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Kinematical Distances to Open Star Clusters

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Abstract. Kinematical distances are estimated for six open star clusters. They agree fairly well with the photometric distances. The kinematical distances cannot, at present, be estimated better than the photometric distances. When more accurate proper motion measurements become available the kinematical distances will improve considerably and may then be used to calibrate the cosmic distance scale.

Key words: open clusters—distances—kinematics

1. Introduction

An accurate estimate of distances to open star clusters is required for many astrophysical investigations. One application is in tracing the spiral arms of the Galaxy because these clusters can be detected to large distances. It is therefore of interest to evaluate their distances by as many different independent methods as possible. The distances to open clusters have generally been estimated by using methods based upon main sequence fitting. Other, less accurate, methods are: the use of variables, stellar evolutionary gaps in the photometric sequences of open clusters, *etc.* In these methods, one must consider reddening/extinction and metallicity corrections, as well as the photometric calibration (Lyngå 1980).

The current best photometric distances of open clusters are correct to within 20–30 per cent. Even so, it would clearly be useful to check them with a totally independent technique when possible, as has been done here. We discuss a method of open cluster distance estimation that is based on the observable kinematical parameters, namely, proper motions and radial velocities of open cluster members. The method is, therefore, independent of interstellar extinction and metallicity corrections as well as of photometric calibration but requires precise measurements of the above-mentioned kinematical parameters which are now becoming available. This technique has been used successfully for the distance estimates of globular clusters (Cudworth & Peterson 1987 and references therein). The method discussed here can, in principle, also be used for calibrating the cosmic distance scale.

2. The Method

If the distribution of the stellar velocities in a cluster could be assumed to be isotropic, the resulting velocity dispersions along three orthogonal axes would be identical. Representing the proper motions along two orthogonal axes (x, y) in the plane of the sky by μ_x and μ_y and the radial velocity along the line of sight, *i. e.* perpendicular to the plane of the sky, by v, one could write

$$\sigma_v = D\,\sigma_{\mu x} = D\,\sigma_{\mu y}, \tag{1}$$

where D is the distance to the cluster, and σ_v, $\sigma_{\mu x}$ and $\sigma_{\mu y}$ are the intrinsic dispersions in v, μ_x and μ_y respectively. The distance D to the cluster can thus be written as

$$D = \sigma_v/\sigma_\mu$$

where $\sigma_\mu = \sigma_{\mu x} = \sigma_{\mu y}$. Expressing the measured quantities in units commonly used, one can write

$$D = 2.1 \times 10^{-2}\ \sigma_v/\sigma_\mu, \tag{2}$$

where D is in kpc, σ_v in $\mathrm{km\,s^{-1}}$ and σ_μ in arcsec/century.

Before applying the method outlined above, it is essential to discuss the question of velocity isotropy in open star clusters. Theoretical predictions about this are subject to a number of uncertainities. As a consequence of dynamical evolution, it is expected that the velocity dispersion would vary inversely as the square root of the stellar mass. But open star clusters are not isolated systems and the effects of encounters with interstellar clouds, tidal forces, mass loss from the massive stars, and vestiges of its initial formation conditions, *etc*., could also be present in the velocity distributions within these objects. In the interior of an open star cluster, relaxation time is short enough to establish isotropy. Galactic tidal forces randomize the velocity directions of outer low-mass stars ejected in eccentric orbits from the cluster centre due to dynamical evolution and consequently, yield a flattened global velocity-mass relation (*cf.* Prata 1971; Mathieu 1983). Under these circumstances, the mass dependence of the velocity dispersion expected from the dynamical evolution of an open star cluster may not be observed. Analyses of the proper motion data by McNamara & Sanders (1977) for M 11; by McNamara & Sekiguchi (1986) for M 35; and by Sagar & Bhatt (1988) for NGC 2287, 2516, IC 2391, NGC 2669, 3532, 4103, 4755 and 5662 open star clusters have shown that $\sigma_{\mu x} \simeq \sigma_{\mu y}$ supporting the basic assumption of the present method. Therefore, the assumption of isotropic stellar velocity dispersions in open star clusters could be justified in general.

3. Distance estimates

Six open clusters for which measurements of both proper motion components and accurate radial velocities of member stars are available are the subject of the present analysis. Ideally, all cluster members should have proper motion and radial velocity measurements together with estimates of the associated errors so that their intrinsic dispersions can be derived. This is because both the intrinsic dispersion and the dispersion due to errors contribute to the observed dispersion (McNamara & Sekiguchi 1986). The distance to the cluster can then be evaluated by using Equation (2). In practice, however, the number of members with proper motion measurements is often considerably larger than of those with radial velocity determinations. Therefore, usually the intrinsic dispersion in radial velocity is determined from available data, which in turn is used with the intrinsic proper motion dispersion estimated from the larger proper motion sample to derive the distance.

3.1 *Estimation of Intrinsic Dispersion*

The procedure given by Jones (1970) is used to estimate here the intrinsic dispersions in proper motion components and radial velocities. The observed proper motion dispersion in one coordinate can be written as

$$\sigma_0^2 = \frac{1}{n-1} \sum_{i=1}^{n} \mu_i^2$$

where μ_i represents the proper motion of star i relative to the mean cluster motion and n is the sample size. Assuming that the proper motion and error distributions are gaussian, one has for the true dispersion σ_κ as

$$\sigma_I^2 = \sigma_0^2 - \frac{1}{n} \sum_{i=1}^{n} \xi_i^2 \quad (3)$$

where ξ_i is the mean error of the proper motion of the ith star. The error in σ_κ is

$$\Delta \sigma_I = \left\{ \frac{1}{4\sigma_I^2} [\varepsilon^2(\sigma_0^2) + \varepsilon^2(\sigma_m^2)] \right\}^{1/2}, \quad (4)$$

with

$$\varepsilon(\sigma_0^2) = \sigma_0^2 \left(\frac{2}{n} \right)^{1/2},$$

and

$$\varepsilon(\sigma_m^2) = \frac{\sqrt{2}}{n} \left(\sum_{i=1}^{n} \xi_i^4 / n_i \right)^{1/2},$$

where n_i is the number of plate pairs on which star i appears.

3.2 *Intrinsic Dispersion in Radial Velocities*

The precise radial velocities (error $\leqslant 1$ km s^{-1}) of members in NGC 2682 and 6705 are given by Mathieu *et al.* (1986) and in NGC 2420 by Liu & Janes (1987). Stars showing no sign of radial velocity variation as well as having proper motion, radial velocity, and *UBV* photometric data compatible for cluster membership are used to estimate the intrinsic dispersion in radial velocities. Gieseking (1981) for NGC 3532 and Mathieu (1986) for NGC 1976 and 2264 have given the values of σ_v. The number of stars (n) used for this purpose and the values of σ_v are listed in Table 1.

3.3 *Dispersion in Proper Motions*

As the clusters under study do not all have the same quality of proper motion data, the error treatments for the estimation of their intrinsic dispersion σ_μ differ. For NGC 3532 and 6705, proper motion components with errors are available, and hence σ_μ is estimated using stars with proper motion membership probability greater than or equal to 70 per cent and having *UBV* data compatible with cluster membership. The assumption of $\sigma_{\mu x} \simeq \sigma_{\mu y}$, is satisfied and their weighted average is considered as σ_μ. For NGC 1976, we adopt σ_μ from McNamara (1976) while for NGC 2682 from McNamara

Table 1. Intrinsic dispersion in proper motions and radial velocities for six open clusters.

		Intrinsic dispersion in					
		Proper motions			Radial velocities		
Cluster name	IAU Designation	σ_μ (arcsec/century)	n	Source	σ_v (km s^{-1})	n	Source
NGC 6705 (M 11)	C 1848–063	0.019 ± 0.003	562	Mathieu (1984, personal communication)	1.4 ± 0.2	25	Mathieu *et al.* (1986)
NGC 3532	C 1104–584	0.08 ± 0.01	410	King (1978)	1.5 ± 0.3	84	Gieseking (1981)
NGC 1976 (Trapezium)	C 0532–054	0.10 ± 0.02	76	McNamara (1976)	2.0	—	Mathieu (1986)
NGC 2682 (M 67)	C 0847 + 120	0.024 ± 017	148	McNamara & Sanders(1978)	0.82 ± 0.07	85	Mathieu *et al.* (1986)
NGC 2264	C 0638 + 099	0.099 ± 0.087	48	Vasilevskis, Sanders & Blaz (1965); Zhao *et al.* (1985)	2.5	—	Mathieu (1986)
NGC 2420	C 0735 + 216	0.013 ± 0.069	22	Altena & Jones (1970)	1.4 ± 0.4	6	Liu & Janes (1987)

& Sanders (1978), where it has been assumed that both proper motion components have the same observed dispersion which has been estimated using the maximum likelihood method of membership estimation (Sanders 1971). The σ_μ for NGC 2264 and 2420 are estimated using the technique given by McNamara & Sanders (1977). For these two clusters, mean errors in both proper motion components are given for groups of stars and the members of the groups with better estimates are used in the analysis. Observed proper motion dispersion estimated using maximum likelihood method is taken from Zhao *et al.* (1985) for NGC 2264 and from Altena & Jones (1970) for NGC 2420. The intrinsic dispersion in proper motion σ_μ and the number of stars (n) used are listed in Table 1.

3.4 *Derived Kinematical Distances and Associated Errors*

Having evaluated the intrinsic dispersions in the radial velocities σ_v and in proper motions σ_μ, use is made of Equation (2) to derive the kinematical distances. The uncertainty in these estimates can be evaluated as:

$$\left(\frac{\Delta D}{D}\right)^2 = \left(\frac{\Delta\sigma_\mu}{\sigma_\mu}\right)^2 + \left(\frac{\Delta\sigma_v}{\sigma_v}\right)^2 \qquad (5)$$

where ΔD, $\Delta\sigma_\mu$ and $\Delta\sigma_v$ are the errors in distance D, proper motion dispersion σ_μ and radial velocity dispersion σ_v respectively. The derived kinematical distances and their uncertainities are given in Table 2. The percentage of error contributed to the kinematical distances due to uncertainities in the dispersions of proper motions and radial velocities are also listed in the table. It should be noted that presently the errors due to uncertainities in the proper motion dispersions are generally larger than those in the radial velocity dispersions.

In the present work, the proper motion data used to derive σ_μ have relatively larger range in stellar mass compared to the data used for the estimation of velocity dispersion. This will not introduce any systematic error in the derived kinematical distances, if there is no dependence of velocity dispersion on stellar mass as pointed out in Section 2. To verify this statement, we estimated σ_μ using the stars in NGC 6705 having radial velocity measurements. All stars are giants and their V magnitude varies from 11.0 to 12.0. Consequently, their mass range is quite narrow. The value of σ_μ comes out to be 0.016 ± 0.005, which is almost equal to the value derived using relatively wider mass range corresponding to $V = 11.0-16.0$ (see Table 1).

However, in the cases where the velocity distribution is not isotropic and dispersion in velocity depends upon stellar mass, the precise radial velocity and proper motion data for the same narrow stellar mass range should be used to derive the kinematical distances. Otherwise, a systematic error will be introduced by the present method. Also the method is applicable only if the cluster is non-rotating. When rotation is present, or even suspected, it is probably best to use stars located in the central region of the cluster because the effect of rotation is almost negligible there (Prata 1971).

3.5 *Comparison with Photometric Distances*

The photometric distances of the open star clusters under discussion are given in Table 2. Their errors listed in the table are mainly due to the inaccuracies in fitting the

Table 2. Kinematical and photometric distances for the open clusters studied here. For NGC 2240, though the mean value of kinematical distance agrees with the photometric one, the formal error in it is very large.

Cluster	Kinematical distances (pc)	Percentage error due to uncertainties in the dispersion of		Photometric distances		
		Proper motions	Radial velocities	Value (pc)	Percentage error	Source
NGC 6705	1550 ± 470	16	14	1900 ± 270	14	Solomon & McNamara (1980)
NGC 3532	400 ± 130	12	20	490 ± 160	33	Fernandez & Salgado (1980)
NGC 1976*	420 ± 90	20	—	472 ± 45	10	Walker (1969)
NGC 2682	720 ± 570	71	9	830 ± 100	12	Nissen, Twarog & Crawford (1987)
NGC 2264*	530 ± 470	88	—	790 ± 75	10	Sagar & Joshi (1983)
NGC 2240	2260:	:	29	1910 ± 190	10	McClure, Forrester & Gibson (1974)

* The error in kinematical distance is only due to the uncertainity in proper motion dispersion.

Hyades sequence to the colour–magnitude diagram of the clusters. Other important sources of error are: ~4 per cent due to the uncertainity in the Hyades distance modulus (Hanson 1980), ~2 per cent due to errors in photoelectric quality photometry; ~5–10 per cent because of using an average value of $R = (A_v/E(B-V)) = 3.1$; and ~5–20 per cent due to not accounting for the variation in metallicity of the open clusters relative to Hyades metallicity (Lyngå 1980; Nissen 1980). Therefore, photometric distances of open star clusters cannot at present be estimated better than ~20 per cent for nearby clusters and ~30 per cent for distant open clusters by fitting the Hyades sequence to the colour–magnitude diagram of the clusters.

A comparison of the kinematical distances with the photometric distances shows a good agreement between them (see Table 2). It would seem from Table 2 that generally the kinematical distances are smaller than the photometric ones but the errors are still quite large. The reality of this can be checked only when more accurate dispersions in proper motions and radial velocities become available in future.

It is unlikely that the current methods will improve the accuracy of photometric distances significantly. On the other hand, it is expected that the accuracy of kinematical distances will improve considerably when more accurate proper motion measurements from the HIPPARCOS space mission or from the Hubble Space Telescope or from ground-based observations become available in future. The error $\Delta\sigma_v$ and $\Delta\sigma_\mu$ are function of σ_I, ξ_i, n and n_i (see Equation 4). Radial velocities of accuracies better than presently available (error $< 1 \text{ km s}^{-1}$) are unlikely to be attainable in the immediate future. However, a larger sample size will improve the accuracy of σ_v; but accuracies better than ~10 per cent in σ_v may not be achieved in the near future (*cf.* Latham 1987). In the case of proper motion, Hubble Space Telescope and HIPPARCOS space mission are expected to improve the measuring accuracy at least by a factor of $\sim 10^2 - 10^3$, which can result in an accuracy of few per cent in σ_μ. For example, in the case of NGC 3532 the proper motion data with mean error of 0.13 arcsec/century (King 1978) yields an accuracy of ~12 per cent in σ_μ (see Table 2). If only measuring errors are improved by a factor of 10 and other parameters are kept constant, an accuracy of ~5 per cent will be achieved in σ_μ. In future, therefore, kinematical distances may be estimated with accuracies better than the photometric distances. As the distances based on the present method are independent of any standard candles, they can be used to calibrate the cosmic distance scale in future.

4. Conclusions

Kinematical distances for six open clusters have been estimated which are in good agreement with the photometric distances. At present the kinematical distances cannot generally be estimated with accuracies better than the photometric distances. It is expected that kinematical distances will improve considerably in future when more accurate proper motion measurements become available. As the method for estimating distances used here is free of the effects of interstellar extinction and other calibrations, it has potential application in the calibration of the cosmic distance scale.

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