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Equivalent Widths of Hydrogen Lyman Alpha Line in an Expanding Spherical Atmosphere

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Abstract. We have calculated the profiles of hydrogen Lyman α line in an expanding spherical atmosphere containing dust and gas. We have investigated the variation of equivalent widths with velocities of expansion of the atmosphere, together with the amount of dust present in the medium. We have drawn curves of growth for different velocities and dust optical depths.

Key words: radiative transfer—Lyman α line—expanding atmospheres —circumstellar dust

1. Introduction

Infrared observations have shown that many stars with extended outer layers as well as planetary nebulae contain dust mixed with the ionized gas (Miller 1974; Osterbrock 1974). Moreover, these objects have complicated internal motions. Dust can influence the structure and dynamics of the planetary nebulae. It may cause reduction in the size of the Strömgren sphere (Mathis 1971; Petrosian, Silk & Field 1972). Calculations of the profiles of lines formed in such media are necessary to investigate the dynamical and structural changes that may be brought out by the presence of dust. Earlier, Peraiah & Wehrse (1978) and Wehrse & Peraiah (1979) have investigated the effects of dust and expansion velocities on the formation of hydrogen Lyman α line. They considered small velocities as the transfer equation was solved in the rest frame. Large velocities can be incorporated only in comoving frame of the fluid (Peraiah, Varghese & Rao 1987). We study here the effects of large velocities on equivalent widths of H Lyman α line.

Curves of growth are generally plotted for a medium which is stationary. However, we find that atmospheres of stars are normally in motion, mostly radial. Therefore it would be difficult to plot the curves of growth for such atmospheres as the optical depth (which contains the absorption coefficient or scattering coefficient) changes with the velocity of expansion, depending upon the thermal velocities of gases. For example, in an electron scattering atmosphere, we have (Mihalas 1978)

$$\tau = \sigma v_{\mathrm{th}} \left(\frac{\mathrm{d}v}{\mathrm{d}r} \right)^{-1}, \tag{1}$$

where τ is the optical depth, σ is the electron scattering coefficient, v_{th} is the thermal velocity of the medium in motion and $(\mathrm{d}v/\mathrm{d}r)$ is the velocity gradient. One should remember that this relation would become invalid in stellar photospheres. In such situations it is difficult to obtain the information about the number density that is influencing line formation. We shall investigate here the effects of expansion velocities and dust on the equivalent widths of resonance lines. For this purpose we choose hydrogen Lyman α line whose parameters are well known. We assume a spherical atmosphere with dust.

2. Calculations

We require the number of neutral hydrogen atoms to calculate the strength of the Lyman α line. For this purpose we need to solve either the statistical equilibrium equation or obtain the number density by using Saha equation of ionization in a simple way. The latter is easy to solve provided we know the temperature structure and electron density. We can specify the electron density distribution in advance while the temperature distribution is calculated by assuming that the Planck function dilutes at the rate of $(r_0/r)^2$ where r_0 and r are the inner and outer radii of the spherical shell. This gives us,

$$B_\nu(T_r)=\left(\frac{r_0}{r}\right)^2 B_\nu(T_{r_0}), \tag{2}$$

where B_ν is the Planck function at frequency ν, T_r is the temperature at the radial point r in the spherical shell and T_{r_0} is the temperature at the inner radius r_0 of the spherical shell. Equation (1) will give us a temperature distribution

$$T(r)=\frac{h\nu}{k}\frac{1}{\ln\left[1+\left(\frac{r}{r_0}\right)^2(e^{h\nu/kTr_0}-1)\right]}, \tag{3}$$

where h and k are the Planck's constant and Boltzmann's constant respectively.

We have assumed a temperature of 15000 K at $r=r_0$ and the distribution is plotted in Fig. 1(a). The assumed electron density is shown in Fig. 1(b).

If we have pure hydrogen gas, then the electron density is given by (Mihalas 1978)

$$n_e(H)=\Phi_H^{-1}[(N\Phi_H+1)^{1/2}-1] \tag{4}$$

where

$$\Phi_H=\frac{U_1(T)}{U_2(T)}c_I T^{-3/2}e^{\chi_I/kT} \tag{5}$$

and χ_I is the ionization potential, $c_I=2.07\times10^{-16}$ cgs units, U_1 and U_2 are the partition functions, $N=N_0+2n_e$, the total number of particles. We derive

$$N_0=\Phi n_e^2. \tag{6}$$

The distribution of neutral atoms is plotted in Fig. 1(c).

We require to calculate the number of hydrogen atoms in the level 1 or N_1 for obtaining the absorption coefficient. This is done by using Boltzmann equation (Aller 1963)

$$\log\frac{N_2}{N_1}=-\theta e+\log\frac{g_2}{g_1} \tag{7}$$

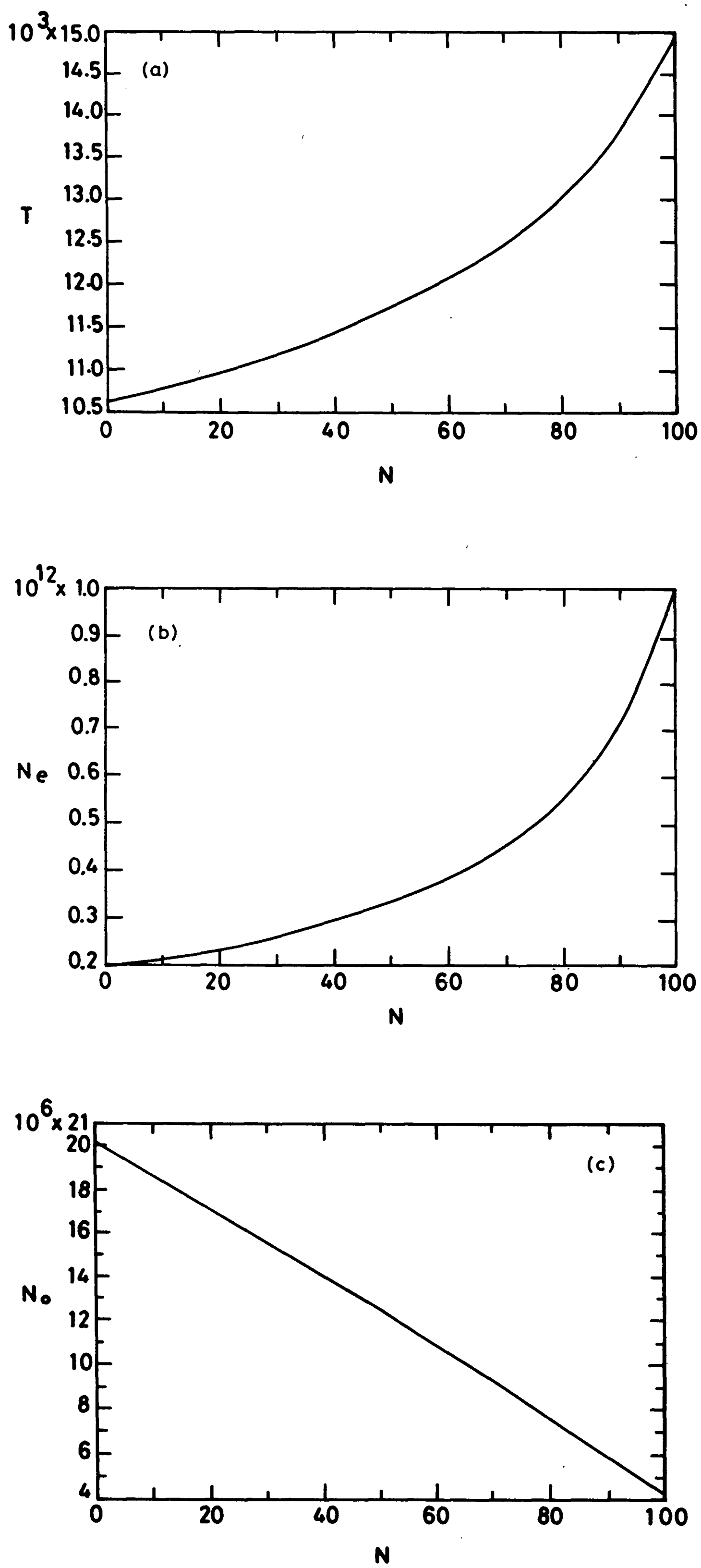

Figure 1. (a) Temperature, (b) electron density, and (c) number of neutral atoms at various points in the spherical shell in which hydrogen Lyman α line is forming. Here $T(r_0)=15000$ K. Shell no. 1 is at $r=r_{max}$ and shell no. 100 is at $r=r_0$.

where $\theta = 5040/T$, e is the excitation potential in eV, and g_1, g_2 are statistical weights of and N_1, N_2 the number of atoms in levels 1 and 2 respectively. With $N_0 = N_1 + N_2$ and $N_2/N_1 = p$, we obtain,

$$N_1 = \frac{N_0}{1+p}. \tag{8}$$

We calculate the absorption coefficient for hydrogen Lyman α line using the formula,

$$\chi_1(\nu) = N_1 A_{21} \frac{g_2}{g_1} \frac{c^2}{2h\nu^3} \frac{h\nu}{4\pi} \phi_\nu \left(1 - \frac{N_2}{N_1} \frac{g_2}{g_1}\right) \tag{9}$$

where A_{21} is the Einstein coefficient for spontaneous emission, c is the velocity of light, ν is the central frequency of H Lyman α line and ϕ_ν is the profile function of the line such that

$$\int_{-\infty}^{+\infty} \phi_\nu d\nu = 1.$$

We have employed a Doppler profile.

The optical depth in each shell of the medium is plotted in Fig. 2(a) and the total optical depth up to every shell is plotted in Fig. 2(b). We assumed that the medium contains dust in addition to hydrogen gas. The amount of dust and its distribution is represented by the dust optical depth. The spherical medium is expanding radially outwards and the line transfer is solved in a comoving frame (see Peraiah, Varghese & Rao 1987):

$$\begin{aligned} \mu \frac{\partial I(r,\mu,x)}{\partial r} + \frac{1-\mu^2}{r} \frac{\partial I(r,\mu,x)}{\partial \mu} &= \chi_L[\beta + \phi(x)]\,[S(r,x,\mu) - I(r,\mu,x)] \\ &+ \left[(1-\mu^2)\frac{V(r)}{r} + \mu^2 \frac{\mathrm{d}V(r)}{\mathrm{d}r}\right] \frac{\partial I(r,\mu,x)}{\partial x} \\ &+ \chi_{\text{dust}}(r)\{S_{\text{dust}}(r,\mu,x) - I(r,\mu,x)\}, \end{aligned} \tag{10}$$

and for the oppositely directed beam,

$$\begin{aligned} -\mu \frac{\partial I(r,-\mu,x)}{\partial r} - \frac{1-\mu^2}{r} \frac{\partial I(r,-\mu,x)}{\partial \mu} &= \chi_L[\beta + \phi(x)]\,[S(r,x,-\mu) - I(r,-\mu,x)] \\ &+ \left[(1-\mu^2)\frac{V(r)}{r} + \mu^2 \frac{\mathrm{d}V(r)}{\mathrm{d}r}\right] \frac{\partial I(r,-\mu,x)}{\partial x} \\ &+ \chi_{\text{dust}}(r)\{S_{\text{dust}}(r,-\mu,x) - I(r,-\mu,x)\}, \end{aligned} \tag{11}$$

where $\mu \in (0, 1)$, and β is the ratio of absorption coefficients in the continuum and line centre. The quantity x is the normalized frequency given by

$$x = \frac{\nu - \nu_0}{\Delta}$$

where Δ is the standard width such as Doppler width and $V(r)$ is the velocity of expansion in units of Doppler width. The source function is given by

$$S(r, \pm\mu, x) = \frac{\phi(x)}{\beta + \phi(x)} S_L(r) + \frac{\beta}{\beta + \phi(x)} S_c(r, x), \tag{12}$$

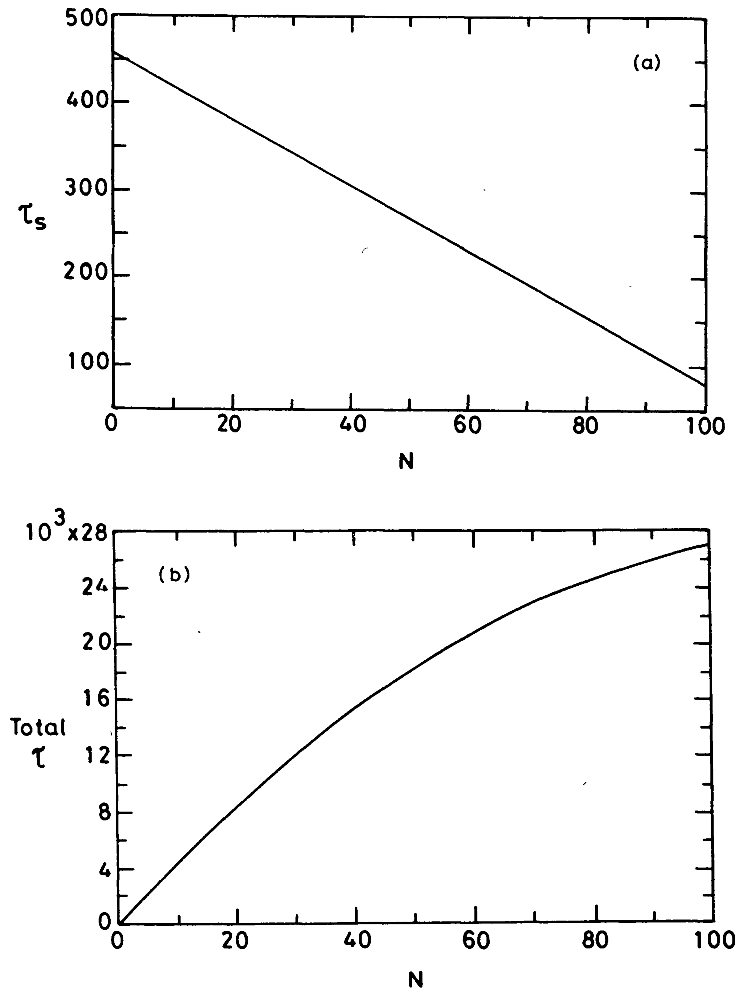

Figure 2. (a) Optical depth in each shell and (b) total optical depth at various layers of the medium.

with

$$S_L(r)=\frac{1-\varepsilon}{2}\int_{-\infty}^{+\infty}\phi(x)\int_{-\infty}^{+\infty}I(r,\mu',x)\mathrm{d}x\mathrm{d}\mu'+\varepsilon B(T(r),x), \tag{13}$$

and

$$S_c(r)=\rho(r)B(T(r),x). \tag{14}$$

Here, ε is the probability per scattering that a photon is thermalized by collisional de-excitation. S_L and S_c are the line and continuum source functions respectively. $B(T(r), x)$ is the Planck function and $\rho(r)$ is an arbitrary factor specified in advance. χ_{dust} is the dust absorption coefficient and S_{dust} the source function for the dust given by,

$$S_{\mathrm{dust}}(r,\pm\mu,x)=(1-\omega)B_{\mathrm{dust}}+\frac{1}{2}\omega\int_{-1}^{+1}P(\mu,\mu',r)I(r,\mu',x)\mathrm{d}\mu', \tag{15}$$

where B_{dust} is the dust source function, ω is the albedo for single scattering and $P(\mu, \mu', r)$ is the phase function of dust scattering which is assumed to be isotropic.

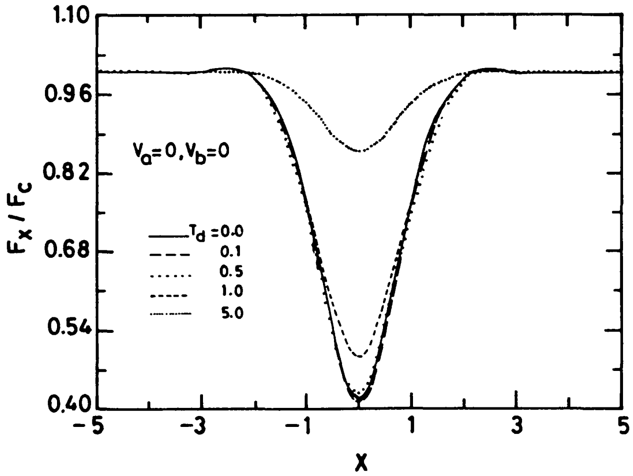

Figure 3. Hydrogen Lyman α line profiles formed in a static medium with different amounts of dust (τ_D). Distribution of dust is constant throughout the atmosphere.

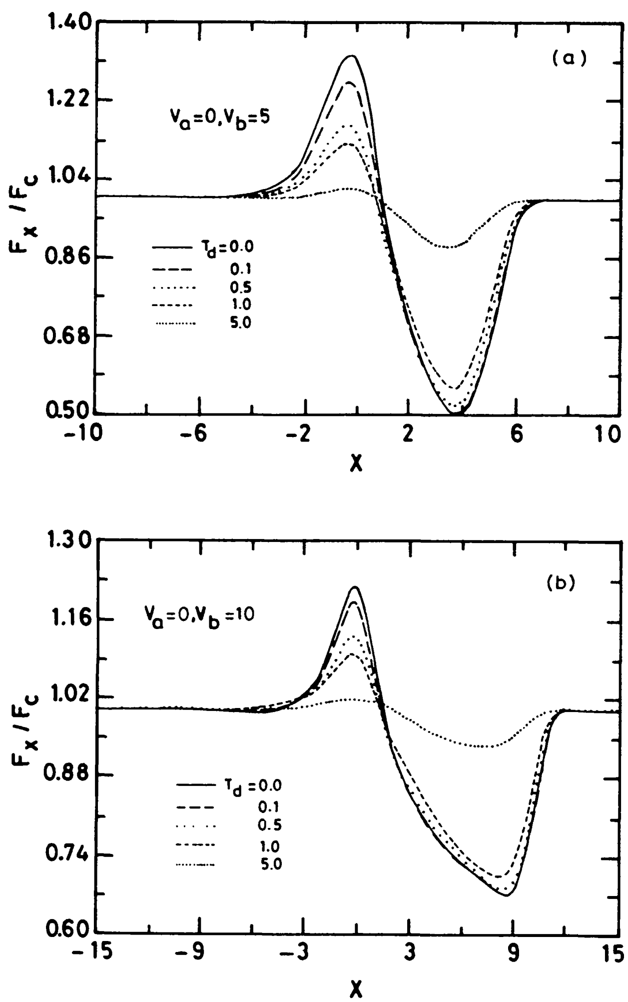

Figure 4. Line profiles of hydrogen Lyman α formed in a medium moving with velocity of expansion (a) $V_b = 5$ mtu and (b) $V_b = 10$ mtu (with velocity gradient).

Equations (10) and (11) are solved according to the procedure given in Peraiah, Varghese & Rao (1987).

3. Results

We have assumed a non-LTE line with a two-level atom approximation. We have set $\varepsilon=0$ and therefore no internal source is assumed. The boundary conditions are given as follows:

$$U^-(\tau=\tau_{\max}, \mu_j, X_i)=1,$$

and

$$\text{for } \varepsilon=0,\ \beta=0 \text{ and } B(T(r), x)=0. \qquad (16)$$

$$U^+(\tau=0, \mu_j, x_i)=0,$$

If a is the inner radius and b is the outer radius of the spherical medium then V_a and V_b represent the velocities in Doppler units at a and b respectively. We consider here two

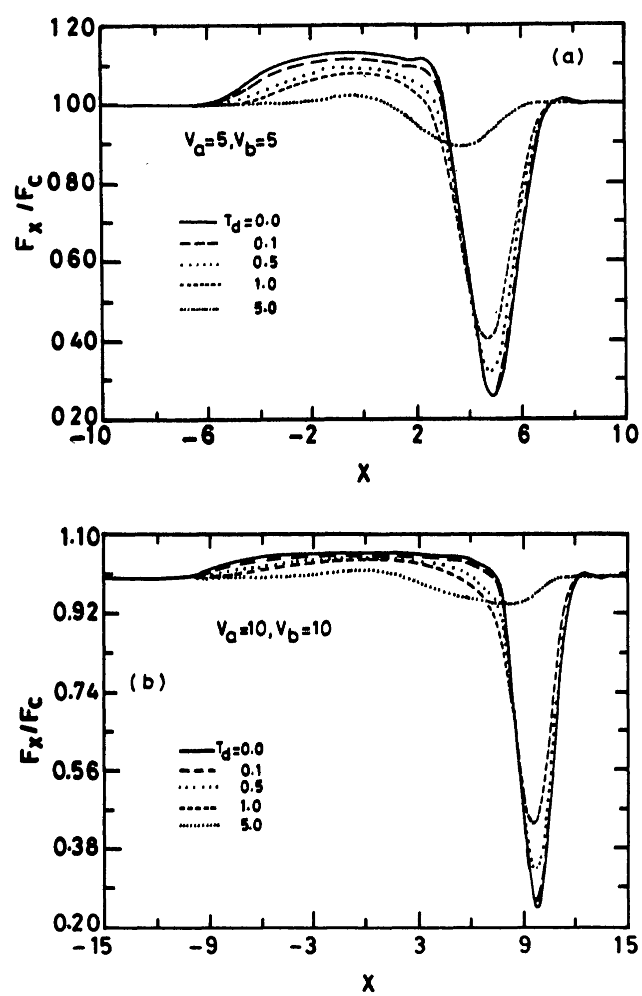

Figure 5. Hydrogen Lyman α line profiles formed in a medium expanding with a velocity of (a) 5 mtu and (b) 10 mtu without velocity gradients.

cases:

$$(1)\ V_a = V_b$$

and

$$(2)\ V_a = 0; \quad V_b > 0.$$

In case (1) we have a uniformly expanding spherical shell and in case (2), we have an expanding spherical shell with velocity gradients. The profiles of hydrogen Lyman α line are given in Figs 3–5. The dust optical depth τ_D is specified in advance and this is shown in the corresponding figures.

In Fig. 3 we plot the ratio of F_x/F_c where

$$F_x = 2\pi \int I_x \mu \mathrm{d}\mu,$$

$$F_c = 2\pi \int I_c \mu \mathrm{d}\mu,$$

I_x and I_c being the intensities in the line and continuum respectively, versus x for a static medium and with various dust optical depths. We obtain symmetric profiles in a static medium and as the dust optical depth is increased more photons are scattered into the centre of the line. In Fig. 4(a) we have introduced the velocity of expansion in which $V_b = 5$ mtu (mean thermal units) while keeping $V_a = 0$. We notice that *P*-Cygni type profiles have developed. The absorption is being shifted towards violet side while the emission peak remains at the centre of gravity of the line. When the dust optical thickness increases the emission reduces considerably while dust scatters more photons into the centre of the absorption core. Therefore it is clear that dust has opposite effects in the emission wings and in the absorption core. In Fig. 4(b) we plot the profiles with the increased expansion velocity of $V_b = 10$ mtu. The effects are similar to those for $V_b = 5$ mtu. Fig. 5 shows profiles from a spherical shell moving with constant velocities of 5 and 10 mtu. Here we note that the wing has become broader while the absorption has become narrow. However, the effects of dust remain the same. The shifts of the absorption core from the centre of the line are almost the

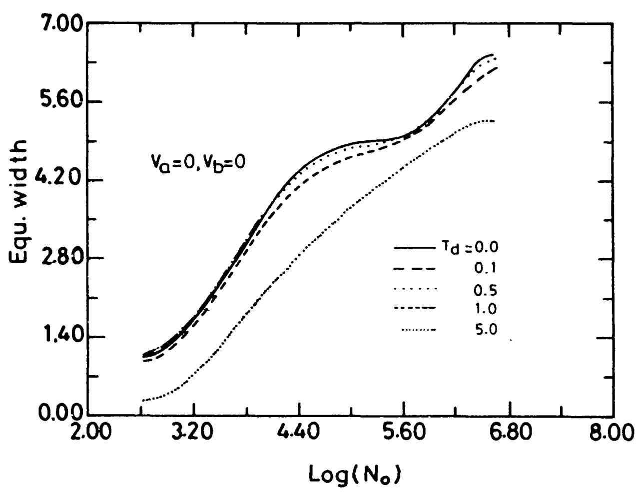

Figure 6. Variation of equivalent widths of hydrogen Lyman α line with the increasing number of neutral atoms (log N_0) for a static medium and for various dust optical depths (τ_D).

same as the expansion velocity used and the emission wing is nearly symmetrically broadened.

In Figs 6 to 8 we plot the equivalent widths versus the number of neutral hydrogen atoms. The equivalent width is defined by

$$\text{Eq.w.} = \int_{-\infty}^{+\infty} \left(1 - \frac{F_x}{F_c}\right) \mathrm{d}x,$$

In Fig. 6 we plot equivalent widths in units of mtu against $\log(N_0)$ for a static medium. It resembles the curve of growth. First we have linearly increasing portion and then a flat portion and again a linearly increasing portion. In Fig. 7 we have plotted the curves of growth for expanding spherical medium with velocity gradient for $V_b = 5$ and 10 mtu. Here we notice a slight change in the flat portion. For a given equivalent width the presence of dust implies larger number of neutral atoms. In Fig. 8 we have considered a spherical shell moving with constant velocity of 5 and 10 mtu. The behaviour is similar to a static case for low values of N_0. However, after

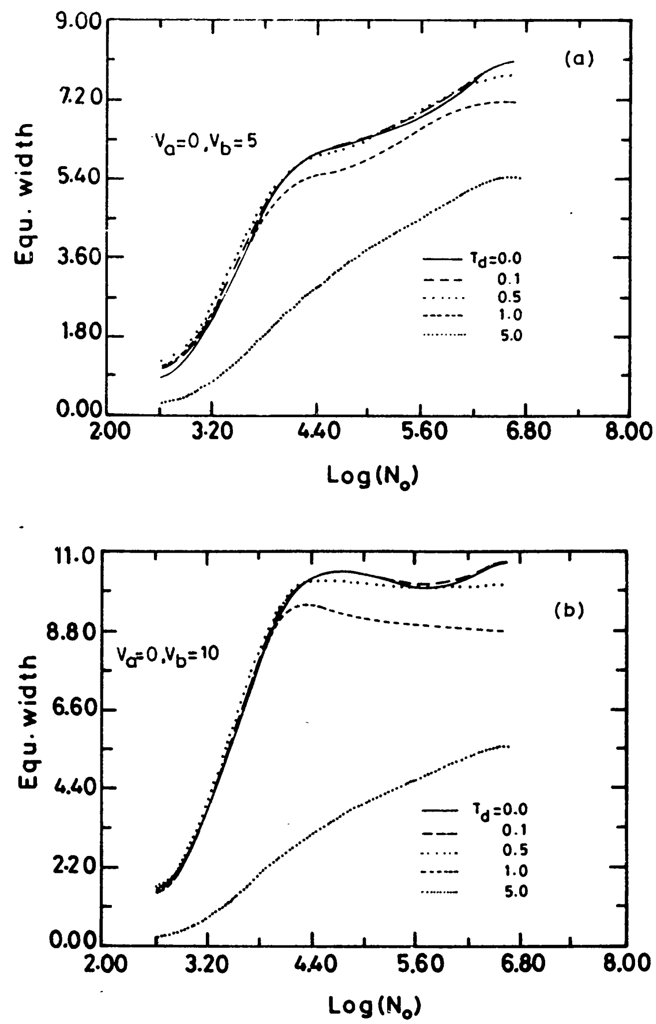

Figure 7. Variation of equivalent widths of hydrogen Lyman α line with increasing number of atoms when the atmosphere is expanding with velocity gradient, and (a) $V_b = 5$ mtu, and (b) $V_b = 10$ mtu.

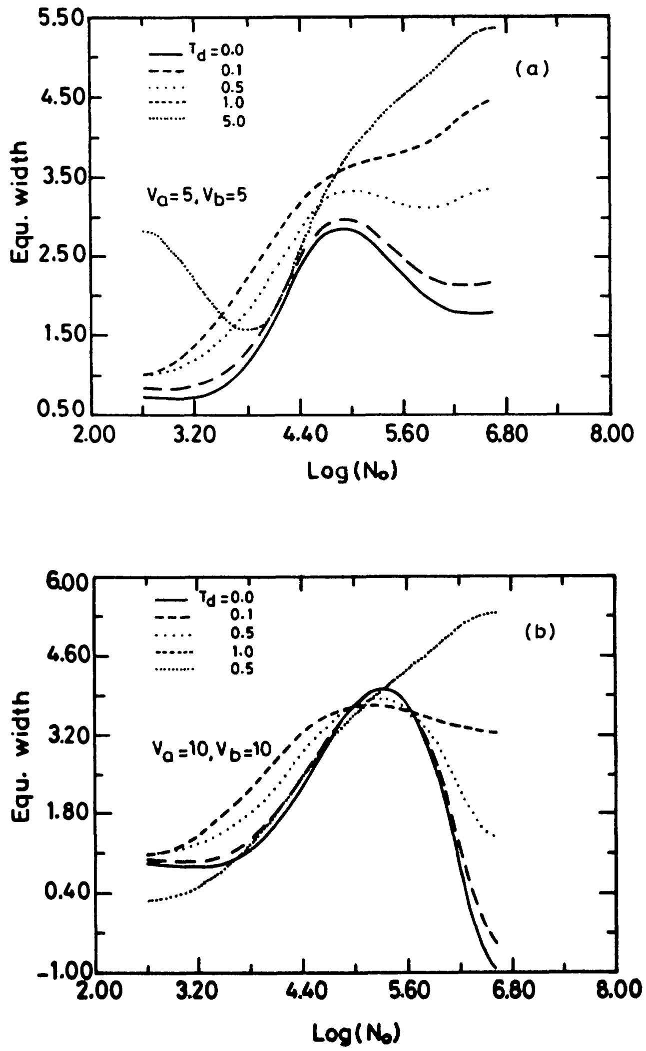

Figure 8. Variation of equivalent widths of hydrogen Lyman α line with increasing number of atoms in a uniformly expanding atmosphere with (a) $V_a = V_b = 5$ mtu and (b) $V_a = V_b = 10$ mtu.

equivalent widths reach a maximum, the width falls and the line appears more in emission. This means that the emission component of the *P*-Cygni profile is larger than the absorption component. The presence of dust increases the equivalent width.

4. Conclusion

In this paper we have calculated the effects of both velocity and dust on the equivalent widths of lines formed in spherically expanding atmospheres. It is found that substantial changes in the equivalent width are caused by the presence of dust in an expanding medium. It is also noticed that dust may increase the equivalent widths and one would overestimate the number of neutral atoms when the effect of dust is ignored.

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