Kodaikanal Obs. Bull. Ser. A (1980) 3, 21-25

ROTATIONAL EFFECTS ON LINE SOURCE FUNCTION

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ABSTRACT

The effects of rotation are investigated on the source functions in an expanding atmosphere. We have considered a non-LTE two-level atom in an extended atmosphere. We have also made use of von Zeipel's theorem in giving the incident radiation at $\tau = \tau_{max}$. Uniform rotation is assumed and the values of the ratios of the centrifugal force and gravity force at the equator are taken to be 0.1, 0.4, 0.8 corresponding to a uniform rotational velocity of 1, 4 and 8 mtu. It is found that rotation will dilute the radiation field which is similar to the effects of expansion.

Key words : source function—von Zeipel's theorem

1. Introduction

In an earlier paper (Peraiah 1980a henceforth called Paper I), we have investigated how rotation changes the line profiles emerging from a radially expanding media in a stellar atmosphere. It is found that substantial changes are introduced into the line shapes when rotation is introduced. Rotation broadens the profile and when there is a radially outward motion it becomes difficult to predict the shape of the emergent line profiles. In paper I, we have treated this problem for small velocities of rotation and expansion by assuming an appropriate radiation field. In it we have shown that the emergent profiles change their shape substantially and indicated that a further detailed study of the radiation field in rotating atmospheres, has to be carried out.

Here we intend to study how the source functions change when the rotational velocities change. We have made use of von Zeipel's law (1924) which states that the emergent flux of total radiation over the surface of a rotationally (and/or tidally) distorted star in radiative equilibrium varies proportionately to the local gravity. This law has been used for obtaining the incident radiation at $\tau = \tau_{max}$ and for calculating the source functions in the medium by using the comoving frame solution of radiative transfer (Peraiah 1980b, c).

2. Results and Discussion

As a preliminary study we have considered only the rotation of a single star. According to von Zeipel's law, the flux of radiation at a given point is proportional to the local gravity. von Zeipel's law is given by (Kopal 1959)

$$F_n = -\frac{C}{K\rho}\frac{dp}{dn} \tag{1}$$

where F_n is the flux of radiant energy across a level surface of constant potential, C is the velocity of light, K is the absorption coefficient, ρ is the density, p is the pressure and dn is the normal distance between adjacent level surfaces. If Ψ is the total potential (*i.e.* sum of potentials due to self gravitation, rotation and tidal interaction) then the radiant flux is given by

$$F_n \sim \frac{d\Psi}{dn} \tag{2}$$

But

$$\frac{d\Psi}{dn} = -g \tag{3}$$

where g is the local gravity. For a rotationally distorted star without tidal forces from external sources (Peraiah 1970)

$$g(r) = \frac{Gm_1}{r^2}\left\{1 - \sin^2\theta\,(f_r^2 - 2 f_r)\right\}^{\frac{1}{2}} \tag{4}$$

where r, θ are the polar coordinates of a point and

$$f_r(\theta) = \frac{r^3\,\Omega^2}{Gm_1} - 2\gamma\left(\frac{r}{r_p}\right)^3 + 4\beta\,\sin^2\theta\left(\frac{r}{r_p}\right)^5 + 6a\sin^4\theta\left(\frac{r}{r_p}\right)^7 \tag{5}$$

$$\gamma = \frac{f}{2x^2}\left(\frac{r_p}{r_e}\right)^3$$

$$\beta = \frac{(x-1)\,f}{2x^2}\left(\frac{r_p}{r_e}\right)^5$$

$$a = \frac{(x-1)^2\,f}{6x^2}\left(\frac{r_p}{r_e}\right)^7 \tag{6}$$

Here x = ratio of angular velocities at the equator and the pole, f = ratio of centrifugal to gravity forces at the equator, r_p and r_e are the radii at pole and equator. At the bottom of the atmosphere, we have introduced the incident radiation proportional to g given by equations (2) and (3) and this is written as

$$U^-_{N+1}(\tau = T, \mu_j) = g/N\,\mu_j\,C_j \tag{7}$$

so that

$$\int U^-_{N+1}(\tau = T, \mu)\,\mu\,d\mu = \Sigma\,U^-(\tau = T, \mu_j)\,\mu_j\,C_j$$

$$= \Sigma\,\frac{g}{N\,\mu_j\,C_j}\,\mu_j\,C_j = g \tag{8}$$

Where μ and C are zeros and weights of angle quadrature and N is the total number of angles and $T = \tau_{max}$. This has been done on the grounds that the radiation is incident on the atmosphere in spherically symmetric approximation. Here U is the specific intensity multiplied by $4\pi r^2$. By assuming a velocity law and an optical depth (T = 1200), we have solved the line transfer in comoving frame with the boundary condition given in (8). While calculating the gravity, g, we have assumed x = 1 (uniform rotation) and the ratio of centrifugal to gravity forces at the equator is set equal to 0.1, 0.4 and 0.8. The geometrical extension of the medium is taken to be B/A = 3 and 10 where B and A are the outer and inner radii of the atmosphere with a velocity with constant velocity gradient. The frequency independent source function is calculated by the formula,

$$S(r) = \int_{-\infty}^{+\infty} S(x, r)\,dx \tag{9}$$

where

$$S(x, r) = \frac{\phi(x)}{\phi(x) + \beta}\,S_L(r) + \frac{\beta}{\phi(x) + \beta}\,S_c(r) \tag{10}$$

S_c is the continuum source function and S_L is the line source function given by

$$S_L(r) = (1 - \epsilon)\int J_x\,\phi x\,dx + \epsilon\,B(r) \tag{11}$$

$$S_c(r) = B(r) \tag{12}$$

$B(\tau)$ is the Planck function, J_x is the mean intensity ϕx is the profile function (Doppler) and ϵ is the probability per scatter that a photon is lost by collisional de-excitation. We have set $\epsilon = \beta = 0$ in all cases. We have taken V_{rot} equal to 1, 4, 8 mean thermal units corresponding to f = 0.1, 0.4 and 0.8. The total source functions plotted with respect to the optical depths are given in Figures 1a and b for B/A = 3 and 10, respectively. We have considered the maximum expansion velocities $V_s = 0$, 3 and 6 as these are sufficient to show the differences due to velocity gradients. The effects of rotation are to reduce the source function considerably and it is interesting to note that the variation of S runs almost parallel to various velocity gradients. The source functions are reduced considerably when rotational velocities are increased. The reduction is almost an order of magnitude from $V_{rot} = 1$ to $V_{rot} = 8$. The main reason for the dilution is that when rotation increases the equatorial parts tend to extend and the density of radiation field decreases. As we are considering uniform rotation, constant velocity gradients would reduce the radiation field uniformly which explains the reason why the source functions are almost parallel to each other for $V_{rot} = 1$, 4 and 8. The source functions

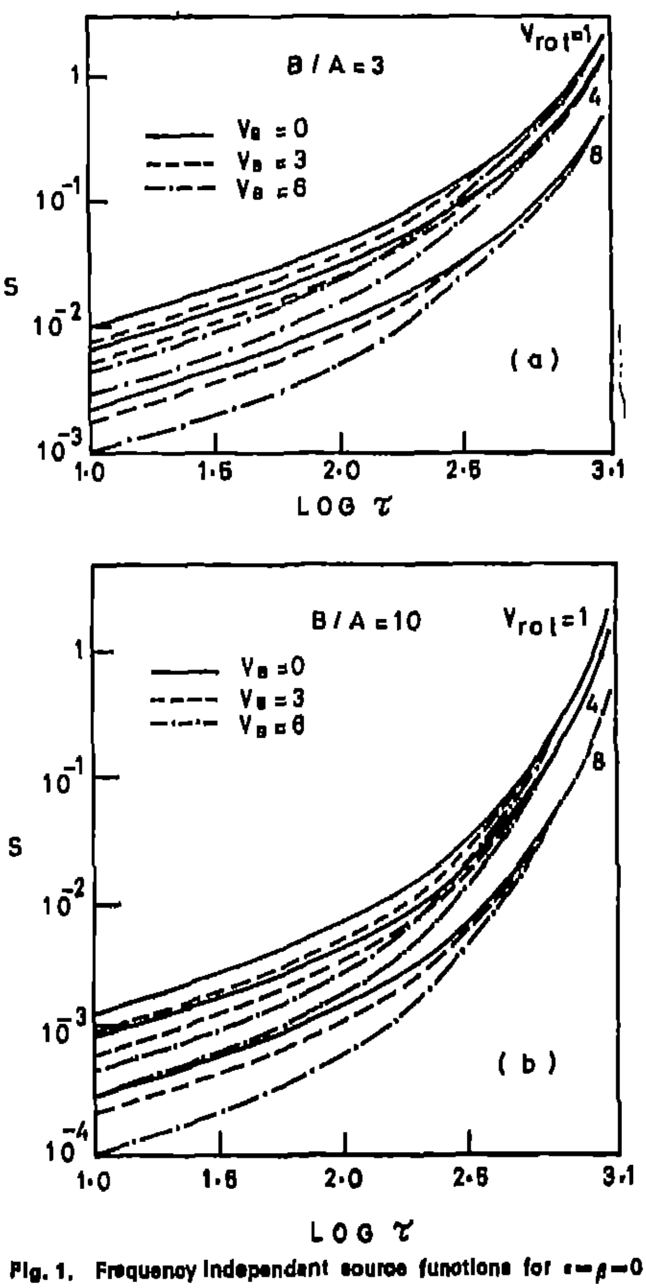

Fig. 1. Frequency independent source functions for $\epsilon = \beta = 0$ and B/A = 3 and 10.

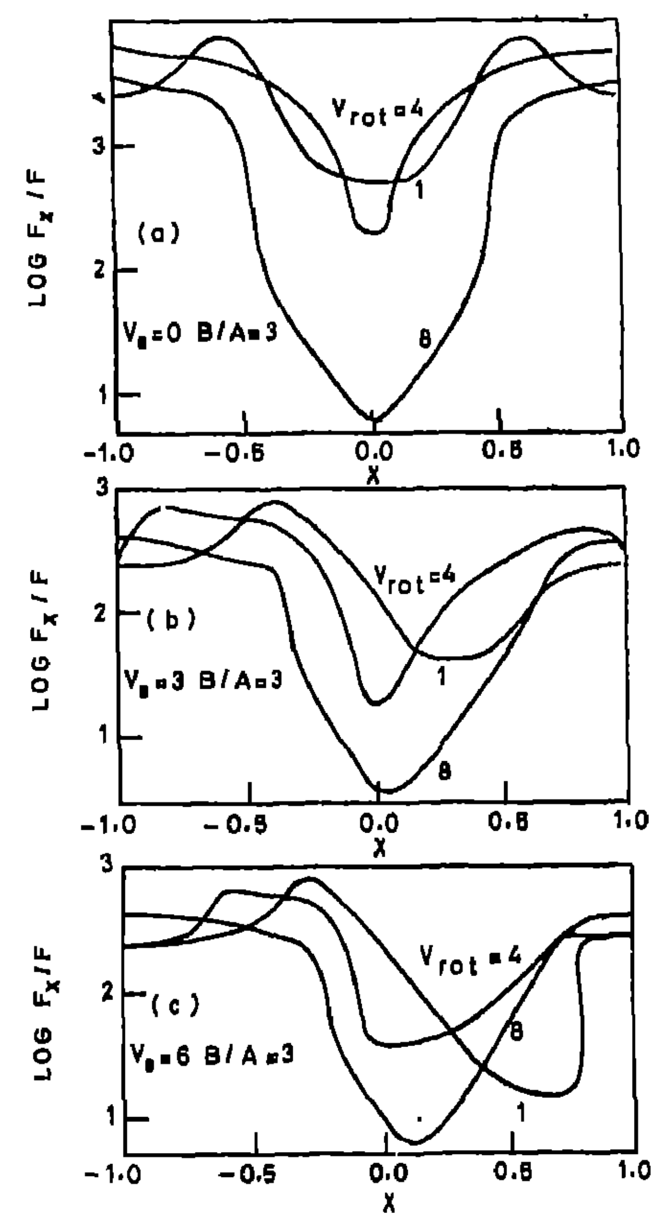

Fig. 2. Flux profiles F at infinity for B/A = 3

very similarly for B/A = 3 and 10. When the geometrical extension is increased the fall in the source functions corresponding to V_{rot} is large in the case of an atmosphere B/A = 10 than that of B/A = 3. This is so because we have chosen the same optical depth in both cases and therefore the density in the atmosphere with B/A = 10 is less than that in the atmosphere with B/A = 3. This effect is enhanced when the rotational velocities are increased.

In Figures 2a, b, c we have presented for B/A = 3 the line profiles observed at infinity for V_s = 0, 3 and 6 mtu respectively (see Paper I for details). In each of these figures we have also presented profiles for V_{rot} = 1, 4 and 8 mtu. In Figure (2a) we note that all profiles are symmetric. For V_{rot} = 1, we obtain a profile with a slight emission in the wings. However when V_{rot} is increased to 4 mtu the wings become broader whereas the

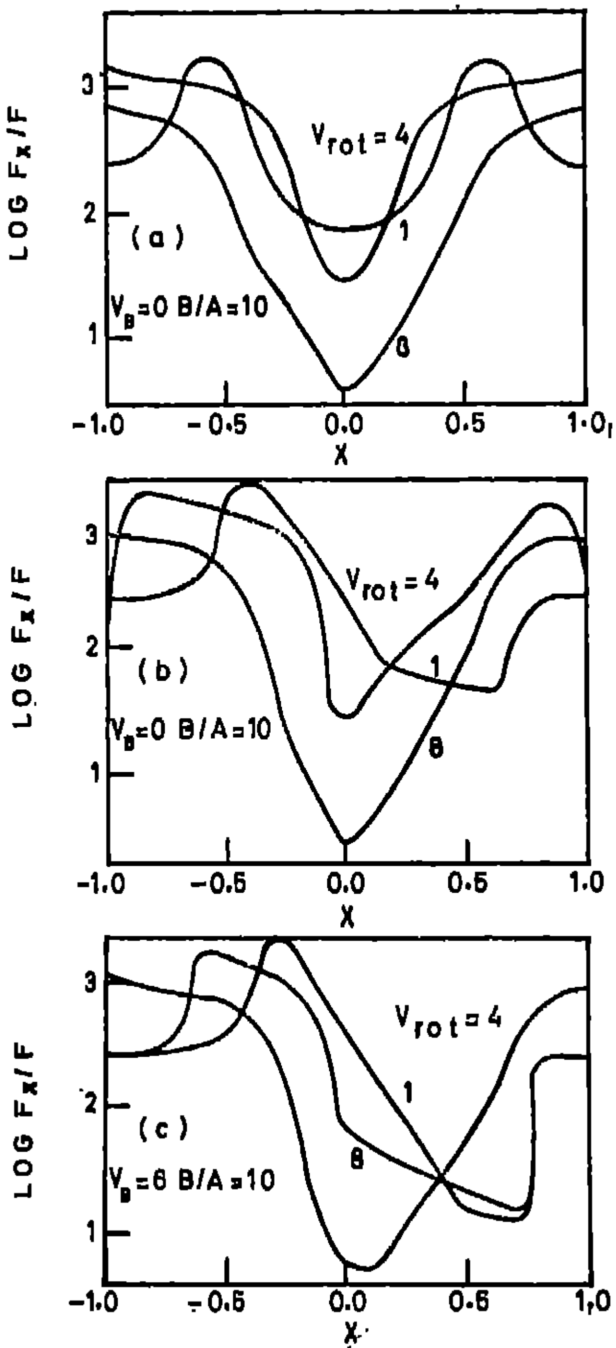

Fig. 3. Flux profiles F at infinity for B/A = 10.

core of the line becomes narrower. When V_{rot} is increased to 6 mtu the line becomes broader uniformly and emission in the wings disappear. When the expansion velocity increases we find a P Cygni type profile which becomes more prominant when $V_s = 6$ (see Figure 2c). In Figures 3a, b, c the flux profiles have been plotted for $B/A = 10$. These are generally similar to those given in Figures 2a, b, c.

M S received on 1st March, 1981.

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