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SIMULTANEOUS SOLUTION OF RADIATIVE TRANSFER EQUATION IN THE COMOVING FRAME AND THE STATISTICAL EQUILIBRIUM EQUATION WITH COMPLETE REDISTRIBUTION

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ABSTRACT

Complete redistribution with Voigt profile function has been employed in obtaining the simultaneous solution of line transfer and the statistical equilibrium equation for a non-LTE two level atom in an extended stellar atmosphere expanding with spherical symmetry. We have taken the geometrical extension of the atmosphere to be 3 and 10 times the stellar radius. We have also estimated the ratio of the number densities N_2/N_1 of the upper and lower levels assuming a velocity law in such a way that it always satisfies the equation of continuity. In the first iteration, we have set the upper level population N_2 equal to zero. In the subsequent iterations this level gets populated considerably although it is still smaller than the equilibrium values. We have set ϵ (the probability of photon destruction by collisional de-excitation) and β (the ratio of continuum to line absorption coefficients) equal to zero in all the cases. We note that velocities do not influence the population densities of the levels as much as the combination of geometrical extension and gas motions.

Key words: radiative transfer—statistical equilibrium equation—voigt function—complete redistribution—level population densities

1. Introduction

In solving the line transfer, it is usual to assume the absorption characteristics in the atmosphere. In addition, we assume the type of geometry, velocity laws, temperature distribution and other parameters. One must try to obtain a consistent solution so that the real physical situation can be described. In an earlier paper (Peraiah 1980), we have treated the radiative transfer equation simultaneously with the statistical equilibrium equation along with the Doppler profile for a resonance line in expanding media. It was found that irrespective of the starting values of population densities, the solution converged to a final unique one. One must take into account the hydrodynamic aspects also so that the final solution is obtained without parameters.

Here, we have introduced the Voigt profile both for emission and absorption of radiation in the line and to evaluate the simultaneous solution between the statistical equilibrium equation and line transfer in the comoving frame.

2. Computational Procedure and Discussion of the Results

We have employed Voigt profile given as

$$\phi(a, x) = \frac{a}{\pi}\int_{-\infty}^{+\infty}\frac{e^{-y^2}\,dy}{(x-y)^2+a^2} \tag{1}$$

where $\phi(a, x)$ is the absorption/emission profile, a is the damping constant ($=10^{-3}$) and x is the normalized frequency at which the emission and absorption is taking place. The quantity x is given by,

$$x = (\nu-\nu_0)/\Delta\nu_D \tag{2}$$

where ν_0, ν are the frequencies at the centre and at any point on the line and $\Delta\nu_s$ is the standard frequency interval. Normally the Doppler width is taken as

$$\Delta\nu_s = \frac{\nu_0}{c} v \tag{3}$$

where c is the velocity of light and v is the mean thermal velocity of the ions. We have calculated the absorption coefficient in the line by the formula,

$$K_s(r) = \frac{h\nu_0}{4\pi\Delta\nu_D}\left(B_{12}N_1(r) - B_{21}N_2(r)\right)\phi(a, x) \tag{4}$$

here B_{12} and B_{21} are the Einstein coefficients and $N_2(r)$ and $N_1(r)$ are the population densities of the upper and lower levels respectively. ν_0 corresponds to the hydrogen Lyman alpha line. These are obtained from the equation

$$\frac{N_1(r)}{N_2(r)} = \frac{A_{21} + C_{21} + \frac{1}{2}B_{21}\int_{-1}^{+1}\int_{-\infty}^{+\infty} dx\, d\mu\, I(\mu, r, x)\,\phi(a, x)}{C_{12} + \frac{1}{2}B_{12}\int_{-1}^{+1}\int_{-\infty}^{+\infty} dx\, d\mu\, I(\mu, r, x)\,\phi(a, x)} \tag{5}$$

A_{21} is the Einstein coefficient for spontaneous emission and C_{12} and C_{21} are the rates of collisional excitation and de-excitation respectively. They are given by (Jefferies 1968)

$$C_{12} \simeq 2.7 \times 10^{-10}\, a_0^{-1.68} \exp(-a_0)\, T^{-3/2} A_{21} \frac{g_2}{g_1}\left(\frac{I_H}{\chi_0}\right)^2 N_e \tag{6}$$

and

$$C_{21} = 2.7 \times 10^{-10}\, a_0^{-1.68}\, T^{-3/2} A_{21}\left(\frac{I_H}{\chi_0}\right)^2 N_e \tag{7}$$

where χ_0 is the excitation energy E_{12} and $a_0 = \chi_0/kT$, T being the temperature. I_H is the ionization potential of hydrogen and N_e is the electron density. We have considered hydrogen Lyman alpha line as a test case. $I(\mu, r, x)$ is the specific intensity of the ray at r making an angle $\cos^{-1}\mu$ with the radius vector. This is obtained from the line transfer in comoving frame given as,

$$\frac{\mu\,\partial I(x, \mu, r)}{\partial r} + \frac{1-\mu^2}{r}\frac{\partial I(x, \mu, r)}{\partial\mu} = K_L(r)\left[\beta + \phi(a, x)\right]\left[S(r) - I(r, \mu, x)\right] + \left\{(1-\mu^2)\frac{v(r)}{r} + \mu^2\frac{dv(r)}{dr}\right\}\frac{\partial I(x, \mu, r)}{\partial x} \tag{8}$$

and

$$-\mu\frac{\partial I(x, -\mu, r)}{\partial r} - \frac{1-\mu^2}{r}\frac{\partial I(x, -\mu, r)}{\partial\mu} = K_L\left[\beta + \phi(a, x)\right]\left[S(r) - I(x, -\mu, r)\right] + \left\{(1-\mu^2)\frac{v(r)}{r} + \mu^2\frac{dv(r)}{dr}\right\}\frac{\partial I(x, -\mu, r)}{\partial x} \tag{9}$$

here β is the ratio of continuum to line centre absorption per unit frequency interval. S (r) is the total source function given by

$$S(r) = \frac{\phi(a, x)}{\beta + \phi(a, x)} S_L(r) + \frac{\beta}{\beta + \phi(a, x)} S_c(r) \tag{10}$$

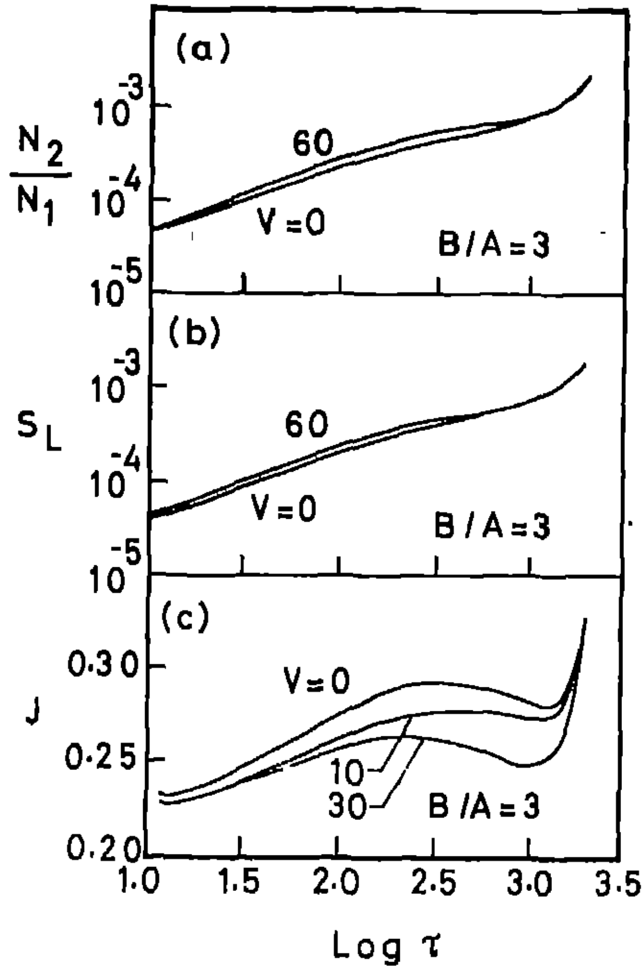

Fig. 1 (a) The ratios of level population N_2/N_1 are plotted versus the total optical depth. (b) The line source function S_L is given against total optical depth. (c) The total mean intensities are given against the total optical depth for B/A = 3.

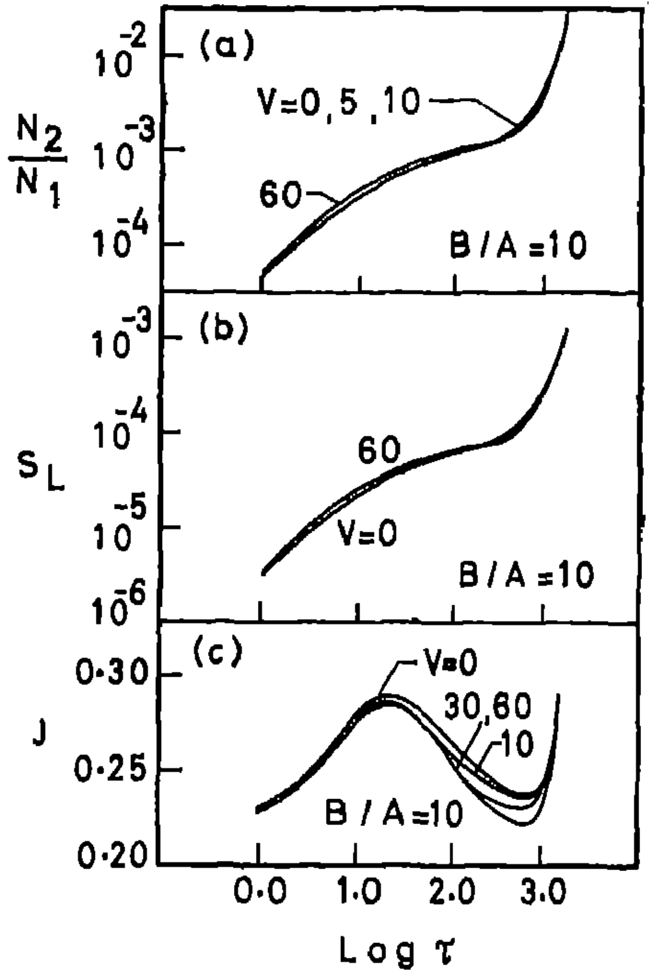

Fig. 2. Same as given in Fig. 1 with B/A = 10.

where $S_c(\tau)$ and $S_L(\tau)$ are the continuum and line source functions. As we are setting $\beta = 0$, equation (10) is reduced to

$$S(\tau) = S_L(\tau). \tag{11}$$

The line source function is calculated by the relation

$$S_L(\tau) = \frac{A_{21} N_2(\tau)}{B_{12} N_1(\tau) - B_{21} N_2(\tau)} \tag{12}$$

and also by the relation,

$$S_L(\tau) = (1-\epsilon)\int_{-1}^{+1}\int_{-\infty}^{+\infty} dx\, d\mu'\, I(x, \mu', \tau)\, \phi(a, x) + \epsilon B(\tau) \tag{13}$$

where $B(\tau)$ is the Planck function and ϵ is the probability per scatter that a photon is thermalized by collisional de-excitation. This parameter has been set equal to zero.

We have assumed a temperature of 30,000 K and an electron density of 10^{12} cm^{-3}. This will decide the number of neutral atoms $N(r) = (N_1(r) + N_2(r))$ through Saha-Boltzman equation. We have taken the inner radius of the atmosphere to be 10^{12} cm. The atmosphere is taken to be 3 and 10 times the inner radius. The velocity and the electron density are varied in such a way that they always satisfy the equation of continuity. The velocity is kept minimum at $\tau = \tau_{max}$, the inner radius A, and is maximum at $\tau = 0$, the outer radius B. Here the velocity increases with the geometrical thickness and decreases as the optical depth increases.

With the above assumptions, we have calculated the optical depth from equation (4) by putting $N_1(r) = N(r)$ and $N_2(r) = 0$. Then, the equation of transfer given in equations (8) and (9) is solved. This solution is used in obtaining new sets of ratio N_1/N_2 from equation (5). The second iteration can be started by calculating the optical depth from equation (4). This procedure is repeated until we reach an accuracy within 1 percent in N_2/N_1 in two successive iterations. Thus we have obtained a convergence with 2-3 iterations.

The results have been presented in Figures 1 and 2. In each figure, we have given the ratio N_2/N_1, S_L and the total mean intensities, J. In Figure 1a the ratio N_2/N_1 is plotted versus the total optical depth for $B/A = 3$. We note that the ratio increases as the optical depth increases and is at its maximum at the innermost boundary. We have used maximum velocities at B equal to 0, 5, 10, 30 and 60 mean thermal units. The results become graphically solvable only when the velocity at B reaches 60. The velocity gradients do not seem to influence the ratio N_2/N_1 very much. In Figure 1b, the line source function is described and its variation is quite similar to that of N_2/N_1. In Figure 1c, the mean intensities are given against τ. The mean intensities attain a maximum value inside the atmosphere. In Figures 2a, b, c, we have given the same quantities for $B/A = 10$ as those given in Figure 1. The extended geometry changes the ratio N_2/N_1, S_L and J much more rapidly.

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References

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