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Tidal transfer of energy and angular momentum to spheroidal galaxies

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SUMMARY
The tidal transfer of energy and angular momentum to a spheriodal test galaxy is studied using tensor virial equations within the framework of impulse approximation. The tidally introduced angular momentum is always along an axis perpendicular to the symmetry axis.

It is shown that, in the case of an initially non-rotating galaxy, more energy is pumped along either of the equatorial axes than along the symmetry axis.

1 INTRODUCTION

The changes in the energy of a star cluster as a result of an encounter with a passing interstellar gas cloud were studied by Spitzer (1958) on the assumption that stars in the cluster remain stationary during the encounter (impulse approximation). This assumption has been extensively used in studying tidal interactions among spherical galaxies (see the review by Alladin & Narasimhan 1982). The problem of transfer of angular momentum during tidal interaction between galaxies has been studied by Peebles (1969) and others with a view to accounting for the spin angular momentum of spiral and elliptical galaxies (see the review by Wesson 1982). Namboodiri & Kochhar (1990) have numerically considered the tidal transfer of energy to a spherical galaxy. In this paper we use the tensor virial equations and the impulse approximation to obtain expressions for changes in the energy and angular momentum of an ellipsoidal galaxy that is perturbed by another galaxy rushing past it.

2 TIDAL TRANSFER OF ENERGY AND ANGULAR MOMENTUM

We consider an ellipsoidal test galaxy (or photo-galaxy) of mass M being tidally perturbed by a galaxy of mass M' (idealized as a mass point) moving with a velocity v high enough for its orbit to be treated as a straight line. The pericentric distance p of the perturber is large compared to the dimensions of the test galaxy so that the tidal effects can be treated as first-order effects.

We define a coordinate system with its origin at the centre of mass of the test galaxy whose equatorial plane is defined to be the x_1–x_2 plane (see Fig. 1). The unit vectors $\hat{\mathbf{p}}$ and $\hat{\mathbf{v}}$ are given by (Knobloch 1978)

$$(\hat{\mathbf{p}}_1, \hat{\mathbf{p}}_2, \hat{\mathbf{p}}_3) = (\cos\phi \sin\theta, \sin\phi \sin\theta, \cos\theta), \tag{1}$$

$$\begin{pmatrix} \hat{\mathbf{v}}_1 \\ \hat{\mathbf{v}}_2 \\ \hat{\mathbf{v}}_3 \end{pmatrix} = \begin{pmatrix} \sin\psi \sin\phi - \cos\psi \cos\theta \cos\phi \\ -\sin\psi \cos\phi - \cos\psi \cos\theta \sin\phi \\ \cos\psi \sin\theta \end{pmatrix}, \tag{2}$$

and $\hat{\mathbf{p}} \cdot \hat{\mathbf{v}} = 0$ as required.

The test galaxy in steady state satisfies the tensor virial equations

$$\frac{dI_{ij}}{dt} = \boldsymbol{L}_{i;j} + \boldsymbol{L}_{j;i} = 0, \tag{3}$$

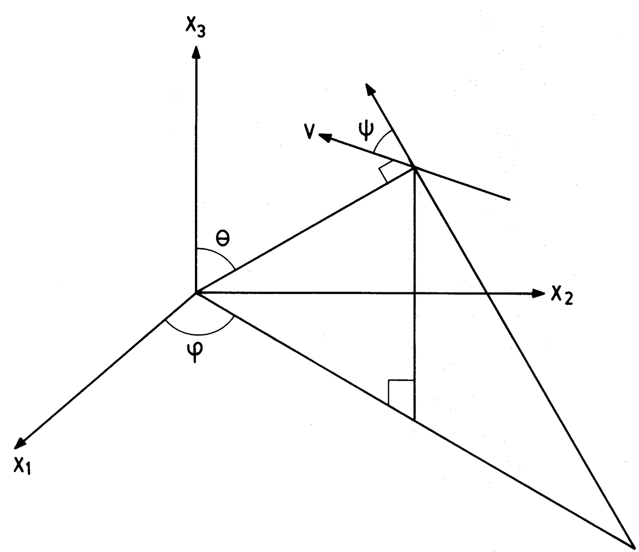

Figure 1. The geometry of the problem.

$$\frac{1}{2}\frac{d^2 I_{ij}}{dt^2}=2K_{ij}+W_{ij}=0. \tag{4}$$

Here $\boldsymbol{I}_{ij}$, $\boldsymbol{L}_{i;j}$, and $\boldsymbol{W}_{ij}$ are respectively the (diagonal) tensors of moment of inertia, momentum and potential energy. $\boldsymbol{K}_{ij}$ is the kinetic energy tensor made up of tensors of ordered motion $\boldsymbol{T}_{ij}$ and random motion $\boldsymbol{\Pi}_{ij}$: $2K_{ij}=2T_{ij}+\boldsymbol{\Pi}_{ij}$ (Som Sunder & Kochhar 1986).

We assume that the stars in the test galaxy remain stationary during the encounter (impulse approximation). Then the resultant velocity increment for a star depends upon its position but not the velocity. For a star at distance x from the centre of the Galaxy, we have to the first order in x_i/p

$$\Delta u_i=\eta\sum_{j=1}^{g}\alpha_{ij}x_j. \tag{5}$$

In this paper the convention of summation over repeated indices is not used; summations are shown explicitly. In equation (5)

$$\eta=\frac{2GM'}{p^2 v}, \tag{6}$$

$$\alpha_{ij}=2\hat{\mathbf{p}}_i\hat{\mathbf{p}}_j+\hat{\mathbf{v}}_i\hat{\mathbf{v}}_j-\delta_{ij}. \tag{7}$$

(Knobloch 1978).

2.1 Kinetic energy

In the impulse approximation, the moment of inertia tensor $\boldsymbol{I}_{ij}$ and the potential energy tensor $\boldsymbol{W}_{ij}$ remain unaffected by the encounter. The change in the kinetic energy tensor of the test galaxy is given by

$$\Delta\boldsymbol{K}_{ij}=\frac{1}{2}\int_\Gamma(\Delta u_i u_j+u_i\Delta u_j)\,f\,dz+\frac{1}{2}\int_\Gamma\Delta u_i\Delta u_j f\,dz. \tag{8}$$

Here Γ is the phase-space volume and dz its small element. Or,

$$\Delta\boldsymbol{K}_{ij}=\frac{1}{2}\eta\sum_m(\alpha_{im}\boldsymbol{L}_{j;m}+\alpha_{jm}\boldsymbol{L}_{i;m})+\frac{1}{2}\eta^2\sum_{mn}\alpha_{im}\alpha_{jn}\boldsymbol{I}_{mn}.$$

The first term on the right-hand side is in general non-zero, but it sums to zero when we take the trace of equation (8); so that

$$\Delta\boldsymbol{K}=\tfrac{1}{2}\eta^2[(1-\hat{\mathbf{v}}_1^2)I_{11}+(1-\hat{\mathbf{v}}_2^2)I_{22}+(1-\hat{\mathbf{v}}_3^2)I_{33}], \tag{9}$$

where we have made use of the relation $\Sigma\alpha_{im}^2=1-v_i^2$ (*cf.* equation 7).

Now assume that the test galaxy is spheroidal ($a_1=a_2\neq a_3$) and let

$$y=1-\frac{a_3^2}{a_1^2}. \tag{10}$$

The parameter y (the 'asphericity') is related to the ellipticity $\varepsilon=1-a_3/a_1$ by $1-y=(1-\varepsilon)^2$. For small ε, $y=2\varepsilon$. The asphericity y is zero for a sphere and +ve (−ve) for oblate (prolate) spheroids. For an oblate spheroid, y is simply the eccentricity squared. Note that a disc formally corresponds to $y=1$.

The change in the scalar kinetic energy then is

$$\Delta\boldsymbol{K}=\eta^2 I_{11}[1-\tfrac{1}{2}y(1-\cos^2\psi\sin^2\theta)], \tag{11}$$

where we have used the relations $I_{33}=I_{11}(1-y)$ and $\Sigma_m\alpha_{im}^2=1-\hat{\mathbf{v}}_i^2$ (*cf.* equation 7). If $\psi=0°$

$$\Delta K=\eta^2 I_{11}\left(1-\frac{y}{2}\cos^2\theta\right). \tag{11a}$$

By virtue of the impulse approximation, this is the total change in the energy of the system. As expected, equation (11) or (11a) reduces to the well-known result of Spitzer (1958) for a spherical test galaxy. Note from equation (11) that, if the perturber is moving perpendicular to the equatorial plane of the test galaxy ($\psi=0°$, $\theta=90°$), the effect of non-sphericity is zero and $\Delta\boldsymbol{K}=\eta^2 I_{11}$. For other orientations of the collision, $\Delta\boldsymbol{K}$ is less than this value for oblate spheroidal test galaxies, and greater than this value for the prolate ones.

One can see physically why this is so. (We thank the referee for pointing this out.) Obviously $\Delta\boldsymbol{K}$ depends on the effective value of $\bar{r}^2$, measured along p. (Hence the θ-dependence of equation 11a.) If the perturber moves parallel to the symmetry axis, I_{11} correctly measures $\bar{r}^2$. For other orientations there is a contribution to the effective value of $\bar{r}^2$ from the symmetry axis, and $\Delta\boldsymbol{K}$ will be smaller or greater than the value one would infer from I_{11} according to whether the system is oblate or prolate.

If we average over all possible orientations of the perturber's orbit, we obtain (*cf.* Knobloch 1978)

$$\langle\Delta\boldsymbol{K}\rangle=\eta^2 I_{11}(1-\tfrac{1}{3}y). \tag{12}$$

Equation (12) tells us that, other things being equal, the energy transferred is less if the test galaxy is a disc ($y=1$) rather than a sphere ($y=0$). Similarly it is less in the case of a sphere than a spindle-shaped galaxy ($y=-2$): the ratio being disc : sphere : spindle $=\tfrac{2}{3}:1:\tfrac{5}{3}$.

In the general case it is not possible to say anything about the non-vanishing tensor components $\boldsymbol{K}_{ij}$. But if we now assume that the test galaxy's angular momentum $J_k=\Sigma_{ij}\varepsilon_{kij}\boldsymbol{L}_{j;i}$ is zero, then by virtue of equation (13) each of the $\boldsymbol{L}_{i;j}$ is zero, and the first term on the right-hand side of equation (8) vanishes so that

$$\Delta\boldsymbol{K}_{jj}=\frac{1}{2}\eta^2\sum_m\alpha_{jm}^2\boldsymbol{I}_{mm}. \tag{13}$$

In the case of a spheroidal test galaxy this reduces to

$$\Delta\boldsymbol{K}_{jj}=\tfrac{1}{2}\eta^2(1-\hat{\mathbf{v}}_j^2-y\alpha_{j3}^2). \tag{14}$$

We see that the total increase in the scalar kinetic energy remains unaffected irrespective of whether the test galaxy is rotating or not.

Averaging over angles gives (*cf.* Knobloch 1978)

$$\langle\Delta\boldsymbol{K}_{11}\rangle=\langle\Delta\boldsymbol{K}_{22}\rangle=\tfrac{1}{3}\eta^2 I_{11}(1-\tfrac{3}{10}y),$$

$$\langle\Delta\boldsymbol{K}_{33}\rangle=\tfrac{1}{3}\eta^2 I_{11}(1-\tfrac{4}{10}y). \tag{15}$$

2.2 Angular momentum

The change in the momentum tensor of the test galaxy is

$$\Delta\boldsymbol{L}_{j;k}=\eta\sum_m\alpha_{jm}I_{km}=\eta\alpha_{jk}I_{kk},$$

so that the change in the angular momentum tensor is

$$\Delta \boldsymbol{J}_i = \sum_{jk} \varepsilon_{ijk} \Delta \boldsymbol{L}_{k;j} = \eta \alpha_{jk} (I_{kk} - I_{jj}). \qquad (16)$$

It is seen from this equation that transfer of angular momentum takes place because of the presence of the gravitational quadrupole moment $I_{kk} - I_{jj}$.

The galaxy's response to the tidal torque is to rotate about a suitable axis so as to orient itself along the direction of closest approach, and thereby reduce the torque. Note that, if any two axes of the ellipsoidal test galaxy are equal, the tidally transferred angular momentum has no component in the perpendicular direction.

In other words, in the case of a spheroidal galaxy $(a_1 = a_2 \neq a_3)$, the angular momentum vector lies in the equatorial plane:

$$\Delta J_1 = -\eta \alpha_{23} (I_{11} - I_{33}),$$

$$\Delta J_2 = +\eta \alpha_{13} (I_{11} - I_{33}),$$

$$\Delta J_3 = 0, \qquad (17)$$

where α_{ij} are given by equation (7). The total angular momentum tidally transferred to the test galaxy is

$$\Delta J = \eta (\alpha_{13}^2 + \alpha_{23}^2)^{1/2} (I_{11} + I_{33}), \qquad (18)$$

where

$$\alpha_{13}^2 + \alpha_{23}^2 = \sin^2 2\theta \sin^2 \psi + \tfrac{1}{4} \sin^2 2\theta \cos^4 \psi + \tfrac{1}{4} \sin^2 \theta \sin^2 2\psi.$$

If we set $\psi = 0°$, the transferred angular momentum is

$$\Delta J = \tfrac{1}{2} \eta I_{11} y \sin 2\theta. \qquad (19)$$

Note that J is zero if $\theta = 0°$ or $90°$.

To facilitate comparison with the earlier works, we rewrite equation (19) as

$$\Delta J = \frac{1}{5} \frac{GMM'}{p^2 v} a_1^2 y \sin 2\theta. \qquad (20)$$

This is similar to the expressions obtained by Peebles (1969) and Wesson (1982) provided we assume that the tidal torque remains effective for time p/v. Note that our numerical factor 0.2 is closer to Wesson's 0.18 than to Peebles's 0.30.

If we average over various orientations in equations (17) (*cf.* Knobloch 1978), we can write the expressions for the angular momentum transferred on an average in a single encounter,

$$\langle \Delta J_1^2 \rangle^{1/2} = \langle \Delta J_2^2 \rangle^{1/2} = \frac{1}{\sqrt{5}} \eta I_{11} y. \qquad (21)$$

3 CONCLUSIONS

We have used the tensor virial equations to derive, under impulse approximation, analytical expressions for the energy and angular momentum transferred to a spheroidal test galaxy as a result of a tidal encounter. We have shown that more energy is transferred along either of the two equatorial axes than along the symmetry axis. If the test galaxy is modelled as a homogeneous Maclaurin spheroid, then we find that $\Delta K/E$ falls off with increasing eccentricity. On the other hand, $\Delta J^2/J^2$ rises till $e = 0.83$ and then falls off, qualitatively mimicking the behaviour of Ω^2 (Som Sunder, Kochhar & Alladin 1989).

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