produces a peak similar to the DZ profile. The Lorentz effect enhances the relative intensity of the DZ emission profile; the reverse of this behaviour appears in the case of absorption lines. Thus we conclude that the Lorentz effect is an important broadening mechanism in the outermost layers of the magnetic white dwarfs, where it leads to an increased saturation of the Zeeman line profile compared to the computation where it is being neglected. Its apparent Doppler like behaviour leads to wider and saturated Doppler cores causing higher temperatures and magnetic fields to be invoked, if this effect is neglected.

5. Conclusions

In the previous sections we have described some mechanisms which are important in a fine analysis of stellar polarization observations. The PPS of low excitation resonance lines of He II in strong magnetic fields (eg. in high gravity magnetic white dwarfs) may be useful in understanding the problems of differential weakening and shifts (some times largely incompatible with the theoretical wavelength positions and strengths of Zeeman subcomponents as given by KEMIC 1974) of the lines in the spectrum of magnetic white dwarfs. As shown in Section 3, a knowledge of the differences in line profiles produced in different atmospheric models helps further in isolating the suitable model required for detailed computations. Small changes in the source function gradients in the outer layers of the stellar atmosphere can lead to a decrease in equivalent width and a large increase in the line polarization. The polarization line transfer calculation in the middle layers of the atmospheres of magnetic white dwarfs can be performed using the conventional profile functions. However, the quadratic Stark effect (in the deepest layers) and the Lorentz effect (in the outermost layers) in the atmosphere considerably influence the Zeeman lines formed in low field magnetic white dwarfs.

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