Magnetic fluctuations and Hall magnetohydrodynamic turbulence in the solar wind

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[1] It is shown that the framework of Hall magnetohydrodynamics (Hall-MHD), which can support three quadratic invariants and allows nonlinear states to depart fundamentally from the Alfvénic, is capable of reproducing in the inertial range the three branches of the observed solar wind magnetic fluctuation spectrum: the Kolmogorov branch $f^{-5/3}$, steepening to $f^{-\alpha_1}$, with $\alpha_1 \simeq 3-4$ on the high-frequency side and flattening to f^{-1} on the low-frequency side. These fluctuations are found to be associated with the nonlinear Hall-MHD shear Alfvén waves. The spectrum of the concomitant whistler-type fluctuations is very different from the observed one. Perhaps the relatively stronger damping of the whistler fluctuations may cause their unobservability. The issue of the anisotropy of the turbulence is addressed briefly. *INDEX TERMS:* 2149 Interplanetary Physics: MHD waves and turbulence; 2164 Interplanetary Physics: Solar wind plasma; 2159 Interplanetary Physics: Plasma waves and turbulence; *KEYWORDS:* solar wind, mhd turbulence, spectral distributions, Hall effect, shear hall waves, generalized helicity

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1. Introduction

[2] The spectral energy distributions of the velocity and the magnetic field fluctuations in the solar wind are now known in a wide frequency range, beginning from much below the proton cyclotron frequency (0.1–1 Hz) and going all the way to hundreds of Hertz. The inferred power spectrum of magnetic fluctuations consists of multiple segments: a Kolmogorov-like branch ($\propto f^{-5/3}$) flanked, on the low-frequency end, by a flatter branch ($\propto f^{-1}$) and on the high-frequency end by a much steeper branch ($\propto f^{-\alpha_1}$, $\alpha_1 \simeq 3-4$) [*Coleman*, 1968; *Behannon*, 1978; *Denskat et al.*, 1983; *Goldstein et al.*, 1994; *Leamon et al.*, 1998]. Attributing the Kolmogorov branch ($\propto f^{-5/3}$) to the standard inertial range cascade, initial explanations invoked dissipation processes (in particular, the collisionless damping of Alfvén and magnetosonic waves [*Leamon et al.*, 1998; *Gary*, 1999; *Marsch*, 1991]) to explain the steeper branch ($\propto f^{-\alpha_1}$, $\alpha_1 \simeq 3-4$). However, a recent critical study has concluded that damping of the linear Alfvén waves via the proton cyclotron resonance and of the magnetosonic waves by the Landau resonance, being strongly k-dependent (wave vector), is quite incapable of producing a power law spectral distribution of magnetic fluctuations [*Li et al.*, 2001]; damping mechanisms lead, instead, to a sharp cutoff in the power spectrum. *Cranmer and von Ballagooijen* [2003] have, however, demonstrated a weaker than exponential dependence of damping on the wave vector by including kinetic effects. However, it is still steeper than that required for explaining the steepened spectrum.

[3] An alternative possibility, suggested by *Ghosh et al.* [1996], links the spectral break and subsequent steepening to a "change" in the "controlling" invariants of the system in the appropriate frequency range. *Matthaeus et al.* [1996] have investigated the anisotropies in the spectral as well as in the variances of the three-dimensional magnetohydrodynamic (MHD) turbulence. *Stawicki et al.* [2001] have invoked the short-wavelength dispersive properties of the magnetosonic/whistler waves to account for the steepened spectrum and christened it as the spectrum in the dispersion range. In this paper we follow and develop these ideas within the framework of Hall magnetohydrodynamics (Hall-MHD). We will harness the three well-known invariants of Hall-MHD [*Mahajan and Yoshida*, 1998; *Krishan and Mahajan*, 2004]. Using dimensional arguments of the Kolmogorov type, we will first derive the fluctuation spectra associated with the velocity and magnetic fields. We then go on to show that in different spectral ranges, different invariants control the energy cascade splitting the inertial range into distinct sections. The steeper and the flatter spectral branches (together with the standard branch), then, are all subparts of the extended inertial range. Invoking the hypothesis of selective dissipation, we then construct the entire magnetic spectrum with its three branches and two breaks by stringing together three spectral segments, each controlled by one of the three invariants.

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[4] The details of the nonlinear Hall-MHD have been presented elsewhere (S. M. Mahajan and V. Krishan, Exact nonlinear Hall-MHD waves, submitted to Physics Review Letters, 2004, hereinafter referred to as Mahajan and Krishnan, submitted manuscript, 2004) (see http://peaches.ph.utexas.edu/ifs/reports2004.html). Here, the aspects relevant to this investigation along with the quadratic invariants are summarized in section 2. In section 3 the respective spectral energy distributions are derived. The derived spectra are shown to account for the observed solar wind spectra in section 4. A short discussion and a summary of the conclusions constitutes section 5.

2. Hall Magnetohydrodynamics (Hall-MHD), Nonlinear Solution, and Invariants

[5] The importance of the nonlinear Alfvénic state for MHD prompts one to speculate if a similar kind of an exact solution exists for Hall-MHD, a system which encompasses MHD but which can sustain a much richer spectrum of plasma states not accessible to MHD.

[6] In the Alfvénic units, with the magnetic field **B** normalized to an ambient field, the velocity **V** normalized to the corresponding Alfvén speed, time and space variables measured in units of the ion gyroperiod $\omega_c^{-1} = m_i c/qB_0$ and the ion skin depth $\lambda_i = c/\omega_{pi}$, respectively, where $\omega_{pi} = (4\pi q^2 n/m_i)^{1/2}$ is the ion plasma frequency, the following dimensionless equations,

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times [(\mathbf{V} - \nabla \times \mathbf{B}) \times \mathbf{B}] \tag{1}$$

$$\frac{\partial (\mathbf{B} + \nabla \times \mathbf{V})}{\partial t} = \nabla \times [\mathbf{V} \times (\mathbf{B} + \nabla \times \mathbf{V})], \tag{2}$$

constitute Hall-MHD. Notice that in equation (2), obtained by taking the curl of the ion force balance equation, the pressure gradient term $\nabla P/n$ has disappeared because it has been assumed to be a perfect gradient (by invoking an equation of state $P = P(n)$, for example); the pressure has not been neglected.

[7] First, we will recount the essential elements of the recently found fully nonlinear wave sustained by Hall-MHD (Mahajan and Krishan, submitted manuscript, 2004). This arbitrary amplitude wave (which contains the standard Alfvénic (whistler) nonlinear state as its long (short) wavelength limit) has the character of a time-dependent ABC flow in the magnetic and velocity fields. For this paper the most important aspect of this wave is the wave number–dependent relationship

$$\mathrm{B}_k = \alpha(k)\mathrm{V}_k \tag{3}$$

between the fluctuating magnetic and velocity fields along with the incompressibility condition $\beta \gg 1$. The proportionality factor turns out to be

$$\alpha_\pm = \left[-\frac{k}{2} \pm \left(\frac{k^2}{4} + 1\right)^{1/2}\right], \tag{4}$$

yielding the nonlinear dispersion relation

$$\omega = \alpha k_s, \tag{5}$$

where k_s is the projection of the wave vector along the ambient field $B_0 = B_0\hat{e}_s$. As stated earlier, in the limit $k \ll 1$ the MHD Alfvénic state

$$\alpha \to \pm 1, \qquad \omega \to \mp k_s, \tag{6}$$

with k independent relationships for both the copropagating and the counterpropagating waves, is dutifully recovered. For $k \gg 1$ it is easy to recognize, in analogy with the linear theory, that the (+) wave is the shear cyclotron branch, while the (−) represents the magnetosonic whistler mode. The frequency of the (+) wave approaches some fraction of the ion gyro frequency (normalizing frequency); it is only when k and B_0 are fully aligned ($\hat{k} \cdot \hat{e}_s = \pm 1$) that the wave reaches the cyclotron frequency asymptotically. The fluctuation relation given by equation (3) will provide a crucial element in the construction of the kinetic and magnetic energy spectra.

[8] The Hall-MHD equations (1)–(2) may be manipulated to extract the well-known invariants [*Yoshida and Mahajan*, 2002]Total energy

$$E = \frac{1}{2}\int \left(V^2 + B^2\right) d^3x = \frac{1}{2}\sum_k |V_k|^2 + |B_k|^2, \tag{7a}$$

Magnetic helicity

$$H_M = \frac{1}{2}\int \boldsymbol{A} \cdot \boldsymbol{B} d^3x = \frac{1}{2}\sum_k \frac{i}{k^2}(\boldsymbol{k} \times \boldsymbol{B}_k) \cdot \boldsymbol{B}_{-k}, \tag{7b}$$

Generalized helicity

$$H_G = \frac{1}{2}\int (\boldsymbol{A} + \boldsymbol{V} \cdot (\boldsymbol{B} + \nabla \times \boldsymbol{V}) d^3x = \frac{1}{2}\sum_k \left[\frac{i\boldsymbol{k} \times \boldsymbol{B}_k}{k^2} + \boldsymbol{V}_k\right] \cdot [\boldsymbol{B}_{-k} - i\boldsymbol{k} \times \boldsymbol{V}_{-k}], \tag{7c}$$

where $\boldsymbol{A}$ is the vector potential. Notice that $H_G - H_M$ is a combination of the kinetic and the cross helicities.

[9] Since the relationship between $\boldsymbol{V}_k$ and $\boldsymbol{B}_k$ in Hall-MHD were just now shown to be k-dependent, it is expected that the current spectral predictions will be substantially different from those of the standard MHD (where $\boldsymbol{V}_k$ and $\boldsymbol{B}_k$ have identical spectra), particularly in the range $k \gg 1$, when the Hall term in equation (3) dominates. The introduction of the Hall term, which brings in an intrinsic scale (the ion skin depth), removes the MHD spectral degeneracy and generates new scale-specific effects.

3. Spectral Energy Distributions

[10] In order to derive the spectral energy distributions, we resort to the Kolmogorov hypothesis, according to which the spectral cascades proceed at a constant rate governed by the eddy turnover time $(k\boldsymbol{V}_k)^{-1}$. For ε_E denoting the constant cascading rate of the total energy E, equation (7a), along with equation (3), yields the dimensional equality

$$(kV_k)\left[1 + (\alpha)^2\right]\frac{V_k^2}{2} = \varepsilon_E. \tag{8}$$

The omnidirectional spectral distribution function $W_E(k)$ (kinetic energy per gram per unit wave vector V_k^2/k), then, takes the form

$$W_E(k) = (2\varepsilon_E)^{\frac{2}{3}}\left[1+(\alpha)^2\right]^{-\frac{2}{3}}k^{-\frac{5}{3}}. \tag{9}$$

Consequently, equation (3) yields

$$M_E(k) = (\alpha)^2 W_E(k), \tag{10}$$

where $M_E(k) = B_k^2/k$ is the similarly defined omnidirectional spectral distribution function of the magnetic energy density.

[11] The cascading of the magnetic helicity H_M (ε_H being the cascading rate for helicity) produces a different dimensional equality,

$$(kV_k)\left(0.5\frac{B_k^2}{k}\right) = \varepsilon_H, \tag{11}$$

resulting in the following different kinetic and magnetic spectral energy distributions:

$$W_H(k) = (2\varepsilon_H)^{\frac{2}{3}}\,(\alpha)^{\frac{-4}{3}}k^{-1} \tag{12}$$

$$M_H(k) = (\alpha)^2 W_H(k). \tag{13}$$

Finally, the cascading of the generalized helicity with a constant rate ε_G gives

$$(kV_k)\left[0.5g(k)V_k^2\right] = \varepsilon_G \tag{14}$$

$$g(k) = (\alpha+k)^2 k^{-1},$$

leading to the spectral energy distributions

$$W_G(k) = (2\varepsilon_G)^{\frac{2}{3}}[g(k)]^{-\frac{2}{3}}k^{-\frac{5}{3}} \tag{15}$$

and

$$M_G(k) = (\alpha)^2 W_G(k).$$

4. Modeling Solar Wind Spectra

[12] The observed frequency spectra of the solar wind are transformed into the wave vector spectra Doppler shifted by the super Alfvénic solar wind flow. Although the anisotropy of the MHD turbulence is now being highly emphasized [*Matthaeus et al.*, 1996], we model the observed reduced omnidirectional spectra with the findings of the isotropic cascade considered in section 3. The primary aim is to highlight the crucial contributions of the Hall effect. This, we believe, is being done for the first time. We will, in addition, indicate briefly how the anisotropy issue can be addressed within the framework of Hall-MHD in section 5. Our intent is to show that the three spectral distributions derived in section 3 can model the three-branch spectrum (k^{-1}, $k^{-5/3}$, $k^{-\alpha_1}\alpha_1 \simeq 3-4$) of the magnetic fluctuations in the solar wind.

[13] If the turbulence is dominated by velocity field fluctuations ($\boldsymbol{V}_k^2 \gg \boldsymbol{B}_k^2$) (which happens, according to equation (3), for ($\alpha \ll 1$) or ($k \gg 1$) for $\alpha \simeq (k^{-1})$), the spectral expressions under the joint dominance of the Hall term and the velocity fluctuations ($k \gg 1$) simplify to

$$W_{E_1}(k) = (2\varepsilon_E)^{2/3}k^{-5/3}, \quad M_{E_1}(k) = (2\varepsilon_E)^{2/3}k^{-11/3}, \tag{16}$$

$$W_{H_1}(k) = (2\varepsilon_H)^{2/3}k^{1/3}, \quad M_{H_1}(k) = (2\varepsilon_H)^{2/3}k^{-5/3}, \tag{17}$$

$$W_{G_1}(k) = (2\varepsilon_G)^{2/3}k^{-7/3}, \quad M_{G_1}(k) = (2\varepsilon_G)^{2/3}k^{-13/3}. \tag{18}$$

In the case where $\alpha = 1$ for $k \ll 1$, one obtains the standard Alfvénic state with $V_k \propto B_k$, and the corresponding spectra are (suffix 1 is used for the Hall-dominant and 2 for the standard MHD limit)

$$M(k) = W(k), \tag{19}$$

$$W_{E_2}(k) = (2\varepsilon_E)^{2/3}k^{-5/3}, \tag{20}$$

$$W_{H_2}(k) = (2\varepsilon_H)^{2/3}k^{-1}, \tag{21}$$

$$W_{G_2}(k) = (2\varepsilon_G)^{2/3}k^{-1}. \tag{22}$$

[14] For the second root of $\alpha \simeq k$, $k \gg 1$ representing the whistler-type fluctuations, we find the following spectra:

$$W_{E_w}(k) = (2\varepsilon_E)^{2/3}k^{-3}, \quad M_{E_w}(k) = (2\varepsilon_E)^{2/3}k^{-1}, \tag{23}$$

$$W_{H_w}(k) = (2\varepsilon_H)^{2/3}k^{-7/3}, \quad M_{H_w}(k) = (2\varepsilon_H)^{2/3}k^{-1/3}, \tag{24}$$

$$W_{G_w}(k) = (2\varepsilon_G)^{2/3}k^{-7/3}, \quad M_{G_w}(k) = (2\varepsilon_G)^{2/3}k^{-1/3}. \tag{25}$$

[15] The observed solar wind magnetic spectrum will be generated if we were to string together the three branches $M_{E_1}(k)(\propto k^{-11/3})$, $M_{H_1}(k)(\propto k^{-5/3})$, and $M_{H_2}(k)(\propto k^{-1})$. The rationale as well as the modality for stringing different branches originates in the hypothesis of selective dissipation. It was first invoked in the studies of two-dimensional hydrodynamic turbulence [*Hasegawa*, 1985]. The idea is that in a given k range the particular invariant which suffers the strongest dissipation controls the spectral behavior (determined, in turn, by arguments a la Kolmogorov). Thus if the k ranges associated with different invariants are distinct and separate, we have a straightforward recipe for constructing the entire k spectrum in the extended inertial range. In two-dimensional hydrodynamic turbulence, for instance, the enstrophy invariant, because of its stronger k

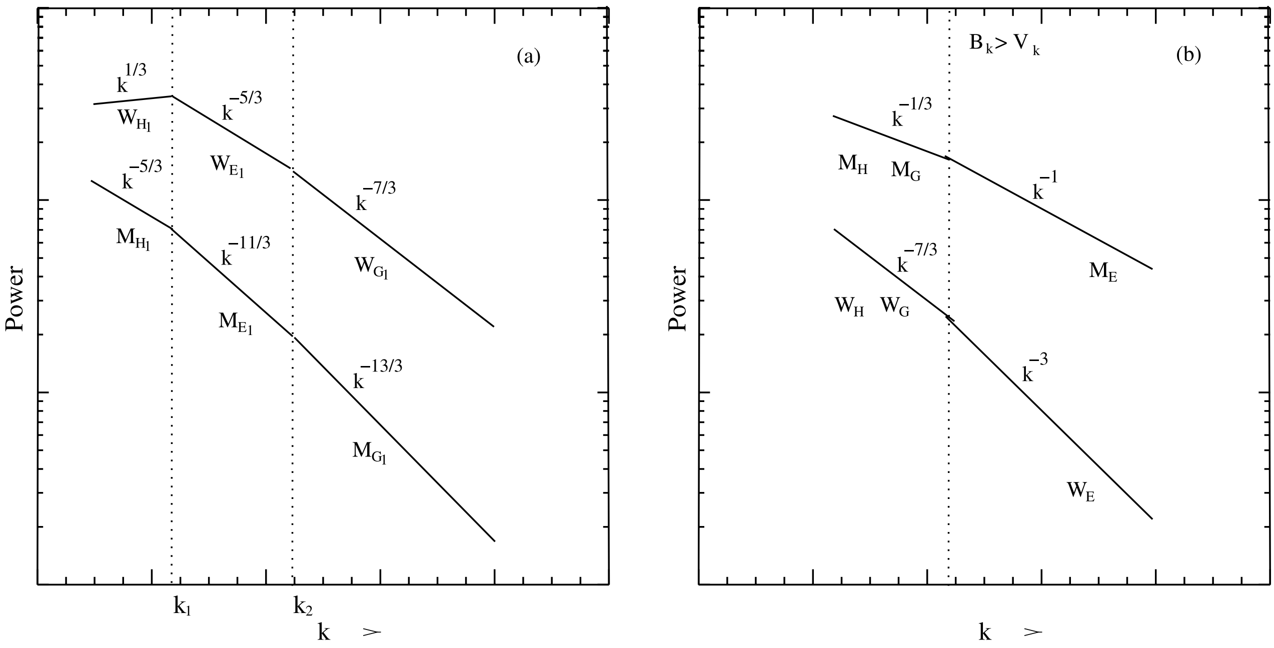

Figure 1. (a) Schematic magnetic (M) and kinetic (W) spectra (shear cyclotron mode) for $\alpha = k^{-1}$ in the Hall region ($k \gg 1$). (b) Schematic magnetic (M) and kinetic (W) spectra (whistler mode) for $\alpha = k$ in the Hall region ($k \gg 1$).

dependence (and hence larger dissipation) compared to the energy invariant, dictates the large k spectral behavior. Therefore the entire inertial range spectrum has two segments: the energy-dominated low k and the enstrophy-dominated high k ($\propto k^{-3}$). The procedure amounts to placing the spectrum with the highest negative exponent at the highest k end and the one with the lowest negative exponent of k at the lowest k end.

[16] The magnetic spectrum $M(k)$ and the kinetic spectrum $W(k)$, constructed by following the procedure delineated above, are shown in Figure 1a for the shear Hall fluctuations (equations (16)–(18)), in Figure 1b for the whistler fluctuations (equations (23)–(25)) for the Hall-dominated regime, and in Figure 2 for the Alfvénic state (equations (20)–(22)).

[17] Notice that the observed solar wind magnetic spectra consisting of the branches $k^{-\alpha_1}$ ($\alpha_1 \sim 3$–4), $k^{-5/3}$, and k^{-1} can be reproduced by stringing the Hall state spectral branches (Figure 1a) at large k with Alfvénic state branches (Figure 2) at small k; the result is displayed in Figure 3. This is rather fortunate because in Hall-MHD it is precisely for large k that the Hall term is dominant, while for small k the standard Alfvénic behavior prevails.

[18] There are three breaks in the spectrum displayed in Figure 3. The break at k_1 is due to the change in the nature of turbulence from Alfvénic (M_{H_2}) to the Hall-dominated state (M_{H_1}). The other breaks are due to changes in the controlling invariant (in the Hall-dominated regime): at k_2 the control is transferred from magnetic helicity H_M to the total energy E and at k_3 from the total energy E to the generalized helicity H_G. The entire spectrum for $k > k_1$ is a consequence of Hall dominance.

[19] We must reiterate that the steepened branches $\propto k^{-11/3}$ and $k^{-13/3}$ are, here, very much a part of the inertial range; they have no connection to the dissipative range invoked in previous studies. The break at k_2 may lie near the observed break near $f \simeq 1$ Hz.

[20] Within the framework of this dimensional Kolmogorov-inspired model, there is another consistent way of constructing the observed magnetic spectrum of the solar wind from the spectral relations we derived. Since the branch $k^{-5/3}$ is common to the Alfvénic and the Hall-

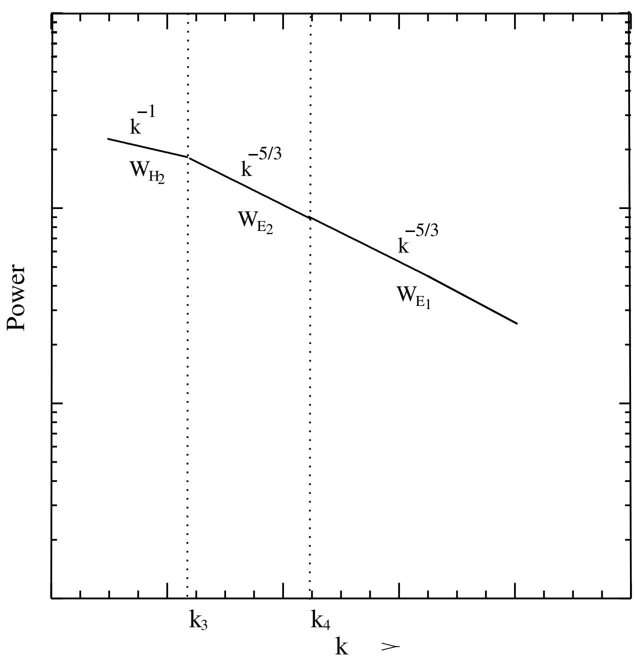

Figure 2. Schematic magnetic (M) and kinetic ($W \equiv M$) spectra (shear Alfvén mode) for $\alpha \simeq 1$ in the Alfvén region ($k \ll 1$).

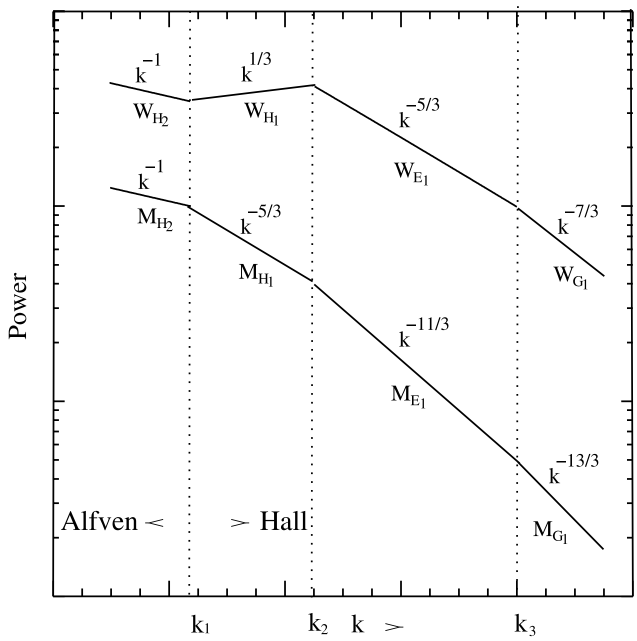

Figure 3. Modeled magnetic (M_1) spectra along with the corresponding kinetic (W_1) spectra.

dominated cases, one could just as well assume that the change from Alfvénic to the Hall-dominated state takes place at k_5 (Figure 4) instead of at k_1, as was assumed for the spectrum of Figure 3. Notice that owing to this replacement, the kinetic energy spectrum of Figure 4 is quite different from that of Figure 3 in the relevant k range. In the literature, k_5 has been identified with the strong damping region of the Alfvén mode via the proton cyclotron resonance [*Gary*, 1993; *Leamon et al.*, 1998].

[21] Thus we find that there are two pathways of reproducing the observed magnetic spectrum, depending upon the location of the spectral breaks. In principle, a somewhat detailed knowledge of the system would allow one to choose the more likely pathway. One would need to find in what range of k the standard Alfvénic description yields to Hall dominance and to what break in the spectrum that k corresponds. The absolute values of the breaks will, naturally, depend upon the numerical values of the parameters of the system.

[22] Within the framework of the Kolmogorov hypothesis, combined with the selective dissipation hypothesis, the positions of the spectral breaks (k_2, k_3, k_4, k_6) indicate the scales of energy injection. The energy injected at k_2, for example, will cascade toward large k as $k^{-11/3}$ and toward small k as $k^{-5/3}$. This is analogous to the two-dimensional turbulence, where the energy cascades to small k as $k^{-5/3}$ and to large k as k^{-3}, a consequence of the two invariants, the energy and the enstrophy. This applies to other breaks at k_3, k_4, and k_6. The breaks at (k_1, k_5), on the other hand, represent smooth transitions between the Hall-dominated and Alfvén states. The observed solar wind magnetic spectrum (M_1) ($k^{-11/3}$, $k^{-5/3}$, k^{-1}), in this context, has two scales of energy injection at (k_2, k_3), while k_1 signifies the change of guard from Alfvén ($k \ll 1$) to the Hall ($k \gg 1$) state; the latter is not a sharp break, but instead the transition is smooth because M_{H_1} (equation (17)) and M_{H_2} (equation (21)) are just limits of the smooth function $M_H(k)$ of equation (13). The break point k_2, determined from $M_{E_1}(k_2) = M_{H_1}(k_2)$, takes on the value $k_2 = (\varepsilon_E/\varepsilon_H)^{1/3}$, reflecting the dependence on the injection rates (also the dissipation rates) of the two invariants. Similar arguments apply to the spectrum M_2.

[23] One would also do well to note that the branch $k^{-5/3}$ exists both in the Alfvén as well as in the Hall state, but in the former it is associated with the invariance of the total energy E and in the latter, with that of the magnetic helicity H_M. The k^{-1} branch, however, exists only in the Alfvén state. As expected, the entire spectrum is dominated by the Hall effect at large k and the Alfvén effect at small k, and the energy injection scales lie at the high k end of the spectrum. This is symptomatic of the inverse or the dual cascade process. It is clear that the spectrum of whistler fluctuations (Figure 1b) cannot account for the observed solar wind magnetic fluctuations. The reason for the unobservability of this spectrum may lie in the stronger damping of the whistler waves. It is also a well-documented fact that out of the two possible types of turbulent fluctuations, namely Alfvénic and magnetosonic, it is the former that is more likely to be observed [*Goldstein et al.*, 1995]. The shear Alfvénic fluctuations suffer strong damping only when their frequency approaches the ion cyclotron frequency, and this happens in Hall-MHD only when k is strictly along the ambient magnetic field. Thus, in general, the Alfvénic fluctuations suffer less damping than the magnetosonic/whistler fluctuations. Although we have here presented an isotropic view of the turbulent fluctuations, the polarization of the Alfvénic fluctuations, i.e., with amplitudes (V, B) perpendicular to the propagation vector k and the nonlinear nature of the cascade time (kV_k), immediately reflects the anisotropy of the turbulence. However, we defer the discussion of this issue until a more quantitative model

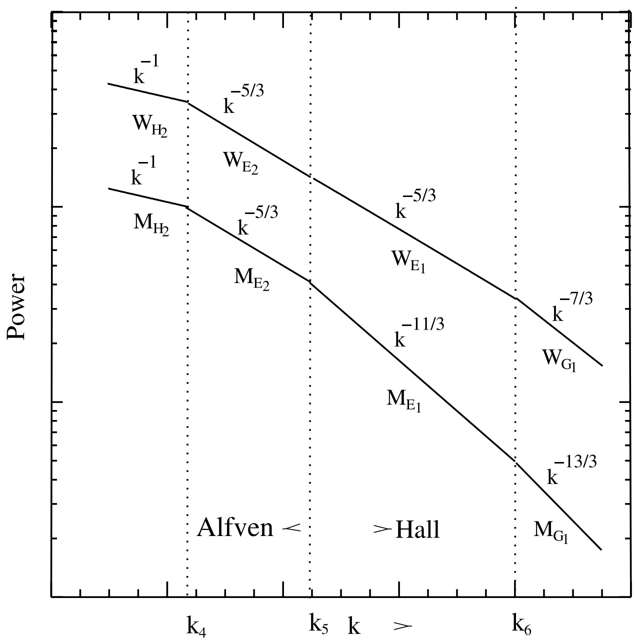

Figure 4. Modeled magnetic (M_2) spectra along with the corresponding kinetic (W_2) spectra.

based on the nonlinear interactions among the fluctuations is developed, and this is underway.

5. Discussion and Conclusions

[24] By including the physics of the Hall current and the fluid vorticity in two-fluid magnetohydrodynamics, the steepened part of the solar wind spectrum is shown to arise in the inertial range as contrasted with the dissipative range invoked in some earlier studies. The steepening in the present model is a consequence of the (***V***, ***B***) relation enshrined in equation (3). This exact nonlinear relationship forbids any coupling between the right-travelling waves with each other or the left-travelling waves with each other. However, the coupling between the left-travelling and the right-travelling waves remains, and this is expected to provide a theoretical model of turbulence, as in the standard Alfvénic turbulence [*Shebalin et al.*, 1983]. There is another way of obtaining the (***V***, ***B***) relation. This is done by invoking the variational principle and the selective decay hypothesis [*Yoshida and Mahajan*, 2002], leading to the double Beltrami conditions, which reduce to the (***V***, ***B***) relation given in equation (3) in the large k limit. We have also shown that this form of the relation is obeyed by the shear wave in the Hall regime. In a final summary, our Hall-MHD model predicts (1) an extended inertial range, with $k^{-11/3}$ along with $k^{-13/3}$ at the high k end, and (2) related but not identical spectra for the kinetic and the magnetic fluctuations. However, the issues of anisotropy and the detailed nature of cascades through mode-mode interactions at a realistic value of β need to be addressed before the model can be taken to represent the reality of the solar wind. It is intriguing that *Stawicki et al.* [2001] have attributed the steepening (limited to k^{-3}) to the higher dispersion of the Alfvén waves at large k_z, using the associated timescale and introducing the term "dispersion range." In contrast, the physics at large k in the framework of Hall-MHD (Hall currents become important, even dominant, at large k) contributes to steepening in a markedly different way: the new non-Alfvénic relationship between ***V*** and ***B*** predicting steepening is a consequence of the shear Hall mode. It is this high k behavior that dictates related but different spectra for the magnetic and kinetic fluctuations, as distinguished from *Stawicki et al.* [2001], where the two spectra are identical. Thus there are at present different ways and approaches of modeling the solar wind spectrum, and further investigations would provide the clues to the real nature of the solar wind turbulence.

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