Kodaikanal Obs. Bull. (1986) 6

TYPE-I SOLAR RADIO BURSTS

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Abstract

A brief review of the recent work done on Type-I radio emission in the Indian Institute of Astrophysics is presented. The most plausible low frequency turbulence needed to generate the type-I bursts is shown to be ion-sound turbulence. A method is presented to evaluate the coronal magnetic field using type-I radio emission data.

1. Introduction

The solar corona is an inhomogenous collisionless hot extensive plasma. It is permeated and structured by the solar magnetic field. The solar activity makes certain regions of the corona to have enhanced density, temperature and magnetic field. Such a magnetic plasma has a variety of collective degrees of freedom, viz., the various types of plasma waves. If a disturbance due to solar activity propagates through the corona, it can be detected by the radio signataures produced by it. The disturbances, in general, can be classified into two categories: (i) charged particle beams, (ii) shock waves. The total plasma system consisting of the ambient plasma and the disturbance is a non-equilibrium system and the system relaxes back to equilibrium by releasing the free energy in the disturbance in the form of plasma instabilities. The particular type of instability that can be excited depends upon the characteristics of the disturbance as well as the ambient plasma that provides the base modes. If the instability is electro static, it cannot propagate outside the coronal plasma to be detected on earth. Then, one has to deal with the mechanisms by which the electrostatic waves are converted into radiation. The electrostatic instabilities are dominant generally in the outer corona (f < 300 Mhz) because the ratio of plasma frequency to the electron gyro frequency exceeds one. When the reverse inequality holds, one normally gets electromagnetic instability which is a direct source of radiation. The most common types of the radio bursts that are produced by these disturbances are

$$\left.\begin{array}{l}\text{Type I}\\ \text{Type II}\end{array}\right\}\ \text{due to shock waves}$$

$$\left.\begin{array}{l}\text{Type III}\\ \text{Type V}\end{array}\right\}\ \text{due to e-beams}$$

Type IV : due to trapped electrons.

In the present work, we deal with the shock waves responsible for the generation of type-I bursts.

The correct understanding of the detected radiation will lead to the correct understanding of the coronal dynamics. If the mechanism of emission is fairly clear then one can draw definite conclusions about the parameters of the corona.

In this review we discuss the particular type of waves which are most plausible under coronal conditions to generate the type-I emission and what information one can get regarding coronal magnetic fields if our understanding of the type-I emission is clear.

2. Theory of Type-I Emission

The type-I emission or Noise storm originates from restricted areas in active regions. This is the only type of non-thermal radio emission that is seldom associated with a solar flare. The emission consists of a continuum component and a burst component. The bursts, sometimes, cluster together to form a 'chain' and the chain has got a particular frequency and time extent. The slope of the chain in the dynamic spectrum shows that the agency responsible for generation of these bursts must be moving slowly in the corona and one can immediately identify that this is a shock wave. The absence of second harmonic radiation and the narrow band character of the type-I bursts indicate that the shock wave must be only slightly superalfvenic. Zaitsev and Fomichev (1973) suggested that the type-I bursts could be caused by perpendicular shocks, without identifying the driving agency. They also assumed shocks of strengths similar to type-II shocks which are flare associated. But type-I bursts are not flare associated and hence the shocks may not be so strong. The shocks causing type-I bursts, therefore correspond to evolving active regions in a quasi-stationary corona where the magnetic field and density are not conducive for flares. Zaitsev and Fomichev (1973) assumed that the plasma frequency (ω_{pe}) is much smaller than the gyrofrequency (ω_{ce}) which is not a realistic approximation. The gyrofrequency can exceed the plasma frequency probably only at frequencies much higher than 300 Mhz which is in the cut off range for type-I bursts (Melrose, 1982). Wentzel (1981) proposed that the interaction of upper hybrid (UH) waves generated by the loss cone distribution of electrons trapped in the closed magnetic fields with the lower hybrid (LH) waves generated by a shock wave gives rise to type-I emission. Spicer et al., (1981) identified the newly emerging flux as the agency which drives the shocks perpendicular to the magnetic field. They also estimate the turbulence level of the LH waves excited by the shocks. The LH turbulence stochastically accelerates the electrons, which in turn generate UH waves, once they develop a loss cone. The interaction between UH and LH waves gives rise to the radiation. The chain is produced when the shock moves out in the corona producing bursts at various places where the resonance conditions are satisfied. The bursts could be generated intermittently due to the random weakening or strengthening of the shock (Spicer et al. 1981) because of fluctuations in the corona (Yakobalev, 1980) or due to the fact that emissions occur at those places in the corona where the plasma frequency matches with integral multiples of the UH frequency during the lifetime of the shock (Wentzel, 1981).

In the ion-acoustic wave model of Benz and Wentzel (1981), ion sound waves are generated by parallel currents due to coronal evolution. But then the slope of the chains in the dynamic spectrum cannot be explained by such a model. In the emerging flux theory of Spicer et al. (1981), the perpendicular currents due to shock gradients feed the LH waves. Keeping in mind the attractive features of ion-sound waves, we explored the possibility of ion-sound waves generated by shock gradients and their interaction with the UH waves to produce the type-I emission.

3. Why Ion-Sound Turbulence is a Better Alternative

We argue below that the ion-sound turbulence generated by the shock gradients is a better alternative as compared to the LH turbulence based on the energy density of the turbulence, generation of hot particles leading to the excitation of UH waves and the wave resonance.

3.1 Energy Densities

Using quasilinear theory of ion-sound waves generated by a weak shock wave, one can estimate the saturation energy density W_s of the ion-sound waves as (Gopalaswamy and Thejappa, 1985)

$$\frac{W_s}{nT_e} = \left(\frac{m_i}{m_e}\right)^{1/2} \left(\frac{\Delta B}{B_0}\right) \left(\frac{C_s V_A}{V_e^2}\right) \left(\frac{M_A^2-1}{M_A}\right) \left(\frac{2}{2-\beta}\right) \left[1+\ln \frac{\omega_{pe}^2}{\omega_{ce}^2}\right]^{-1} \quad (1)$$

where

nT_e - thermal energy density
n, T_e - electron density and temperature of the corona
m_i, m_e - ion, electron mass
$B_0, \Delta B$ - background magnetic field and its, jump across the shock
C_s, V_A, V_e - ion sound, Alfven and electron thermal velocities
M_A - Alfvenic mach number of the shock
β - plasma beta of the corona, and
ω_{pe}, ω_{ce} - electron plasma and cyclotron frequencies.

Under similar conditions the energy density W_{LH} of LH waves has been derived as (Spicer et al. (1981))

$$\frac{W_{LH}}{nT_e} = \left(\frac{\omega_{ce}^2}{\omega_{pe}^2}\right) \left(\frac{V_A}{C_s}\right) \left(\frac{\Delta B}{B_0}\right) \left(\frac{M_A^2-1}{M_A}\right) \left(\frac{m_e}{m_i}\right)^{1/2} \quad (2)$$

Comparison of (1) and (2) gives the ratio of the two energy densities.

$$\frac{W_{LH}}{W_s} = \frac{\omega_{ce}^2}{\omega_{pe}^2} \left(1-\frac{1}{2}\beta\right) \left(1+\ln \frac{\omega_{pe}^2}{\omega_{ce}^2}\right) \quad (3)$$

In those regions of the solar corona where the type-I emission is predominant, β, $\omega_{ce}^2/\omega_{pe}^2 \ll 1$. This is because the magnetic field is small (to be shown later, see also Gopalswamy, et al. (1985)) and is of the order of 1 G. Then for 100 MHz plasma level, one gets $\omega_{ce}^2/\omega_{pe}^2 \sim 2.8 \times 10^{-2}$ so that equation (3) implies that $W_{LH}/W_S \sim 4.5 \times 10^{-3}$ and hence the ion sound turbulence is dominant. The importance of this statement will be clear when we look at the wave-wave interaction for type-I emission. For the UH waves to provide adequate brightness temperature, the low frequency waves should have an energy density,

$$\frac{W^\sigma}{nT_e} \gtrsim \frac{6\sqrt{3}}{\pi} \frac{V_e c}{\omega_{pe} L_N V_\phi^\sigma} \quad (4)$$

where σ represents any suitable low frequency waves that interact with the UH waves to produce the radiation; L_N is the scale height of coronal electron density variation and V_ϕ^σ is the phase velocity of the low frequency waves. For a coronal plasma frequency of 100 MHz, $L_N \sim 10^{10}$ cm. For typical phase velocities of the low frequency waves in the million degree corona, one gets from (4),

$$\frac{W^\sigma}{nT_e} \gtrsim 1.3 \times 10^{-6} \quad (5)$$

This condition is satisfied only marginally by the LH waves whereas the energy density of IS waves is much larger than the limit (5).

3.2 Generation of Hot Electrons

Now, the low frequency turbulence should generate energetic electrons. The energetic electrons should develop a loss cone distribution to generate the necessary high frequency waves. The LH waves can stochastically accelerate the electrons to high energies while the IS waves cannot (Lampe and Papadopoulos, 1977; Kaplan et al. 1974). But there is another efficient process by which the IS waves can produce energetic particles with a loss cone distribution in order to generate UH waves. Whistler waves and IS waves have the same range of frequencies. Hence, the IS waves can get converted into whistler waves through non-linear scattering from the ions and electrons. The characteristic time of conversion is

$$\tau_{\text{Scatter}} = \frac{20}{\beta}\left(\frac{W_s}{nT_e}\right)^{-1} \omega_{ce}^{-1} ,$$

where $\beta = 8\pi nT_e/B_0^2$ and ω_{ce} is the electron cyclotron frequency.

These whistlers have electric field normal to the magnetic field and hence increase the transverse energy of the electrons as they are absorbed by the electrons. Once the transverse velocity of the particle exceeds certain threshold value determined by the mirror ratio, the electrons are trapped. The characteristic time over which this trapping occurs is

$$\tau_{\text{heat}} = \frac{6}{\pi}\left(\frac{\omega_{pe}}{\omega_{ce}}\right)\left(\frac{V_h}{V_e}\right)^2\left(\frac{W_w}{nT_e}\right)^{-1} \omega_{ce}^{-1} ,$$

where W_w is the energy density of whistler waves. For a V_h (velocity of the heated electron due to whistler absorption) of ~7 V_e and $W_\omega \sim 0.5\ W_s$ one gets $\tau_{\text{scatter}} \sim 0.01$s and $\tau_{\text{heat}} \sim 0.5$ s. Both these time scales are well within the collisional damping time (Kaplan and Tsytovich 1973).

$$\tau_{\text{coll}} = \left(\frac{T_i}{T_e}\right)^{3/2} \frac{N_D}{\omega_{pi}} ,$$

where T_i, ω_{pi}-ion temperature and ion plasma frequency, N_D-debye number $= n\lambda_e^3$, λ_e being electron debye radius. Hence the ion sound turbulence provides an alternative mechanism to produce anisotropic distribution of energetic electrons.

3.3 Resonance Condition

For the efficient interaction of the high and low frequency waves, the following resonance conditions should be satisfied:

$$\underset{\sim}{k}_\ell + \underset{\sim}{k}_\sigma = \underset{\sim}{k}_t , \tag{6}$$

$$\omega_t + \omega_\sigma = \omega_t , \tag{7}$$

where $(\underset{\sim}{k},\omega)$ are the wave number and frequency, and ℓ, σ, t represent the UH, low frequency and transverse waves respectively. Since $k_t \ll k_i$, k_σ one needs $k_i \sim k_\sigma$.

For maximum growing modes (Wentzel 1981),

$$k_\ell \simeq 2k_e\left(\frac{V_e}{V_h}\right) , \tag{8}$$

$$k_{LH} \lesssim k_e \left(\frac{\omega_{ce}}{\omega_{pe}}\right) \tag{9}$$

and for the ion sound waves,

$$k_s = k_e \left(\frac{\omega_{ce}}{\omega_{pe}}\right) \quad \text{to} \quad k_e \tag{10}$$

Since $\omega_{ce}/\omega_{pe} \ll 1$ for the coronal plasma level at 100 MHz, we see that there is a better overlap in the k-space in the case of IS waves compared to the LH waves. (9) demands that V_h must be atleast 20 V_e to satisfy the resonance condition while moderate electron heating is sufficient in the case of ion sound wave because of its wider range of maximum growth.

It is clear from the above discussions that the IS turbulence is more likely candidate for the generation of type-I bursts in the solar corona. There is one comment in order. Though one have compared the LH and IS turbulence under identical conditions, we have assumed non-isothermality. But the solar corona is usually isothermal in which case the ion-sound waves are heavily damped. But if one takes a closer look at the instabilities generated in the shock front one can come to the conclusion that the isothermality will be broken locally. It has been pointed out (Galeev, 1976; Tidman and Krall, 1971) that a variety of instabilities are excited at the shock front and most of them essentially heat the electrons and quenched at the initial portion of the shock front itself. Deeper into the shock, the electron temperature exceeds the ion temperature and the ion sound waves grow and dominate the structure of the shock waves, by limiting the perpendicular current quasilinearly. The Buneman instability heats the electrons in the steep initial portion of the shock front where the electron drift velocity is of the order of electron thermal velocity. Deeper into the shock, the drift velocity falls below the thermal velocity, quenching the Buneman instability, but exciting ion-sound instability as now the electron temperature exceeds the ion temperature.

4. Coronal Magnetic Fields

4.1 The Method

The notable feature of the type-I chains are

(i) the slow frequency drift rate, and
(ii) the extremely narrow bandwidth.

These two characteristics are observable and in what follows we deduce the coronal magnetic field using these as input in our calculation. It is to be noted that our calculation gives values of the magnetic field for the corona above mild active region because the type-I emission is not correlated with flares. Moreover, the formula we derive applies to that part of the corona which is type-I active. This can probably be extrapolated into other regions provided the corona is free from flares. We assume (i) the emission is at local plasma frequency (the gyrofrequency is much smaller than the plasma frequency), (ii) the density jump across the shock remains almost constant throughout the life time of the shock, (iii) the coronal temperature remains constant throughout the region of occurrence of type-I chains.

The crucial relation that describes the shock jumps is the Rankine-Hugoniot relation which relates the density, velocity or magnetic field jump across the shock to the Alfven and ion-sound velocities in the ambient coronal plasma and the upstream shock velocity. The Alfven velocity in the ambient plasma gives the magnetic field of the corresponding coronal layer. The ion-sound velocity is given by the coronal temperature. Since the corona is isothermal with a million degree temperature, the sound velocity is fixed. The shock velocity is determined from the drift rate of the type-I chain assuming a particular coronal density model. It is assumed that the emission is taking place at plasma frequency. Since the coronal density is proportional to the square of the plasma frequency, the shock veloicity is determined once the coronal density gradient is chosen. Thus, the relative density jump $\Delta n/n$, obtained from the relative bandwidth the ion sound velocity C_s (from the coronal temperature) and the shock velocity V_s (from the drift rate) are known in the modified form of the Rankine-Hugoniot relation:

$$\frac{V_A^2}{V_1^2} = \frac{8\left(1-\frac{\Delta n}{n}\right) - 2\left(1-\frac{\Delta n}{n}\right)\left(1+5\frac{C_s^2}{V_A^2}\right)}{1+5\left(1-\frac{\Delta n}{n}\right)} \tag{11}$$

Hence the Alfven velocity V_A and the magnetic field B corresponding to the layer of density n is determined.

The type-I chain data were collected from the literature which correspond to the frequency range 300 MHz to 40 MHz. We have discarded those data which gave negative values for the right hand side of equation (11) because the left hand side is positive.

Though the range of occurrence of type-I emission is from about 300 MHz down to about 40 MHz, each chain occurs only over a very small frequency range which is evident from their observed small bandwidth. Translated into distance over which a particular shock (corresponding to a particular chain) exists radiating, this distance is a tiny fraction of the scale length of coronal density variation. Therefore, the density jump across the shock over the life time of the shock remains approximately constant. The magnetic field estimation from each chain, therefore, gives the value corresponding to the central point of the distance the shock travels during its life time.

To study the influence of the density model assumed on the estimated values of the magnetic field, we considered two cases, corresponding to x=2 and x=4 in the following formula

$$n(\rho) = 4.2x\ 10^{4+4.32/\rho} \tag{12}$$

This formula is due to Newkirk (1967) for plasma above active regions x=2 corresponds to mild active regions and x=4 corresponds to active regions with fast changes. Actually, this value of x can be anywhere between 2 and 5 for any particular active region. For the sake of comparison, we followed the method ($V=V_A$) used by Wild and Tlamicha (1964) to determine the magnetic field corresponding to both density models.

4.2 Results

The calculated magnetic fields using our method ($V_S > V_A$) and that of Wild and Tlamicha (1964) are presented in Table I. The corresponding coronal height is also found in the table. Notice that a particular frequency corresponds to different layers in the corona for different density models. It is clearly found that the magnetic field values obtained by $V_S > V_A$ method is always less than the values obtained from $V_S=V_A$ method.

The least square fits of the tabulated values are given in figures (1) and (2) for x=2 and x=4 models respectively. Also plotted in the same figures is the empirical curve obtained by Dulk and MacLean (1978):

$$B = 0.5(\rho-1)^{-1.5} \tag{13}$$

The empirical formulae are

$$\left.\begin{array}{ll} B = 0.6(\rho-1)^{-0\cdot94}, & V_S = V_A \\ B = 0.41(\rho-1)^{-0\cdot82}, & V_S > V_A \end{array}\right\} \quad x = 2 \qquad (14)$$

$$\left.\begin{array}{ll} B = 0.9(\rho-1)^{-0\cdot92}, & V_S = V_A \\ B = 0.7(\rho-1)^{-1\cdot1}, & V_S > V_A \end{array}\right\} \quad x = 4 \qquad (15)$$

In Fig.(3) we have drawn the curves obtained by $V_S > V_A$ method for $x=2$ and $x=4$ density models. Fig.(3) shows that the values obtained by $V_S = V_A$ method are larger than both $V_S > V_A$ method and the values obtained by Dulk and MacLean (1978). In fig.(2) values obtained by all the methods tend to become closer. This probably may be because we have assumed more density ($x=4$) which might not actually present in the mild active regions. In fact, we should remember that the empirical formula of Dulk and McLean (1978) is derived from data of flare associated bursts. From Fig.(3) it is clear that magnetic field is enhanced when the density is enhanced.

5. . Conclusions

In this review we have discussed two aspects of type-I solar radio emission. The first one deals with the driving agency of the shock responsible for generating type-I

Table I

The estimated magnetic fields for various cases

Sl. No.	Fre-quency (MHz)	ρ	x=2 B(G) V > VA	B(G) V=VA	ρ	X=4 B(G) V>VA	B(G) V=VA	References
1	250	1.09	7.510	8.444	1.177	8.960	10.02	De Groot (1966)
2	244.5	1.093	2.810	3.836	1.183	3.750	4.493	Karlicky and Jiricka (1982)
3	230	1.108	5.001	5.581	1.201	6.020	6.645	De Groot (1976)
4	219	1.121	2.630	3.157	1.215	3.250	3.766	De Groot (1976)
5	197.5	1.147	1.410	2.298	1.247	1.820	2.710	Wild and Tlamicha (1964)
6	158.5	1.208	0.663	1.347	1.319	0.992	1.605	Karlicky and Jiricka (1982)
7	150	1.225	1.060	1.594	1.339	1.490	1.932	Elgaroy and Ugland (1970)
8	109	1.329	1.350	1.607	1.465	1.730	1.980	Tlamicha (1982)
9	68	1.520	0.613	0.843	1.700	0.525	0.620	Aurass et al (1982)
10	60.5	1.578	0.168	0.820	1.772	0.830	1.036	Wild and Tlamicha (1964)
11	60	1.582	0.369	0.538	1.780	0.824	0.950	Aubier et al (1978)
12	55	1.627	0.752	0.993	1.834	.990	1.263	Wild and Tlamicha (1964)
13	54.5	1.632	0.813	1.106	1.840	1.070	1.409	-do-
14	52.25	1.654	1.125	1.222	1.869	1.460	1.558	-do-
15	52.0	1.657	0.403	0.621	0.873	0.570	0.793	-do-
16	51.75	1.660	0.340	0.602	1.876	0.497	0.763	-do-
17	50.0	1.679	1.090	2.090	1.901	1.402	2.577	-do-
18	47	1.715	0.402	0.616	1.947	0.564	0.793	-do-
19	46.5	1.721	0.575	0.880	1.955	1.020	1.134	-do-
20	46	1.728	0.750	1.013	1.963	0.990	1.308	-do-

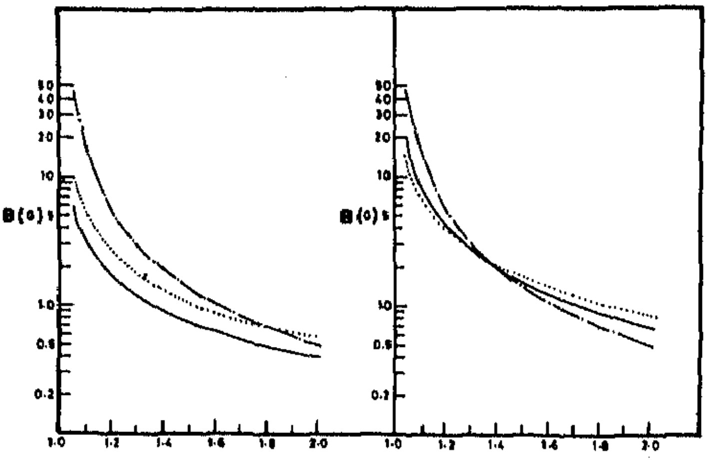

FIG.1. FIG.2

Fig.1. (for x=2) the magnetic field B, versus radial distance, ρ in units of solar radius $R_\odot$. The least square fits for the $V_1 > V_A$ and $V_1 = V_A$ cases, are represented by ——— and . . . . . respectively. The empirical fit of Dulk and McLean (1978) is represented by .-.-.-.-.

Fig.2. (for x=4) The magnetic field B, versus radial distance, ρ in units of solar radius $R_\odot$. The least square fits for the $V_1 > V_A$ and $V_1 = V_A$ cases, are represented by ——— and . . . . . respectively. The empirical fit of Dulk and McLean (1978) is represented by .-.-.-.-.

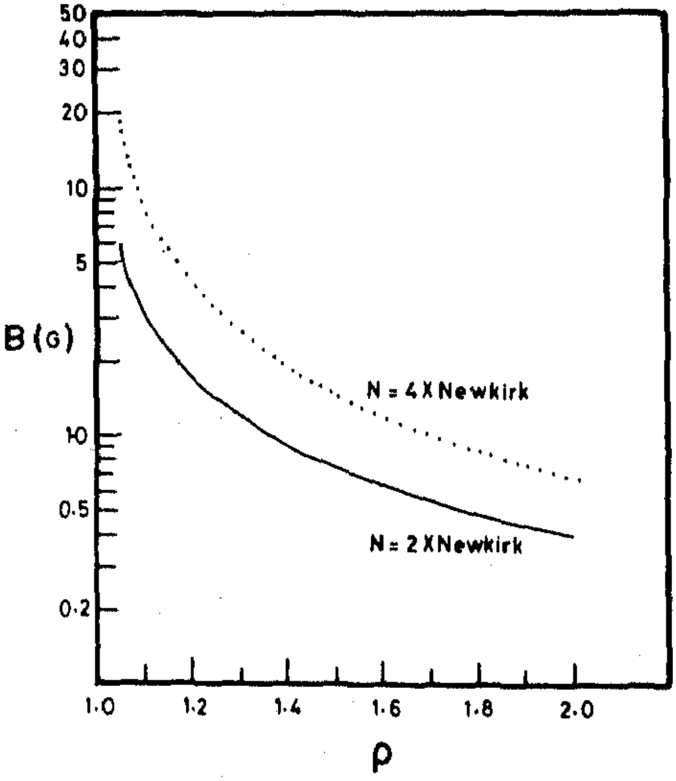

Fig.3. The magnetic field versus radial distance, ρ in units of $R_\odot$ for the case $V_1 > V_A$.

(Newkirk = $4.2 \times 10^{4+4.32/\rho}$)

radiation. Here we discussed the three wave interaction and pointed out that the ion-sound turbulence is the most plausible low frequency turbulence under coronal conditions for interacting with upper hybrid waves and generating the radiation. The second one concerns with the estimation of macroscopic parameters of the corona with a realistic model of type-I radiation. Specifically, we estimated the coronal magnetic field in the region 1-2 solar radii above the photsphere. We derived empirical relation of the coronal magnetic field as a function of radial distance above photosphere.

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