Kodaikanal Obs. Bull. (1986) 6

SOLAR CONVECTION AND OSCILLATIONS

S.M. Chitre
TATA INSTITUTE OF FUNDAMENTAL RESEARCH
HOMI BHABHA ROAD
BOMBAY 400 005

Abstract

A wide variety of gravity and acoustic wave motions have been detected at the surface of the Sun as Doppler shifts of spectral lines. These velocity fields provide a valuable probe for studying the internal solar structure and its dynamics. The role of convection in driving the observed motions is discussed in the framework of a linear stability theory. The most unstable convective modes excited in the solar envelope are found to be in reasonable agreement with the features associated with granulation and supergrannulation. It is demonstrated that when the mechanical and thermal effects of turbulent convection are incorporated in the analysis, a linear superposition of statistically independent unstable convective modes can reproduce the model convective flux profile of the mixing-length formalism. The stability of acoustic modes is investigated to find many of them to be overstable, with the most rapidly growing modes occupying a region centred predominantly around 3.3 mHz and spread over a broad range of length scales. It is argued that these five-minute oscillations are largely driven by the effects of turbulent convection.

1. Introduction

The inside of the Sun is clearly not directly accessible to observations; nevertheless, it is possible to construct a reasonably accurate picture of its interior with the help of equations governing its mechanical and thermal equilibrium. the Sun is, in fact, the primary calibrator of the theory of stellar structure. Moreover, the Sun is sufficiently near to Earth for its surface phenomena to be closely scrutinized and its atmosphere to be studied in detail.

The observations of the solar atmosphere reveal a wide range of velocity fields with two dominant components: a non-oscillatory part which is believed to be a manifestation of the sub-photospheric convective motions and an oscillatory component with a period predominantly centred around 300 sec. One of the remarkable properties of the solar convective motion is the discrete spectrum of horizontal length scales: granulation with a characteristic cell size of ~2000 Km and having an average lifetime 8-10 min (Beckers and Canfield, 1976) and supergranulation, with an average diameter ~30,000 Km and lasting for ~1-2 days (Simon and Leighton, 1964). In addition, global convection comparable in size to the thickness of the convection zone (~200,000 Km) and lasting for several months has also been detected (Howard, 1971). Amongst the oscillatory motions the five-minute oscillations have been the subject of most extensive study since their discovery by Leighton, Noyes and Simon (1962).

Just as geoseismology provides a wealth of information about the interior of Earth by analysing the frequencies of terestrial oscillations, solar seismology, with its origin in the work of Deubner (1975) who successfully resolved the spatial and temporal structure of these oscillations, is expected to be a valuable diagnostic probe for the structure of the Sun. A power-spectrum of the five-minute oscillations of high degree (spherical harmonic degree $\ell \gtrsim 150$) was produced by Deubner, Ulrich and Rhodes (1979). The observations using integrated sunlight by Claverie et al (1979) and Grec et al (1980) revealed the existence of five-minute oscillations of low degree ($\ell \leq 3$). Recently, Durall and Harvey (1983) provided the power spectrum of these oscillations in the intermediate degree ($1 \leq \ell \leq 150$) thus bringing the gap between the observations of low and high degree.

Any reasonable theoretical model must not only account for the existence of preferred length scales on the solar surface, but should also explain the observed features of the solar five-minute oscillations. For this purpose, Antia, Chitre and Narasimha (1983) calculated the growth rates of linear convective modes by perturbing a realistic solar envelope model computed using the mixing-length prescription. The main thrust of the calculation was to incorporate the effects of turbulence on the mean flow through the eddy transport coefficients. The equilibrium solar envelope model does not get sensibly altered by the inclusion of the turbulent pressure, but the convective modes are significantly damped by the Reynolds stresses. The growth rate of the fundamental mode then exhibits a double peak when plotted against the horizontal wave number which may be interpreted as yielding the two distinct scales of convection corresponding to granulation and supergranulation. We calibrate the turbulent Prandtl number (P_t= turbulent viscosity/turbulent conductivity) which gives the most reasonable accord with the granular and supergranular motions and adopt it for examining the stability of non-radial acoustic modes. We find many of these trapped acoustic modes to be overstable with the most rapidly growing modes occupying a region centred around a period of five-minutes and spread over a wide range of length-scales.

2. Basic Mathematical Scheme

We adopt the usual hydrodynamical equations governing the conservation of mass, momentum and energy applicable to a viscous, thermally conducting fluid.

$$\text{(mass)} \quad : \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0$$

$$\text{(momentum)} \quad : \rho \frac{\partial \mathbf{v}}{\partial t} + \rho(\mathbf{v} \cdot \nabla)\mathbf{v} = \rho \mathbf{g} - \nabla P - \nabla P_{tub} - \frac{2}{3}\mu_{tub} \nabla(\nabla \cdot \mathbf{v})$$

$$- \frac{2}{3}(\nabla \cdot \mathbf{v}) \nabla \mu_{tub} + \nabla [\mu_{tub}(\nabla \mathbf{v} + \mathbf{v}\nabla)]$$

$$\text{(energy)} \quad : \rho T \frac{\partial S}{\partial t} + \rho T \mathbf{v} \cdot \nabla S + \rho \frac{\partial E_{tub}}{\partial t} + \rho \mathbf{v} \cdot \nabla E_{tub} + P_{tub} \nabla \cdot \mathbf{v} = - \nabla \cdot \mathbf{F}$$

In this we have neglected the viscous dissipation in the energy equation and adopted the usual notation with S as the specific entropy, P the thermodynamic pressure (gas + radiation), P_{tub} the turbulent pressure, E_{tub} the turbulent energy density (= $(3/2)(P_{tub}/\rho)$) and $\mathbf{F}$, the total flux (=$\mathbf{F}^{rad} + \mathbf{F}^{conv}$). The above equations are supplemented by an equation of state for a perfect gas undergoing ionization. We compute the radiative flux using the Eddington approximation (Unno and Spiegel, 1966):

$$\mathbf{F}^{rad} = - \frac{4}{3\kappa\rho} \nabla J \quad ,$$

where $J = \sigma T^4 - (\nabla \cdot \mathbf{F}^{rad}/4\kappa\rho)$ is the radiation intensity. For calculating the convective flux we adopt the mixing-length prescription and write

$$\mathbf{F}^{conv} = -K_{tub}(\nabla T - \nabla_{ad} \frac{T}{P} \nabla P) \quad ,$$

where $\nabla'_{ad} = \nabla_{ad} \frac{d\ln P}{d\ln(P+P_{tub})}$ and the coefficient of turbulent conductivity is expressed in the form $K_{tub} = \alpha\rho C_p WL$. Here α is an efficiency factor of order unity, the mixing length L is taken as $z+486$ Km, z measured downwards from the level $\tau=1$ and W is the mean convective velocity (cf, Shaviv and Chitre, 1968) which includes the turbulent drag experienced by moving elements (Antia, Chitre and Narasimha, 1984). The turbulent dynamic viscosity is then written as

$$\mu_{tub} = P_t \alpha\rho WL \quad ,$$

where P_t is the turbulent Prandtl number which we treat as a free parameter to be calibrated from the best fit of maximally growing convective modes with the observed features of granulation and supergranulation.

For the present sun we adopt the following set of parameters:

$$X=0.74, \quad Z=0.018, \quad T_c=1.52\times10^7 \text{ K}, \quad \rho_c=145 \text{ gm/cm}^3, \quad \text{and } X_c=0.36.$$

We assume spherical symmetry and express the eigenfunctions for any perturbed scalar physical variable in the spherical geometry (r,θ,ϕ) as

$$f(r,\theta,\phi,t) = f_0(r) + f_1(r) Y_\ell^m(\theta,\phi)\, e^{\omega t} \quad ,$$

where the subscripts 0 and 1 refer respectively to the unperturbed and perturbed quantities, Y_ℓ^m is the spherical harmonic of degree ℓ and ω the eigenvalue which may be real or comlex. The velocity and flux perturbations are taken to have the following form:

$$\begin{matrix} \underset{\sim}{v}(r,\theta,\phi) \\ \underline{F}(r,\theta,\phi) \end{matrix} = \begin{bmatrix} v_r(r), & v_h(r)\frac{\partial}{\partial\theta}, & v_h(r)\frac{1}{\sin\theta}\frac{\partial}{\partial\phi} \\ F_r(r), & F_h(r)\frac{\partial}{\partial\theta}, & F_h(r)\frac{1}{\sin\theta}\frac{\partial}{\partial\phi} \end{bmatrix} Y_\ell^m(\theta,\phi)$$

The equations governing the perturbed quantities form a set of eight first order differential equations, while in the adjoining inviscid layers bounding the convection zone the order of the equations reduces to four. We therefore need two boundary conditions at each boundary and six connection conditions at the interfaces.

We invoke free boundary conditions at both the boundaries where the Lagrangian pressure perturbation is assumed to vanish. We adopt the thermal boundary condition requiring the radiation not to enter the layer from infinity at the upper boundary and for the lower boundary we impose the condition of the vanishing of Lagrangian flux perturbation. The six connection conditions at both the interfaces relate to the continuity of the perturbed horizontal and vertical components of the viscous stress tensor, vertical components of velocity and total flux, entropy and radiation intensity. The linearized system of equations together withy the boundary and connection conditions constitute a generalized eigenvalue problem where the eigenvalues (real/complex) and the associated eigenfunctions are determined numerically for a specified value of the horizontal harmonic number ℓ using a finite-difference scheme (Antia, 1979).

3. Convective Modes and Consistency of Mixing Length Theory

We first attempt to address the question of the preferred length scales observed on the solar surface and in particular why there are two distinct scales of motion granulation and supergranulation. To investigate the convective modes we compute the real growth rate ω as a function of ℓ which is related to the horizontal wavelength $\lambda = 2\pi R_\odot/\sqrt{\ell(\ell+1)}$ ($R_\odot$ = solar radius). For a given ℓ, there exists a series of eigenvalues ω and we call the one with the highest eigenvalue the fundamental convective mode.

A little way below the solar surface the temperature gradient becomes unstable to convection and save the top several tens of Kilometers, a major fraction of the total flux in the solar envelope is transported by convection. Furthermore, the raditive conductivity is also very small as compared to the turbulent heat conductivity. The turbulence is therefore expected to have a significant influence both in the way it modulates the heat flux and through the direct effect of the Reynolds stresses. We incorporate the mechanical and thermal effects of turbulence in a very approximate manner in our calculation through the eddy transport coefficients and the effects of momentum transfer are included through P_{tub}.

The numerical results of our computation are summarized in Table 1 which shows the preferred length scales and associated time scales ($1/\omega_{max}$) for three values of the tubulent Prandtl number P_t = 0.1, 0.2, 0.3.

Table I

P_t	Primary maximum		Secondary maximum	
	t(min)	(km)	t(hr)	(km)
0.1	13	1600	82	68 000
0.2	17.8	2000	135	77 000
0.3	24.4	2200	215	110 000

For the choice of parameters adopted, the e-folding time and horizontal wavelengths for the most unstable fundamental mode lie respectively in the range of 10-25 min and 1600-2400 km. The length scale associated with the primary maximum is not particularly sensitive to the variation of the parameters, and the preferred length and time-scales are evidently in reasonable accord with the observed granular features. The secondary maximum, however, is rather sensitive to the choice of the parameters, and the corresponding e-folding time and horizontal wavelength vary over a wide range. It is tempting to identify the convective modes corresponding to $P_t \approx 1/3$ as yielding the length- and time-scales that are not in too unreasonable agreement with the observed cell sizes and lifetimes of granulation and supergranulation.

It is instructive to note that the dissipative mechanisms arising from turbulent transport coefficients which are effective at short wavelengths manifest their damping influence for large values of ℓ. This is evident from the role of the second-order derivatives of velocity and temperature in the momentum and energy equations in assuming importance for large values of ℓ. The turbulent pressure, on the other hand, appears only as a first-order derivative in the momentum equation and its influence is therefore expected to be felt for intermediate values of ℓ; the modes with large length scales are clearly not affected by the presence of grad P_{tub}, while for intermediate values of ℓ, the growing rates may even become negative because of the damping by the turbulent pressure. This seems to be responsible for production a double peak in the plot of ω vs ℓ, a behaviour which seems to be in contrast to the case without turbulent pressure which yields only one maximal growth rate for the fundamental mode. Such a behaviour is highly suggestive of mode coupling (Antia, Chitre and Narasimha, 1983).

The mixing length theory has been extensively used in constructing convection zone models for stars. But the question whether these models are consistent with models incorporating convection dynamics was wide open. Hart (1973) convincingly demonstrated that no linear superposition of inviscid, adiabatic modes can even remotely reproduce the convective heat flux of a stellar envelope. This was a consequence of all the convective modes having a sharp peak in the strongly superadiabatic layer near the top of the convection zone, and as a result of the model convective flux could not be consistently represented by a combination of statistically independent unstable non-dissipative convective modes. This situation was remedied by Narasimha and Antia (1982) who explicitly included the effects of eddy transport coefficients on the eigenfunctions of unstable convective modes. The eigen-functions then no longer peak only in the sub-photospheric superadiabatic region, but rather they are sperad over the entire convection zone: those with large values of ℓ peak in the sub-surface layers, while those with small peak near the botton of the convection zone.

The energy flux, $F_\ell(r)$, transported by any given mode may be calculated by using the expression,

$$F_\ell(r) = \rho_0 v_r (T_0 S_1 + P_1/\rho_0)$$

Clearly the eigenfunctions, v_r, S_1 and P_1 are arbitrary in a linearized theory to the extent of a constant multiple. In order to fix the normalization of the multiplicative constant is so chosen that the maximum of the convective flux, $F_\ell(r)$, over the entire convection zone equals the actual model convective flux at the depth. For a linear superposition of statistically independent linear modes, we have

$$\sum_\ell a_\ell^2 F_\ell(r) = F^{conv}(r) \quad ,$$

a_ℓ being real constants which are supposed to fix the amplitudes of corresponding modes in the solar envelope. If one succeeds in finding a set of real constants, a_ℓ such that the above equation is reasonably satisfied over the entire convection zone, then one would conclude that the mixing-length theory is self-consistent. As is clear from Fig.1 it turns out that the maxima of $F_\ell(r)$ occur at different depths for different values of ℓ, and it is possible for a suitable superposition of statistically independent unstable convective modes to reproduce the convective flux profile of the mixing-length theory in a consistent manner.

The feature of the flux profiles due to convective eigenmodes with different values of ℓ peaking at different depths can be profitably employed to identify the mixing length at a particular depth with some length scale associated with the dominant convective mode which has a peak in the flux profile at that depth. We choose the full width at half maximum for that eigenmode at the natural choice for the length scale. This is illustrated in Fig.2 which shows the mixing length as a function of log P for the solar envelope mode along with the full width at half-maximum for the fundamental mode. The general agreement between the two curves is certainly noteworthy!

4. Excitation of Five-Minute Oscillations

The important question of the excitation mechanism for five-minute oscillations was addressed by Ando and Osaki (1975) who investigated the stability of non-radial oscillations of intermediate and high degree for a realistic solar envelope model. This work included the full effects of radiative exchange in the stability analysis. However, the interaction between turbulent convection and oscillation was neglected in this work; this situation was later remedied by Goldreich and Keeley (1977) who sought to incorporate the influence of turbulence on the stability of acoustic modes. Antia, Chitre and Narasimha (1982) investigated the stability of acoustic modes and the question of their excitation mechanism utilizing the framework employed for studying the convective modes and adopting the same set of parameters, in particular, the turbulent Prandtl number $P_t \approx 1/3$.

The numerical results for the acoustic modes of low, intermediate and high degree yield eigenfrequencies which are in reasonable agreement with the observed frequencies of five-minute oscillations (cf. Fig.3). Thus, for the intermediate and low values of ℓ, the departure of theoretically calculated eigen-frequencies is within 15 μHz, i.e. no more than 0.7% of the observed frequencies. The contours of constant stability coefficient η ($\equiv$ growth rate/frequency) are displayed in Fig.4 in a frequency (ω) vs. spherical harmonic degree (ℓ) plot. In this figure the outermost contour represents the marginally stable case (η=0) within which all the eigenmodes are unstable (indicated by 0), while the modes outside this region are stable (denoted by X). Interestingly, the high-frequence cut-off around 4 mHz appears to be more or less independent of ℓ and is presumably a reflection of damping in the atmosphere, which is in rough agreement with the observations of Duvall and Harvey (1983). Moreover, for low ℓ, the lower harmonics are found to be stable or have an extremely small growth rate, and this is consistent with the low power observed in these harmonics.

Our results are in general agreement with the earlier analysis of Ando and Osaki (1975), except for a significant difference that we get closed contours of the stability coefficient η, with a distinct peak at $\eta=1.6\times10^{-3}$ around ℓ=475 and ω=3.5 mHz, while Ando and Osaki get open contours with η increasing with ℓ. The pronounced peak of the stability coefficient is situated almost near the central region of the observed power spectrum of five-minute oscillations. This difference is clearly due to the effect of turbulent viscosity which produces increased damping for larger values of ℓ and hence decreses the growth rate at high ℓ. We find several acoustic modes trapped in the solar interior to be overstable and the most rapidly growing modes occupy a region centred predominantly around 300 sec with a wide range of horizontal length scales.

In order to understand the possible excitation mechanism responsible for driving the five-minute oscillations, we must recognize that both the radiative and turbulent diffusion mechanisms have their origin in the strongly superadiabatic layers in the sub-photospheric region. But, as Unno (1976) has pointed out, the efficiency of the turbulent diffusion mechanism is significantly larger by a factor, $\frac{F^{conv}}{F^{rad}}\,\frac{\nabla}{\nabla-\nabla_{ad}}$ compared to the purely radiative exchange mechanism. This prompted us to compute the growth rates of acoustic modes in the solar envelope both with and without the turbulent diffusion mechanism. The numerical results are summarized in Table 2 which shows the viscous acoustic modes for the solar envelope model for ℓ=200 with P_t=1/3 when (a) the turbulent diffusion mechanism is suppressed, (b) both the radiative processes (κ-mechanism) and the turbulent mechanism operate.

Table 2

Mode	Period (sec)	η (a)	η (b)
f	714	- 3.25 (-4)	- 1.71 (-4)
P1	513	- 4.83 (-4)	- 5.67 (-5)
P2	416	- 1.03 (-3)	4.33 (-4)
P3	357	- 1.11 (-3)	1.03 (-3)
P4	315	- 1.34 (-3)	1.50 (-3)
P5	281	- 1.48 (-3)	1.47 (-3)
P6	257	- 1.66 (-3)	1.31 (-3)
P7	237	- 1.72 (-3)	8.17 (-4)

It is clear that the degree of instability of acoustic modes, as measured by their growth rates, is considerably stronger when the turbulent diffusion mechanism is included than when only the κ-mechanism operates. We conclude that the five-minute oscillations are driven by a simultaneous action of the κ-mechanism and the radiative and turbulent diffusion mechanisms, but the turbulent diffusion seems to be the process which makes a dominant contribution to the excitation of acoustic waves in the sun.

5. Role of Solar Physics in Broader Context of Astrophysics

The aspiration of solar physicists is to seek answers to questions that have hitherto been in the realm of theoretical speculation. We would like to know about the conditions in the deep interior of the sun, and test the hypothesis that the hydrogen burning reactions really occur in the solar core and the energy generation is mainly by the proton-proton chain. We should like to find out if the solar core is rotating rapidly and if there is a mixing of the nuclear products with the surrounding layers of the core. We ought to have a knowledge of the extent of the outer convection zone and the physical conditions near the base of this zone. We should be able to infer the variation of the speed of sound in the solar interior and also the chemical composition of the sun.

Helioseismology must surely contain an enormous amount of information about the solar interior and one should be able to extract it from the detailed observations of eigenfrequencies of acoustic and gravity modes detected at the solar surface. A complementary probe is provided by the neutrinos produced in the core which, we had hoped, would enable us to to 'see' the interior of the sun. Unfortunately, the measured neutrino flux has consistently turned out to be a factor 3-4 lower than the theoretical prediction of the Standard Solar Model. We therefore need an independent probe of the Sun to reveal the physical and chemical conditions in its interior and the observed eigenfrequencies seem just appropriate for this purpose. We outline below some of the broad physics problems to which helioseismology might hopefully provide valuable clues.

(a) An important inference of most comological models is the amount of helium produced in the early universe. The distribution of helium inside the present Sun may be inferred from the observed frequencies of the global modes of oscillation It turns out outside the core of the Sun, and beneath the subsurface convection zone, the helium abundance in about 25 per cent by mass. It is generally believed that this intermediate region in the Sun is relatively uncontaminated by the nuclear products generated in the core and by material that may have been accreted by the Sun during the lifetime. If this indeed should be the case, we have an access to the protosolar helium abundance of the gas from which the Sun condensed some 4-6 billion years ago. This may provide the most reliable estimate of helium abundance in the early universe.

(b) There is convincing evidence present in the observed eigenfrequencies about the internal rotation of the Sun. The fine structure could also be contributed by the presence of strong magnetic fields of primordial origin buried deep in the core. The variation of solar rotation with depth has an important bearing on solar dynamics. A rapidly rotating core is liable to induce a circulation which might lead to the helium produced by the nuclear reactions to be mixed with the surrounding material. Furthermore, a knowledge of the solar rotation is vital in understanding the solar activity cycle, and also the dynamics of the convection zone.

(c) The observed solar eigenfrequencies have implications for the Sun's gravitational potential. One of the classical tests of general relativity involves the precession of the orbit of planet Mercury. It has been known that, after correcting for the planetary interactions, the residual orbit of Mercury was processing about the Sun at 43" per century, and this was successfully explained by general relativity assuming the Sun to be spherically symmetric. The rotation of the Sun would, however, cause oblateness which would modify the Sun's gravitational field to have a quadrupole moment. It appears from the observed rotational splitting of the eigenfrequencies that the oblateness does not contribute more than one percent to the precession of the orbit of Mercury. This may resolve questions of the accuracy of the theory of general relativity.

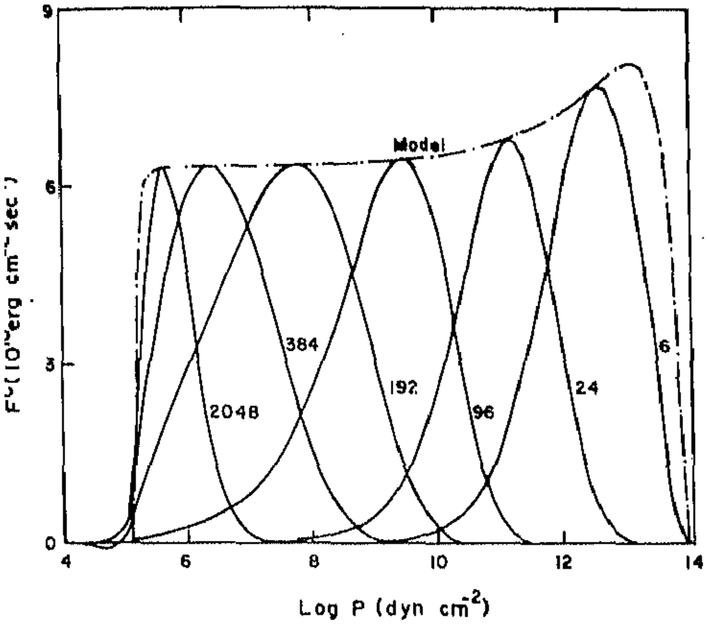

Fig.1. Convective flux carried by fundamental mode for ℓ=2048, 384, 192, 96, 24 and 6 is plotted against the logarithm of pressure. The convective flux profile given by the mixing-length model is indicated by the dot-dashed curve.

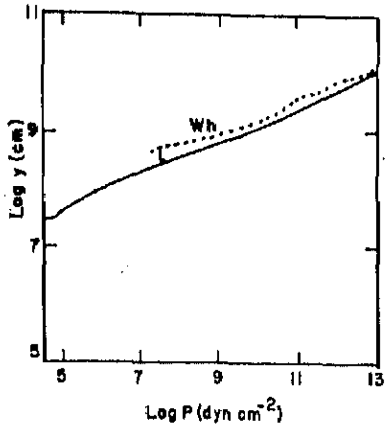

Fig.2. Mixing length L = z+486 Km and, W_H are displayed as functions of the logarithm of pressure.

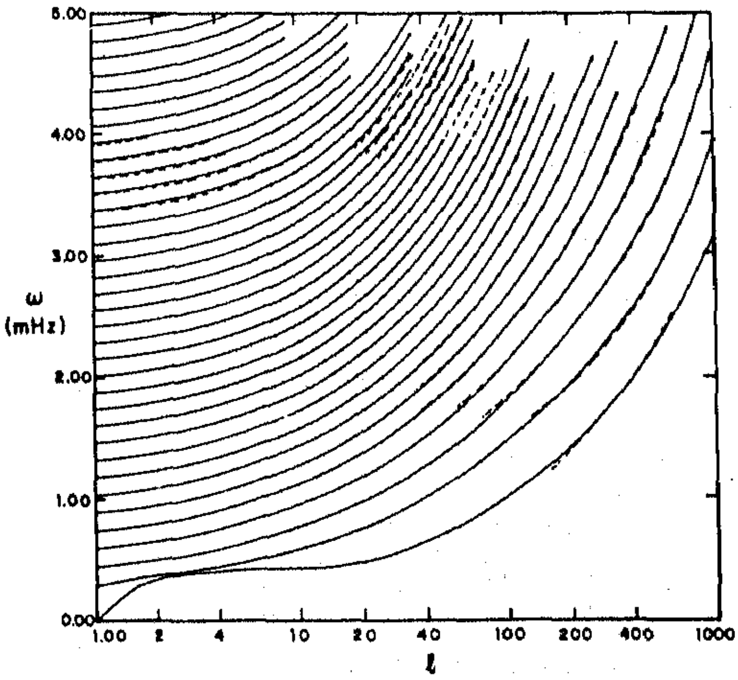

Fig.3. Frequencies ω as a function of ℓ for the first 35 acoustic modes. The observed frequencies are denoted by dashed curves.

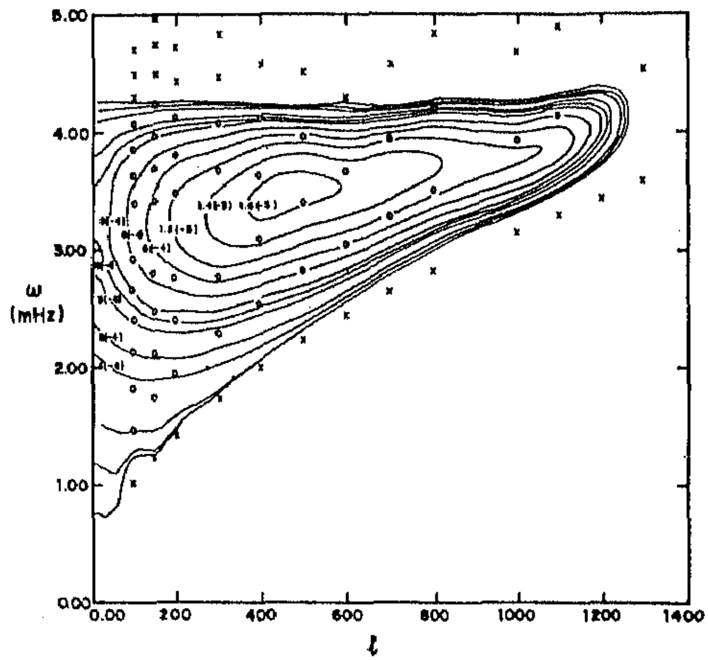

Fig.4. Contours of equal stability coefficients η are shown in the (ω-ℓ) plot. Stable modes are indicated by crosses (X) and unstable by open circles (O); outermost contour refers to the marginally stable case η=0.

References

Ando, H. and Osaki, Y. 1975. Publ. Astron. Soc. Japan, **27**, 581.

Antia, H.M. 1979. J. Comp. Phys. **30**, 283.

Antia, H.M., Chitre, S.M. and Narasimha, D. 1982. Solar Phys. **77**, 303.

Antia, H.M., Chitre, S.M. and Narasimha, D. 1983. Mon. Not. R. Astron. Soc. **204**, 865.

Antia, H.M., Chitre, S.M. and Narasimha, D. 1984. Astrophys. J. **282**, 574.

Beckers, J.M. and Canfield, R.C. 1976 in Physique des Mouvements dans les Atmospheres Stellaires, 207 (CNRS No.250, eds. Cayrel and Steinberg).

Claverie, A., Isaak, G.R., McLeod, C.P., Van der Raay, H.B., Rocca Cartes, T. 1981. Nature, **293**, 443.

Deubner, F.-L. 1975. Astron. Astrophys. **44**, 371.

Deubner, F.-L., Ulrich, R.K. and Rhodes, D.J. Astron. Astrophys. **72**, 177.

Duvall, T.L. and Harvey, J.W. 1983. Nature, **302**, 24.

Goldreich, P. and Keeley, D.A. 1977. Astrophys. J. **211**, 934.

Grec, G., Fossat, E. and Pomerante, M. 1983. Solar Phys. **82**, 55.

Hart, M.H. 1973. Astrophys. J. **184**, 587.

Howard, R. 1971. Solar Phys. **16**, 21.

Leighton, R.B., Noyes, R.W. and Simon, G.W. 1962. Astrophys. J. **135**, 474.

Narasimha, D. and Antia, H.M. 1982. Astrophys. J. **262**, 358.

Shaviv, G. and Chitre, S.M. 1968. Mon. Not. R. Astron. Soc. **140**, 61.

Simon, G.W. and Leighton, R.B. 1964. Astrophys. J. **140**, 1120.

Unno, W. 1976 in Problems of Stellar Convection, 315 (IAU Colloqium **38**, eds. Spiegel and Zahn).

Unno, W. and Spiegel, E.A. 1966. Publ. Astron. Soc. Japan, **18**, 85.