

## Analysis of Ca<sup>+</sup>K Plage Area for Short Period Variation

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**Abstract.** We have analysed the observations of Solar Ca<sup>+</sup>K daily plage area for the period 1951–1977 to find evidence for the existence of short period (around 12–13 days) variation in the data. We divided the data in three groups—two corresponding to 10–20°N and 10–20°S latitude belts, and one corresponding to the total plage area—and used the power spectrum and autocorrelation techniques for the analysis. Both the techniques clearly show the 27-day periodicity due to solar rotation modulation in all the sets. A 12–13 day periodicity is seen in only 3, out of a total of 57 data sets when autocorrelation technique is used. A generally weak peak around 12–13 days is, however, seen in the power spectrum of all the data sets. The relative power in the 12–13 day peak is found to be significantly higher in those three data sets where the autocorrelation also shows this periodicity. On these two epochs the sunspot area distribution showed the existence of two distinct active longitudes separated by about 140–170 degrees. This seems to be the cause for the existence of a periodicity around 12–13 days in the autocorrelation and enhancement in the relative power of the 12–13 days peak in the power spectrum of these two epochs.

*Key words:* Sun, calcium plage area—Sun, rotation period

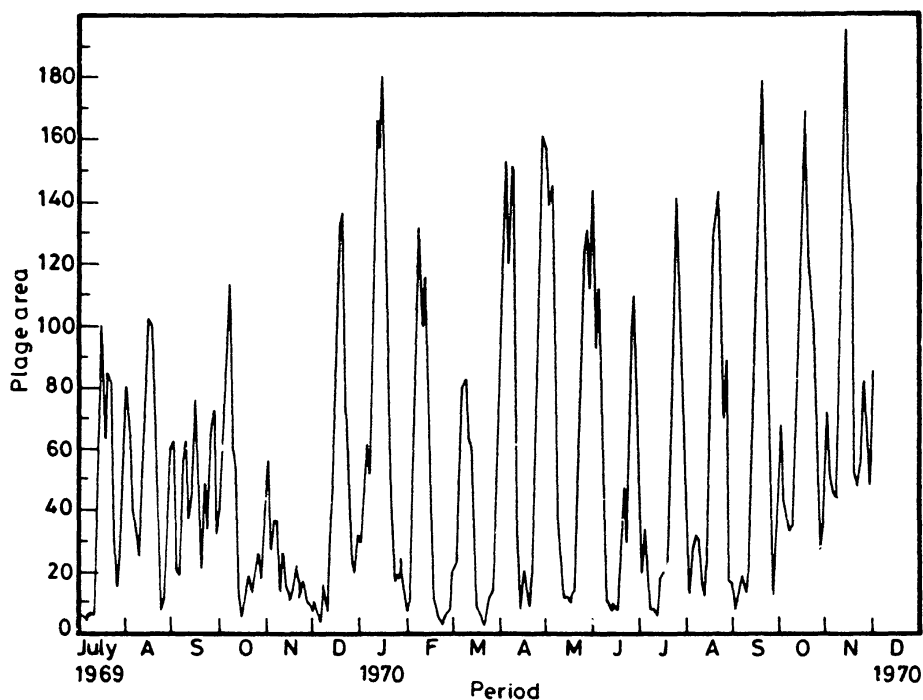
### 1. Introduction

In a recent work, Singh & Livingston (1987) have analysed daily Ca<sup>+</sup>K index in integrated sunlight and find a variation of 27 days due to solar rotation modulation. Knight, Schatten & Sturrock (1979) also reported the rotation rate peak at 27 days in the power spectra of the 122 years Zurich sunspot numbers; in addition, they found a significant peak at 12.07 day period. Earlier, Dicke (1976) had reported a 12.2 day (sidereal) periodicity in the measurements of solar oblateness. Hughes & Kesteven (1981), after analyzing the  $\lambda$ 10 cm solar flux data of 32 years, however, reported periodicities with period of about 25 and 31, 17 and 20, and 9 and 10 days but no evidence for the 12.6 days (synodic) period. Similarly, Stimets & Londono (1982) find no evidence of a periodicity around 12 days in their Ca<sup>+</sup>K flux of integrated sunlight measured over short period (about 8 months). Singh & Prabhu (1985) have analyzed the Ca<sup>+</sup>K plage area data spread over 30 years (1951–1981) for the study of variations in the solar rotation rate. They found a distinct peak in the power spectrum around 27 day period; in addition, a weak peak is noticed around 13 day period. We have further analysed the data of Ca<sup>+</sup>K plage area by autocorrelation and power spectrum

techniques to study the stability of the peak around 12–13 day period, and investigate the possible causes, such as presence of two active longitudes separated by around 180 degree, for this peak.

## 2. Data analysis and results

The details of the data are given in an earlier paper by Singh & Prabhu (1985). For the present study the plage area was considered in three groups—(i) over 10–20°N latitude belt, (ii) over 10–20°S latitude belt, and (iii) the total plage area over the visible disc. The total period of 27 years was divided into 19 sets, each of 512 days. A typical data set is shown in Fig. 1. Autocorrelation coefficients and FFT power spectrum for each set were computed. The results of autocorrelation analysis are given in Table 1, and those of FFT power spectrum analysis in Table 2. In Table 1, column 1 gives the period of observations, columns 2–6 give periods derived from the successive peaks in the auto-correlation, column 7 gives the mean periodicity along with the standard deviation. The periods in columns 2–6 have been derived from the corresponding peaks with respect to the peak at zero phase. In Table 2 the period of observations is given in column 1, the prominent peak around 27 days due to solar rotation modulation is given in column 2, and the weak peak around 13-day period is listed in column 3. The relative power in the 13-day peak with respect to the power in the corresponding 27-day peak is also listed in column 3 within parenthesis. Columns 2–3 correspond to the results of 10–20°S latitude belt and columns 4–7 correspond to the results of the other two groups.



**Figure 1.** Ca + K daily plage area—a typical raw data set. The area is given in millionths of the visible solar disc.

**Table 1.** Periods (in days) derived from autocorrelation peak separations. Dash implies that the separation could not be determined unambiguously from the corresponding peak.

Period	Periodicity derived from peak					
	I	II	III	IV	V	Mean
<i>10–20° S latitude</i>						
1951 Jan–1952 May	—	—	—	—	—	—
1952 Jun–1953 Oct	26.5	27.2	27.8	27.5	27.6	27.3 ± 0.5
1953 Nov–1955 Mar	26.3	27.5	24.0	24.4	24.9	25.4 ± 1.3
	14.0	—	15.1	12.0	12.3	13.3 ± 1.3
1955 Apr–1956 Aug	26.0	26.3	27.2	26.5	26.4	26.5 ± 0.4
1956 Sep–1958 Jan	27.0	28.3	27.8	27.9	28.2	27.8 ± 0.5
1958 Feb–1959 Jun	27.5	28.5	29.0	29.0	—	28.5 ± 0.6
1959 Jul–1960 Nov	26.2	25.6	23.4	24.4	—	24.9 ± 1.1
1960 Dec–1962 Apr	24.0	24.6	23.4	24.4	24.4	24.2 ± 0.4
1962 May–1963 Oct	26.2	26.2	26.7	—	26.4	26.4 ± 0.2
1963 Nov–1965 Mar	26.2	27.3	25.5	—	—	26.3 ± 0.7
1965 Apr–1966 Aug	27.0	27.3	27.3	27.5	26.8	27.2 ± 0.2
1966 Sep–1968 Jan	27.3	27.3	27.3	26.8	27.0	27.1 ± 0.2
1968 Feb–1969 Jun	28.0	29.0	28.0	28.3	27.2	28.1 ± 0.6
1969 Jul–1970 Nov	27.0	27.2	27.3	27.1	27.2	27.2 ± 0.1
1970 Dec–1972 Apr	25.3	24.8	—	—	—	25.1 ± 0.2
1972 May–1973 Sep	25.4	26.2	24.3	24.7	24.7	25.1 ± 0.7
1973 Oct–1975 Feb	27.3	28.3	—	—	—	27.8 ± 0.5
1975 Mar–1976 Jul	29.4	—	—	—	27.4	28.4 ± 1.0
1976 Aug–1977 Dec	26.5	27.2	27.8	27.8	—	27.3 ± 0.5
<i>10–20° N latitude</i>						
1951 Jan–1952 May	26.8	27.0	27.1	—	—	26.9 ± 0.1
1952 Jun–1953 Oct	27.0	27.5	26.9	27.2	27.4	27.2 ± 0.2
1953 Nov–1955 Mar	26.0	27.0	27.3	26.9	27.4	26.9 ± 0.5
1955 Apr–1956 Aug	26.2	27.0	28.3	29.0	28.6	27.8 ± 1.0
1956 Sep–1958 Jan	26.5	26.5	27.2	27.5	27.6	27.1 ± 0.5
1958 Feb–1959 Jun	26.5	27.3	27.6	27.0	27.0	27.1 ± 0.4
1959 Jul–1960 Nov	26.5	27.2	27.2	27.3	27.0	27.0 ± 0.3
1960 Dec–1962 Apr	26.5	27.5	26.7	26.7	26.4	26.8 ± 0.4
1962 May–1963 Oct	26.0	27.8	26.5	26.5	25.9	26.5 ± 0.7
1963 Nov–1965 Mar	25.0	25.3	24.7	—	26.8	25.5 ± 0.8
1965 Apr–1966 Aug	26.0	27.5	27.0	27.0	27.0	26.9 ± 0.5
	14.0	13.0	—	—	—	13.5 ± 0.5
1966 Sep–1968 Jan	27.0	27.5	26.6	26.6	—	26.9 ± 0.4
1968 Feb–1969 Jun	28.5	28.3	28.0	27.8	—	28.2 ± 0.3
1969 Jul–1970 Nov	26.0	26.3	25.0	25.5	25.8	25.7 ± 0.3
1970 Dec–1972 Apr	25.5	22.8	22.3	23.6	—	23.6 ± 1.2
1972 May–1973 Sep	26.5	26.5	27.0	26.8	26.6	26.7 ± 0.2
1973 Oct–1975 Feb	28.5	—	—	—	—	28.5
1975 Mar–1976 Jul	26.0	—	—	—	—	26.0
1976 Aug–1977 Dec	26.0	26.8	27.2	27.3	26.3	26.7 ± 0.5
<i>Total plage area</i>						
1951 Jan–1952 May	27.0	28.5	28.4	27.6	26.9	27.7 ± 0.7
1952 Jun–1953 Oct	—	—	—	—	—	—
1953 Nov–1955 Mar	30.0	28.2	27.2	—	—	28.5 ± 1.2
1955 Apr–1956 Aug	—	—	—	—	—	—
1956 Sep–1958 Jan	27.0	27.3	27.8	27.6	27.8	27.5 ± 0.3

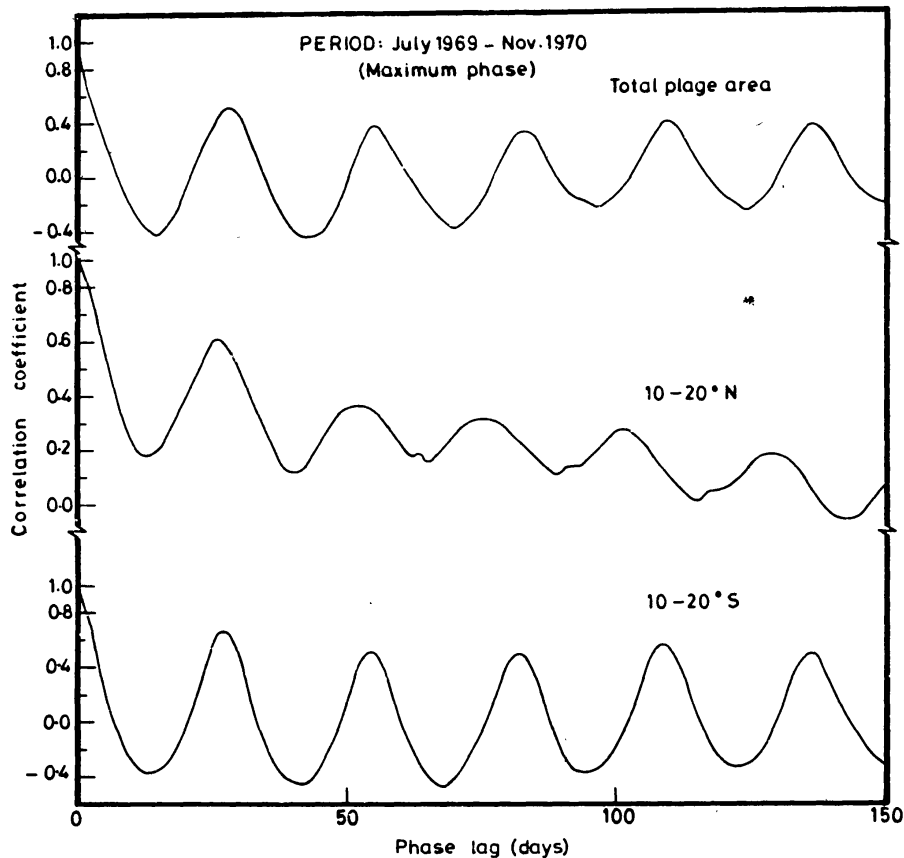
Table 1. Continued.

Period	Periodicity derived from peak					
	I	II	III	IV	V	Mean
1958 Feb–1959 Jun	27.0	27.8	28.3	28.9	29.0	28.2±0.7
1959 Jul–1960 Nov	27.0	27.0	—	—	—	27.0±0.0
1960 Dec–1962 Apr	25.0	28.0	27.3	27.5	27.0	27.0±1.0
1962 May–1963 Oct	27.5	29.0	27.5	26.8	27.1	27.6±0.7
1963 Nov–1965 Mar	25.0	27.2	24.9	—	27.0	26.0±1.1
1965 Apr–1966 Aug	28.0	27.5	27.2	27.2	26.8	27.3±0.4
	14.0	14.3	—	13.8	13.2	13.8±0.4
1966 Sep–1968 Jan	—	—	—	—	—	—
1968 Feb–1969 Jun	29.0	27.5	27.2	28.3	—	28.0±0.7
1969 Jul–1970 Nov	28.0	27.5	27.6	27.4	27.3	27.6±0.2
1970 Dec–1972 Apr	25.0	24.0	24.3	23.7	24.3	24.3±0.4
1972 May–1973 Sep	—	—	—	—	—	—
1973 Oct–1975 Feb	—	—	—	—	—	—
1975 Mar–1976 Jul	27.0	26.0	26.0	27.0	28.1	27.2±0.8
1976 Aug–1977 Dec	25.5	28.5	25.3	—	26.5	26.4±1.3

Table 2. Periods (in days) derived from the power spectrum analysis; the relative power in the peak around 13 days is given within parentheses.

Period	10–20°S		10–20°N		Total plage area	
	I	II	I	II	I	II
1951 Jan–1952 May	—	—	26.9	13.8(0.19)	27.8	13.8(0.13)
1952 Jun–1953 Oct	27.7	13.6(0.68)	26.9	13.8(0.10)	28.4	13.5(1.70)
1953 Nov–1955 Mar	24.4	13.8(0.48)	26.9	13.8(0.14)	26.9	13.6(0.28)
1955 Apr–1956 Aug	27.0	13.5(0.18)	28.4	13.8(0.24)	25.6	13.8(1.80)
1956 Sep–1958 Jan	28.1	13.5(0.09)	26.9	13.5(0.22)	28.4	13.5(0.15)
1958 Feb–1959 Jun	28.4	13.8(0.14)	26.9	13.5(0.36)	27.7	13.5(0.24)
1959 Jul–1960 Nov	26.9	13.7(0.18)	27.7	13.5(0.25)	27.7	13.3(0.36)
1960 Dec–1962 Apr	24.4	12.1(0.15)	26.9	13.1(0.07)	26.9	13.5(0.15)
1962 May–1963 Oct	26.3	14.6(0.47)	28.4	13.8(0.25)	27.8	14.6(0.19)
1963 Nov–1965 Mar	24.6	13.8(0.14)	26.6	13.8(0.13)	26.9	13.8(0.12)
1965 Apr–1966 Aug	26.9	13.8(0.27)	26.9	13.5(0.65)	26.9	13.8(0.61)
1966 Sep–1968 Jan	26.9	13.8(0.16)	26.9	13.5(0.04)	—	—
1968 Feb–1969 Jun	29.2	13.7(0.14)	27.7	14.4(0.30)	27.7	13.5(0.16)
1969 Jul–1970 Nov	26.9	13.5(0.14)	26.3	13.3(0.07)	27.7	13.5(0.18)
1970 Dec–1972 Apr	25.6	14.2(0.30)	25.6	13.8(0.16)	25.6	13.5(0.19)
1972 May–1973 Sep	24.4	13.8(0.33)	26.9	13.5(0.14)	26.3	14.2(0.62)
1973 Oct–1975 Feb	30.1	13.5(0.33)	25.6	13.5(0.07)	28.4	14.2(0.15)
1975 Mar–1976 Jul	28.4	13.8(0.30)	26.9	13.5(0.62)	26.9	12.8(0.30)
1976 Aug–1977 Dec	28.4	13.8(0.08)	26.9	13.8(0.08)	26.9	13.8(0.26)

Autocorrelation coefficients as a function of phase lag for the three groups at one of the maxima and one of the minima of solar cycle are shown, respectively, in Figs 2 and 3. The autocorrelation coefficients corresponding to the maximum phase go through well-defined cycles which may be due to the occurrence of large number of



**Figure 2.** Autocorrelation function of the daily plage area for the period 1969 July–1970 November (maximum phase) for  $10\text{--}20^\circ\text{S}$ ,  $10\text{--}20^\circ\text{N}$  latitude belts and total plage area.

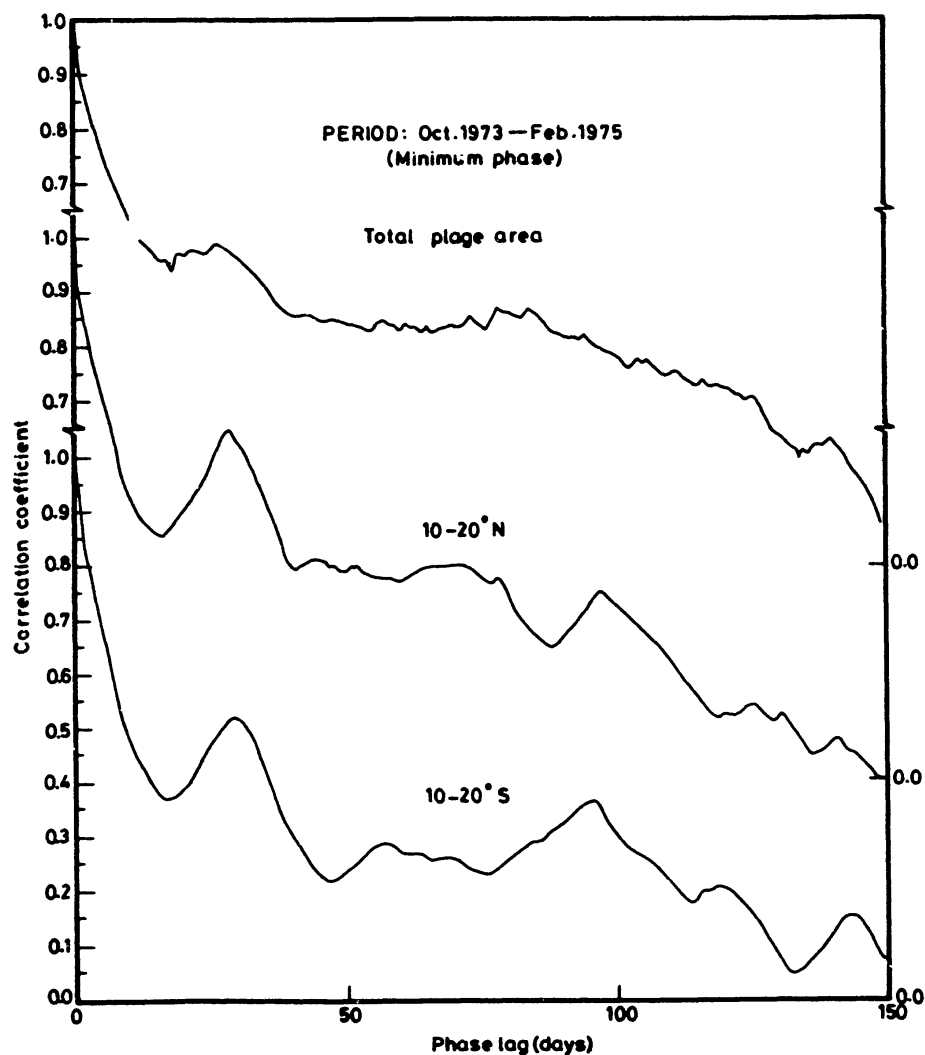
long-lived plages during this phase. The behaviour of the autocorrelation coefficients at the minimum phase is, however, not so well-defined though it yields the period of rotation. This may be due to the occurrence of a small number of plages, that too short-lived, during this phase.

### 3. Discussion

The autocorrelation analysis (Table 1) shows that only one data set out of a total of 19 sets has a 13-day periodicity in addition to the 27-day periodicity in each of the three groups. For example, during the period 1965 April–1966 August, the 13-day periodicity is seen in the  $10\text{--}20^\circ\text{N}$  latitude belt and in the integrated plage area but not in the  $10\text{--}20^\circ$  belt. Similarly, during the period 1953 November–1955 March, the 13-day periodicity is seen in only  $10\text{--}20^\circ\text{S}$  belt but not in the other two groups.

The results of the power spectrum analysis (Table 2) show that power in the 13-day peak is generally very low as compared to the power in the 27-day peak. The relative error in the power spectra at the 27-day and 13.5-day periods were estimated using the following expression (Hoyng 1976):

$$\text{Relative error, } \frac{\varepsilon_p}{|a_p|^2} \simeq (2x - x^2)^{1/2},$$

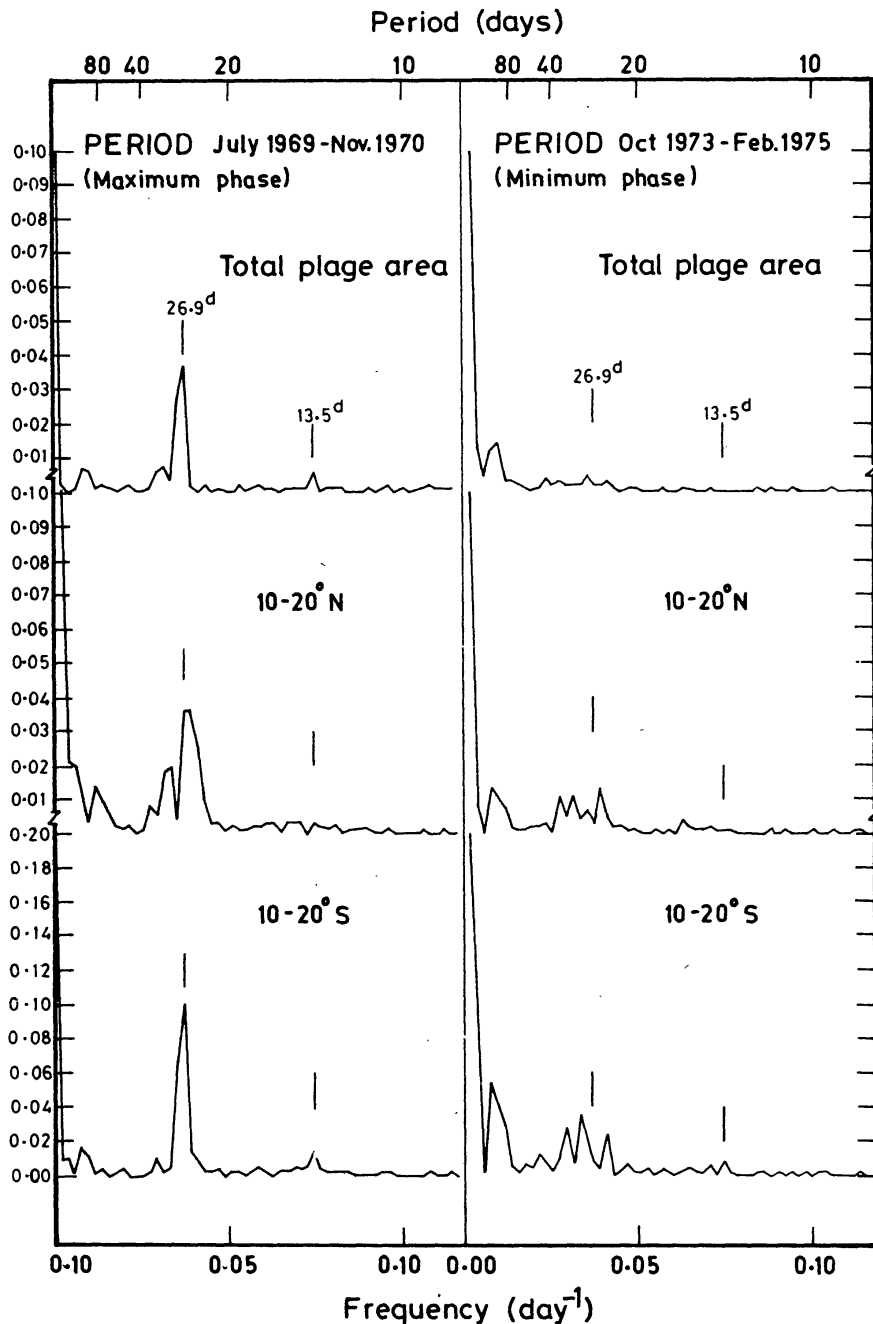


**Figure 3.** Autocorrelation function of the daily plage area for the period 1973 October–1975 February (minimum phase) for 10–20°S, 10–20°N latitude belts and the total plage area.

where  $x$  equals the ratio of average high-frequency power and the power at frequency  $p$ .

The computed values of relative error range between 0.08 to 0.19 at the 27-day period, and 0.28 to 0.75 at the 13.5-day period. These values indicate that whereas the peak at 27 days in all the spectra is quite real, the 13.5-day peak may not be real in most of the spectra as the relative error exceeds 0.50 in those cases.

In some cases, however, the relative power (*i.e.*, the power with respect to the power in the 27-day peak) in the 13-day peak is appreciable. This is particularly noticeable in the periods 1953 November–1955 March and 1965 April–1966 August—the periods in which the 13-day periodicity is seen clearly in the autocorrelation coefficients as well. To investigate the cause of the appearance of 13-day periodicity during these periods, we have studied the frequency distribution of sunspot area (Royal Greenwich Annals, Photoheliographic data) as a function of heliographic longitude. Two longitudes,



**Figure 4.** Power spectra of the daily plage area for the data sets corresponding to Figs 2 and 3.

approximately  $140\text{--}170^\circ$  apart, were found to be more active.\* This is likely to have given higher power in the 13-day peak in the power spectrum and a periodicity of 13 days in the autocorrelation coefficients on these two epochs.

\* This observation can be fruitfully exploited in stellar work. In stellar observations, one generally measures the  $\text{Ca}^+ \text{K}$  flux and does power spectrum analysis to study the temporal variation due to rotation and other causes. An enhancement in the higher-frequency components in the power spectrum can, possibly, be attributed to the presence of other active longitudes on the star.



Donnelly (1988) finds that power spectra of short-term variations of the solar ultraviolet UV flux consistently include two peaks, the largest peak falling in the 27–28 day range and the second largest falling in the 13–14 day range. The ratio of the 13-day power and the 27-day power for the wavelength region 165–260 nm and for the Mg I line fall in the range from 0.4 to just above 0.5, which are very high and uniform compared to the values 0.31 and 0.05 for the sunspot number  $R$  and  $\lambda 10.7$  cm radio flux, respectively. He shows that most of the power in the 13–14 day peak comes from two peaks in the active region distribution per solar rotation (Figure 2 in Donnelly 1988). Earlier, using isolated sets of Ca<sup>+</sup>K plage data, each of 2–4 months duration, Donnelly *et al.* (1983) showed that the 13-day periodicity is caused by the concentration of plages at solar longitudes nearly 180° apart.

Further, one notices in Table 2 that relative power in the 13-day peak, particularly in the total plage area, is very high at another three epochs (1952 June–1953 October, 1955 April–1956 August and 1972 May–1973 September) as well. The power spectra at these epochs are very noisy and contain multiple peaks around the rotation modulation peak. The autocorrelation (Table 1) at these epochs also fails to give any peak, including the rotation rate, for the total plage area. All the five epochs represent the ‘minimum’ phase of solar cycle. Occurrence of a small number of short-lived and small size plages during the ‘minimum’ phase appears to be responsible for the noisy power spectra and absence of rotation peak in the autocorrelation of these epochs.

It is thus seen that, barring the five epochs discussed above, a generally weak peak at 13-day period is found in the power spectra of all the data sets. Power spectrum analysis is known to produce harmonics in addition to a peak at the fundamental frequency in case of non-sinusoidal distribution of data. One may note from Fig. 1 that the Ca<sup>+</sup>K plage area data is non-sinusoidal, and this might be responsible for the weak peak at 13-day period in the power spectrum. In fact, the autocorrelation technique was used additionally to differentiate between the harmonics and the real peaks.

Thus, our analysis of the data of Ca<sup>+</sup>K plage area spanning over 27 years show that (i) autocorrelation technique is more suited for the analysis of such data, and (ii) the 13-day periodicity, at two epochs (1953 November–1955 March and 1965 April–1966 August) is due to the occurrence of enhanced activity at two different longitudes at a given time.

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